# CLASSROOM CAPSULES

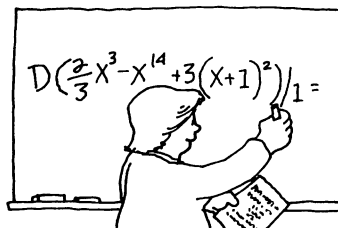A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Nazanin Azarnia, Department of Mathematics, Santa Fe Community College Gainesville, FL 32606-6200

## How Does the NFL Rate the Passing Ability of Quarterbacks?

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A list [1], [2], [3] of the NFL's "All-Time Leading Passers" is given in Table 1 below. In this list, which is restricted to players with a minimum of 1,500 pass attempts, players are ranked by a single numeric score. This rating system has been widely published in newspapers during recent NFL seasons to gauge the relative performance of quarterbacks for the current season. While it is known that this

**Table 1**
All-Time Leading Passers (through 1989–1990 season)

| Player | Att | Comp | Yards | TD | Int | Rating |
|--------|-----|------|-------|-----|-----|--------|
| Joe Montana | 4059 | 2593 | 31054 | 216 | 107 | 94.0 |
| Dan Marino | 3650 | 2174 | 27853 | 220 | 125 | 89.3 |
| Boomer Esiason | 2285 | 1296 | 18350 | 126 | 76 | 87.3 |
| Dave Krieg | 2842 | 1644 | 20858 | 169 | 116 | 83.7 |
| Robert Staubach | 2958 | 1685 | 22700 | 153 | 109 | 83.416 |
| Bernie Kosar | 1940 | 1134 | 13888 | 75 | 47 | 83.415 |
| Ken O'Brien | 2467 | 1471 | 17589 | 96 | 68 | 83.0 |
| Jim Kelly | 1742 | 1032 | 12901 | 81 | 63 | 82.74 |
| Neil Lomax | 3153 | 1817 | 22771 | 136 | 90 | 82.68 |
| Sonny Jurgensen | 4262 | 2433 | 32224 | 255 | 189 | 82.6 |
| Len Dawson | 3741 | 2136 | 28711 | 239 | 183 | 82.6 |
| Ken Anderson | 4475 | 2654 | 32838 | 197 | 160 | 81.9 |
| Danny White | 2950 | 1761 | 21959 | 155 | 132 | 81.7 |
| Bart Starr | 3149 | 1808 | 24718 | 152 | 138 | 80.5 |
| Fran Tarkenton | 6467 | 3686 | 47003 | 342 | 266 | 80.4 |
| Tony Eason | 1536 | 898 | 10987 | 61 | 50 | 80.3 |
| Dan Fouts | 5604 | 3297 | 43040 | 254 | 242 | 80.2 |
| Jim McMahon | 1831 | 1050 | 13335 | 77 | 66 | 79.2 |
| Bert Jones | 2551 | 1430 | 18190 | 124 | 101 | 78.2 |
| Johnny Unitas | 5186 | 2830 | 40239 | 290 | 253 | 78.2 |

rating is computed as a "percentage of completions, percentage of touchdown passes, percentage of interceptions, and average gain per pass attempt" [3], there seems to be no published account of exactly how it is computed.

As we will see, the NFL apparently computes its ratings of quarterbacks by taking a linear combination of these four statistics (see Table 2), including a nonzero constant term. That is, rating seems to be computed using

$$\text{Rating} = a + b(\%\ \text{Comp}) + c(\%\ \text{TD}) + d(\%\ \text{INT}) + e(\text{YDS}/\text{ATT}) \qquad (1)$$

**Table 2**
Statistics the Rating is Based Upon

| Player | % Comp | % TD's | % Int's | Yds/Att |
|---|---|---|---|---|
| Joe Montana | 63.8827 | 5.32151 | 2.63612 | 7.65065 |
| Dan Marino | 59.5616 | 6.02740 | 3.42466 | 7.63096 |
| Boomer Esiason | 56.7177 | 5.51422 | 3.32604 | 8.03063 |
| Dave Krieg | 57.8466 | 5.94652 | 4.08163 | 7.33920 |
| Roger Staubach | 56.9642 | 5.17241 | 3.68492 | 7.67410 |
| Bernie Kosar | 58.4536 | 3.86598 | 2.42268 | 7.15876 |
| Ken O'Brien | 59.6271 | 3.89137 | 2.75638 | 7.12971 |
| Jim Kelly | 59.2422 | 4.64983 | 3.61653 | 7.40586 |
| Neil Lomax | 57.6277 | 4.31335 | 2.85442 | 7.22201 |
| Sonny Jurgensen | 57.0859 | 5.98311 | 4.43454 | 7.56077 |
| Len Dawson | 57.0970 | 6.38867 | 4.89174 | 7.67469 |
| Ken Anderson | 59.3073 | 4.40223 | 3.57542 | 7.33810 |
| Danny White | 59.6949 | 5.25424 | 4.47458 | 7.44373 |
| Bart Starr | 57.4151 | 4.82693 | 4.38234 | 7.84948 |
| Fran Tarkenton | 56.9971 | 5.28839 | 4.11319 | 7.26813 |
| Tony Eason | 58.4635 | 3.97135 | 3.25521 | 7.15299 |
| Dan Fouts | 58.8330 | 4.53248 | 4.31834 | 7.68023 |
| Jim McMahon | 57.3457 | 4.20535 | 3.60459 | 7.28291 |
| Bert Jones | 56.0564 | 4.86084 | 3.95923 | 7.13054 |
| Johnny Unitas | 54.5700 | 5.59198 | 4.87852 | 7.75916 |

