

We apply implicit differentiation or the inverse function rule to the defining equation for  $\theta$ , which gives

$$-\sin \theta \frac{d\theta}{dt} = 1.$$

Then, using  $\sin \theta = \sqrt{1 - t^2}$  (from the figure or from  $\sin \theta = \sqrt{1 - \cos^2 \theta}$ ), we arrive at

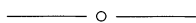
$$-\frac{d\theta}{dt} = \frac{1}{\sin \theta(t)} = \frac{1}{\sqrt{1 - t^2}}.$$

Simplifying and combining, we find

$$A'(t) = \frac{1}{2} \left( \frac{1}{\sqrt{1 - t^2}} + \frac{1 - t^2}{\sqrt{1 - t^2}} - \frac{t^2}{\sqrt{1 - t^2}} \right) = \frac{1 - t^2}{\sqrt{1 - t^2}} = \sqrt{1 - t^2} = f(t),$$

just as promised by the Fundamental Theorem of Calculus.

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## More Coconuts

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The linear Diophantine problem “Dividing Coconuts,” solved by Singh and Bhat-tacharya [2], is thought-provoking and prompted me to find several other solutions. Here is my favorite one. Recall that the problem, first posed by Paul Halmos [1], is stated as follows:

After gathering a pile of coconuts one day, five sailors on a desert island agree to divide them evenly after a night’s rest. During the night one sailor gets up, divides the nuts into five equal piles with a remainder of one, which he tosses to a conveniently nearby monkey, and, secreting his pile, mixes up the others and retires. The second sailor does the same thing, and so do the third, the fourth, and the fifth. In the morning the remaining pile of coconuts (less one) is again divisible by 5. What is the smallest number of coconuts that the original pile could have contained?

Why should the monkey be getting coconuts that the sailors worked to gather? Since the original number  $N$  has a remainder of 1 upon division by 5, let us suggest that the sailors add four big round stones to the pile and pretend they are coconuts. Now the number is  $N + 4$  and it is divisible by 5.

When the first sailor gets up in the night and removes one-fifth of the pile, taking no stones, he gets to keep the coconut that would otherwise have been thrown to the monkey. The remaining  $\frac{4}{5}(N + 4)$  coconuts and stones in the pile is still divisible by 5 because it still contains the four stones. In the same way, the other sailors get up in the night. Each removes a fifth of the existing pile, taking no stones and keeping the coconut that would have gone to the monkey.

By morning, the pile contains  $\left(\frac{4}{5}\right)^5 (N + 4)$  coconuts and stones. Since this number still includes the four stones, it is still divisible by 5, so it must have the form  $5m$

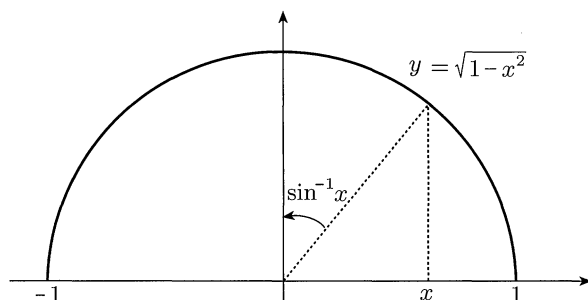
for some positive integer  $m$ . It follows that  $N + 4 = \left(\frac{5}{4}\right)^5 5m$ . The smallest integer  $N$  of this form would have  $m = 4^5$  and then  $N = 5^6 - 4 = 15621$ . In the morning, the depleted pile of  $\left(\frac{4}{5}\right)^5 (N + 4) = 4^5 \cdot 5 = 5120$  coconuts and stones still includes the four stones so, presumably, the sailors will take 1023 coconuts each and throw one coconut and four stones to the monkey. The solution is easily generalized to the case of  $k$  sailors and  $k - 1$  stones.

## References

1. Paul Halmos, *Problems for Mathematicians Young and Old*, MAA, Washington, DC, 1991, p. 22.
2. Sahib Singh and Dip Bhattacharya, On dividing coconuts, *College Mathematics Journal* 28:3 (May 1997) 203–204.

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### The Derivative of the Inverse Sine



By the integral formula for arc length,  $\sin^{-1} x = \int_0^x \frac{1}{\sqrt{1-t^2}} dt$ . Thus,

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}.$$

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