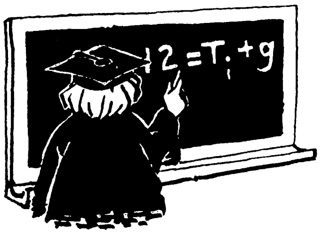

CLASSROOM CAPSULES

Edited by
Warren Page

Classroom Capsules serves to convey new insights on familiar topics and to enhance pedagogy through shared teaching experiences. Its format consists primarily of readily understood mathematics capsules which make their impact quickly and effectively. Such tidbits should be nurtured, cultivated, and presented for the benefit of your colleagues elsewhere. Queries, when available, will round out the column and serve to open further dialog on specific items of reader concern.

Readers are invited to submit material for consideration to:

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Income Averaging Can Increase Your Tax Liability

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The results of the article by Henry & Trapp regarding Income Tax Averaging and Convexity [CMJ 15 (June 1984) 253–255] are valid when and only when the taxable income (I) is \$50,000 or more. In this case, the tax must be computed by using the Tax Rate Schedules (See bottom of page 31 of the 1982 Tax Tables) which is the same as the manner in which the tax is computed using Income Averaging, that is, Income Averaging requires the use of the Tax Rate Schedules also (See lines 26, 27, and 28 of Schedule G). Thus, the function represented by T on the right side of

$$T(I) \geqslant 5T(0.2I + 0.24B) - 4T(0.3B) \qquad \text{for all } I, B$$

in the article is the same as the function represented by T on the left side, namely, the continuous piecewise linear and increasing function induced by the Tax Rate Schedules:

$$T(x) = a_i + r_i(x - b_i) \qquad \text{for} \quad b_i \leqslant x < b_{i+1},$$

where $r_{i+1} > r_i$ and $a_i + r_i(b_{i+1} - b_i) = a_{i+1}$. However, if the taxable income is less than \$50,000 and at the same time eligibility for Income Averaging is met, then the taxpayer has two options available for computing the tax liability.

(i) Because the taxable income is less than \$50,000, the taxpayer is eligible to use the Tax Tables.

(ii) Because of eligibility for Income Averaging, the taxpayer can use the Tax Rate Schedules required by Income Averaging.

Unfortunately, this is not stated clearly in a number of tax manuals.

In this case, the function represented by T on the right side of (5) is still the function induced by the Tax Rate Schedules, but the function represented by T on the left side is the discontinuous Tax Table step function which jumps every \$50 increase in taxable income. In this case, Income Averaging may yield a lower tax, the same tax, or a higher tax than that obtained from the Tax Tables.

To show that Income Averaging can yield a higher tax, suppose I (taxable income for 1982) falls in the range

$$49{,}685 \leqslant I \leqslant 49{,}699 \tag{1}$$

and B (the sum of the taxable incomes for the four preceding years) falls in the range

$$152{,}658 \leqslant B \leqslant 152{,}666. \tag{2}$$

Assume further that the filing status is MFJ (Married Filing Jointly). Recall that eligibility for Income Averaging requires that $I-.3B$ must exceed 3,000. From (2) we get

$$45{,}797.40 \leqslant .3B \leqslant 45{,}799.80. \tag{3}$$

Subtracting the largest value for $.3B$ from the smallest value for I, we get the minimum value for $I-.3B$ which is 3,885.20. Thus, every choice for I and B above meets the eligibility requirement for Income Averaging. Next, observe that

