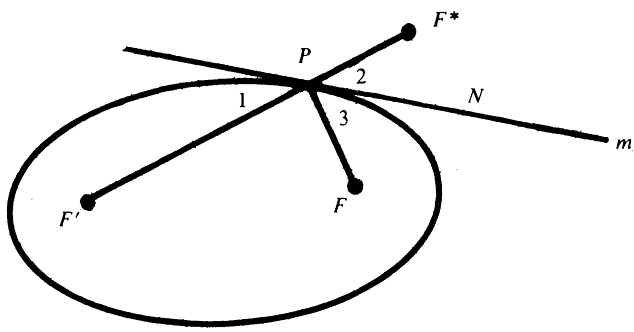
A Pretrigonometry Proof of the Reflection Property of the Ellipse

Zalman P. Usiskin, University of Chicago, Chicago, IL

In any ellipse, the segments from the foci to any point on the ellipse make equal angles with the tangent. This is equivalent to the reflecting property applied in whispering galleries. The reflecting property is often proved using equations of lines, eccentricity, trigonometry, and/or calculus. All of that machinery disguises the important ideas, namely that this property is fundamentally geometric, not algebraic. To use coordinates distorts one's understanding.



Suppose line m is tangent to the above ellipse at P, and let $F'P + FP = 2a$. Since m is a tangent line, any point N on m other than P lies outside the ellipse; so any other point N satisfies

$$F'N + FN > 2a.$$

Let F^* be the reflection image of F over line m. Since reflections preserve distance, $FP = F^*P$ and $FN = F^*N$. Since they preserve angle measure, angles 2 and 3 have the same measure. We now show that P must be on $F'F^*$. We do this by showing that the distance from F' to F^* is minimized by going through P. Putting all the above together, we see that for all $N \neq P$:

$$F'P + F^*P = F'P + FP = 2a < F'N + FN = F'N + F^*N.$$

Thus, angles 1 and 2 are vertical angles and have the same measure. So angles 1 and 3 have the same measure, which was to be proved.

Numerical Integration via Integration by Parts

Frank Burk, California State University, Chico, CA

In this note we illustrate how integration by parts can be used to obtain the familiar rectangular, trapezoidal, midpoint, and Simpson approximations of integrals. Our approach can serve to further enrich students' appreciation of the relationships among these numerical approximations.

Suppose $P(x)$ is a polynomial (to be determined), and $f(x)$ is continuously differentiable on $[a, b]$. From integration by parts,

$$\int P'f\,dx = Pf - \int Pf'\,dx. \tag{1}$$

If (1) is to provide an approximation to $\int f\,dx$, we must require $P'(x) = 1$. So assume $P(x) = x + B$.