for particular choices of constants $a, b, c, d, e$. In our case, as rating is not given to us precisely, but generally only to the nearest 0.1, we cannot simply take 5 of our 20 quarterbacks and solve a system of 5 equations in the 5 unknowns $a, b, c, d, e$. It is well known, however, how to chose $a, b, c, d, e$ to minimize the sum of squared deviations between the given (rounded) ratings and the right-hand side of (1). Let $r$ denote the 20 by 1 vector of rounded ratings, and let $A$ be the 20 by 5 matrix whose first column contains ones and whose second through fifth columns are those given in Table 2. From [4], for example, the solution $x = (a, b, c, d, e)$ minimizing the sum of squared deviations $(r - Ax)^t(r - Ax)$ satisfies

$$(A^tA)x = A^tr.$$

In our case, the above expression is nearly

$$
\begin{bmatrix}
20.00 & 1162.79 & 100.01 & 74.69 & 149.38 \\
1162.79 & 67672.34 & 5809.64 & 4328.66 & 8684.06 \\
100.01 & 5809.64 & 511.25 & 379.69 & 749.24 \\
74.69 & 4328.66 & 379.69 & 288.91 & 559.34 \\
149.38 & 8684.06 & 749.24 & 559.34 & 1117.10
\end{bmatrix}
x =
\begin{bmatrix}
1655.4 \\
96336.2 \\
8294.2 \\
6155.6 \\
12370.3
\end{bmatrix}
$$

which, solving for $x$, approximately yields

$$x = (1.91, 0.84, 3.33, -4.16, 4.14).$$

Using the solution for $x$ as a starting point we find that the expression

$$\text{Rating} = \frac{50 + 20(\% \text{ Comp}) + 80(\% \text{ TD}) - 100(\% \text{ INT}) + 100(\text{YDS/ATT})}{24}$$

gives the ratings of Table 1 up to rounding. Note, for example, that touchdown percentage is deemed to be four times as important as completion percentage with this rating scheme.

Rather than find the solution vector $x$ which minimizes the sum of squared deviations (the "$L_2$" solution) one could find $x$ to minimize the sum of absolute deviations (the "$L_1$" solution) or to minimize the maximum absolute deviation (the "$L_\infty$" solution) by solving the appropriate linear programming problem (in our case these three solutions are nearly identical). For those familiar with linear programming the details are given, for example, in [5].

## References

1. *The 1991 Information Please Almanac*, Houghton Mifflin, Boston, 1990 p. 889.
2. John W. Wright, (Ed.), *The Universal Almanac 1991*, Andrews & McMeel, Kansas City, 1990, p. 602.
3. *The World Almanac and Book of Facts 1991*, Pharos Books, New York, 1990 p. 910.
4. Ben Noble and James Daniel, *Applied Linear Algebra*, 2nd Edition, Prentice Hall, New Jersey, 1977, p. 60.
5. Vašek Chvátal, *Linear Programming*, W. H. Freeman, New York, 1980, pp. 222–226.

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## Stacking Ellipses—Revisited

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I recently read the September 1991 issue of the *CMJ* and the article *Stacking Ellipses*. The point of the article wasn't expressly stated, but I surmised that it was to determine the area of an ellipse without resorting to calculus, given Cavalieri's principle and the area of a circle. If this is the case, a much simpler proof in the same vein exists, which also does not need to distinguish between rational and irrational values for the semiaxes. Merely inscribe the ellipse $x^2/a^2 + y^2/b^2 = 1$ in its circumscribing circle $x^2/a^2 + y^2/a^2 = 1$ and compare $y$ values. Since they are in the ratio $b/a$, the areas must be likewise. Hence the area of the ellipse is $\pi ab$.

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## Quadratic Confidence Intervals

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In an introductory statistics course, students learn from the central limit theorem that for binomial data with sample proportion $\hat{p}$ and large enough sample size $n$, $(\hat{p} - p)/\sqrt{p(1-p)/n}$ is approximately distributed as a standard normal variable