$$46{,}574.92 \leqslant .2I+.24B \leqslant 46{,}579.64. \tag{4}$$

Using the Tax Rate Schedules for MFJ (page 32 of the Tax Tables), we find

$$T(46{,}574.92) = 11{,}457 + .44(46{,}574.92 - 45{,}800) = 11{,}797.96$$

$$T(46{,}579.64) = 11{,}457 + .44(46{,}579.64 - 45{,}800) = 11{,}800.04.$$

Since T is increasing, we get

$$59{,}989.80 \leqslant 5T(.2I+.24B) \leqslant 59{,}000.20. \tag{5}$$

Now using (3) and the Tax Rate Schedules for MFJ again, we find

$$T(45{,}797.40) = 7{,}323 + .39(45{,}797.40 - 35{,}200) = 11{,}455.99$$

$$T(45{,}799.80) = 7{,}323 + .39(45{,}799.80 - 35{,}200) = 11{,}456.92.$$

Again using the fact that T is increasing, we get

$$(9) \quad 11{,}455.99 \leqslant T(.3B) \leqslant 11{,}456.92. \tag{6}$$

Thus, the tax computed by Income Averaging satisfies

$$13{,}162.12 \leqslant 5T(.2I+.24B) - 4T(.3B) \leqslant 13{,}176.24. \tag{7}$$

Since $I \subset [49{,}650;\ 49{,}700)$ [using the 1982 Tax Tables (page 31)] the tax for MFJ is 13,162. Thus, the tax computed by Income Averaging exceeds the Tax Table tax anywhere from 12¢ to \$14.24, depending on the choice for I and B.

It has been my experience, as a tax practitioner, that of those taxpayers eligible for Income Averaging and whose taxable incomes are less than $50,000, approximately 20% of the time the tax computed by Income Averaging exceeds the tax from the Tax Tables. So this is not an infrequent occurrence, and the warning by the Government in Publication 17 should be heeded.

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Medical Cozenage on Fermat's Last Theorem

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Recently a new form of the disease craniosis has surfaced which may have a profound impact on a famous problem in mathematics due to Fermat. Craniosis is a disease whereby the stricken individual is unable to distinguish between left and right—this being due to a neurochemical malfunction in the medulla oblongata, the organ connecting the left and right hemispheres of the brain. The new form of the disease may be identified by the brain's inability to distinguish between up and down. Although the causes are not well understood, medical researchers are making inroads and several outstanding articles in medical journals have already appeared.

There is now strong evidence to support the belief that Fermat suffered from up/down craniosis. A thorough study of his writings, particularly his earlier writings, indicates a rather frequent transposition of mathematical expressions. For instance, there are at least twelve cases where 2^x was written when x^2 was really intended. Several transposition errors in fractions also occurred, though many of these types of errors were probably caught and corrected by Fermat's printer and publisher, Henri von Blatt, a knowledgeable mathematician in his own right. In total, there are no less than 45 up/down transpositions in Fermat's earlier writings.

With the medical evidence available, it seems highly probable that Fermat's "marginal proof' of his Last Theorem was based on a transposition of the famous equation

$$x^n + y^n = z^n.$$

It is likely that Fermat actually had in mind a proof of the theorem: *There are positive integer solutions to*

$$n^x + n^y = n^z \tag{1}$$

only when $n = 2$. The proof is as follows. Since $z \geqq \max(x, y)$, we see from (1) that $n^x \mid n^y$ and $n^y \mid n^x$. Therefore, $x = y$ and it follows that $z = x + 1$ and $n = 2$. An obvious generalization of (1) is to ask for which positive integers n there exist positive integers x, y, u, z satisfying

$$n^x + n^y + n^u = n^z. \tag{2}$$

The only possible solutions of (2) occur for $n = 2, 3$. To see this, first assume $u \geqq \max(x, y)$. Then $n^x \mid n^u$. And since $n^x \mid n^z$, we also have $n^x \mid n^y$. By symmetry, $n^y \mid n^x$. So $x = y$ and equation (2) reduces to

$$2n^x + n^u = n^z. \tag{3}$$

Thus, $n^u \mid 2n^x$ and $2n^{x-u}$ is integral for $x \leqq u$. If $n > 2$, then $x = u$ and equation (3) becomes

$$3n^x = n^z,$$

from which it follows that $n = 3$ and the solution of (2) is $x = y = u = z - 1$. For