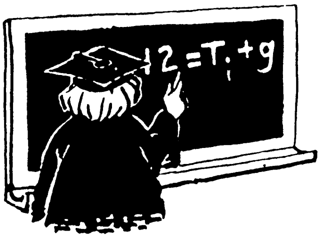

CLASSROOM CAPSULES

Edited by Frank Flanigan

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A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics.

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To View an Ellipse in Perspective

Charles G. Moore, Northern Arizona University, Flagstaff, AZ

"If an ellipse is viewed at a distance from any angle, is the image presented to the eye of the viewer actually an ellipse?" The casual answer usually is in the negative, based upon the assumption that the image is some egg-shaped distortion of an ellipse. We shall show that this question may be answered in the affirmative, by using Pascal's theorem on inscribed hexagons. [See, for example, H. S. M. Coxeter's *Introduction to Geometry*, John Wiley, New York, 1961, pp. 252–255, and Howard Eves' *A Survey of Geometry*, Revised Ed., Allyn and Bacon, Boston, 1972, pp. 67–70.]

Pascal's Theorem. *If a hexagon is inscribed in a conic, then the points of intersection of the pairs of opposite sides are collinear, and conversely.*

Converse of Pascal's Theorem. *If, for any hexagon inscribed in a curve the intersections of opposite pairs of sides are collinear, then that curve is a conic.*

The line of intersections of the three pairs of opposite sides is called a *Pascal line*. Figure 1 shows a hexagon ABCDEFA inscribed in a circle. The Pascal line is shown as a bold line. The inscribed hexagon need not be convex. In Figure 2, a nonconvex hexagon is inscribed in a parabola and the Pascal line is again shown as a bold line.

In Figure 3, an ellipse in a horizontal plane S is viewed by an eye at point ξ. Rays are projected from all points of the ellipse to ξ. A plane P orthogonal to the line of sight intersects the family of rays from ξ to the ellipse. The plane P is called a *picture plane*, and the projection of the ellipse onto P is that which the eye sees when viewing an ellipse. We wish to determine if the projection of the ellipse onto P is also an ellipse.

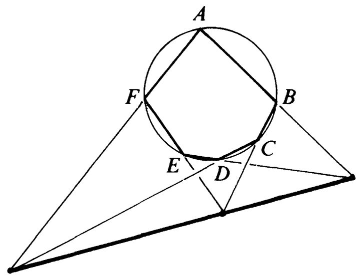

Figure 1.

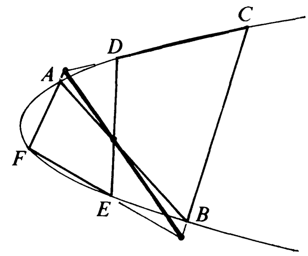

Figure 2.

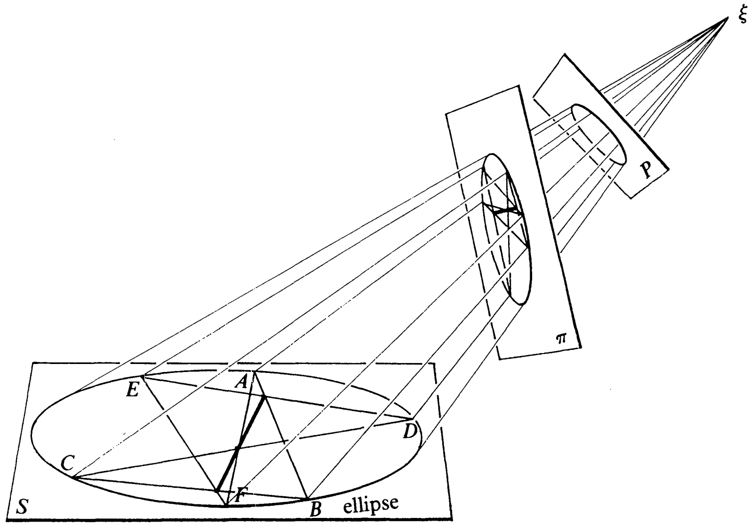

Figure 3.

A plane π situated obliquely with respect to the line of sight also intersects the family of rays from the ellipse in S to ξ in such a manner that these rays intersect π in a closed curve. A hexagon ABCDEFA is inscribed in the ellipse, and the intersections of the three pairs of opposite sides are identified.

The Pascal line is shown as a bold line in the plane S. We now consider the projection of the entire hexagon ABCDEFA in plane S onto the oblique plane π. Since linearity is preserved under a central projection, the hexagon ABCDEFA inscribed in the ellipse in S projects into a hexagon inscribed in the closed projections of the ellipse onto π. In particular, the Pascal line in S projects into a straight line in plane π. The converse of Pascal's theorem asserts that the projection of the ellipse in S onto π is an ellipse. (Note that an ellipse may be projected into any conic with the proper location of ξ. However, the only closed conic is an ellipse. If, as indicated, the plane π intersects the family of rays from ξ to the given ellipse in a closed curve then that intersection is an ellipse.) Thus, the projection of the ellipse in S onto a plane π is an ellipse. Since π can be taken as P, the projection of the ellipse in S onto the picture plane P is an ellipse. Thus, the question addressed by this paper is answered in the affirmative.

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What's Significant about a Digit?

David A. Smith, Duke University, Durham, NC

The question of the title might well have been (but wasn't) posed by a student wondering about instructions to report numerical answers to, say, "four significant digits." (At Duke, we routinely give such instructions to our calculus students.) It will come as no surprise that our freshmen enter with little or no idea as to what makes a digit significant.

In spite of the fact that practically every standard calculus book assumes that students will use calculators at least some of the time, you will look in vain for one that defines significant digit. Small wonder then that students make quite arbitrary decisions, reporting everything displayed on a calculator as a final answer, but discarding digits to suit their convenience if an intermediate result must be copied down and rekeyed.

What then are significant digits? A reliable source [*The American Heritage Dictionary of the English Language*, *New College Edition* (W. Morris, ed.), Houghton Mifflin, 1978] defines them as:

> The digits of the decimal form of a number beginning with the leftmost nonzero digit and extending to the right to include all digits warranted by the accuracy of measuring devices used to obtain the number.

That definition, while clearly aimed at laboratory science applications, may be the only one our students have seen in secondary school (if, indeed, they have seen any definition at all). It's actually a workable definition for a mathematics course, if we expand "measuring" to "measuring and calculating." It is still an intuitive definition, however, because of the imprecision of "warranted."

The natural place to look for a mathematical definition of significant digit would be in a numerical analysis book—indeed, it is the "creeping down" of topics from numerical analysis into the calculus sequence that makes this topic important at this time. But not even the authors of these books agree on the "correct" definition. On page 5 of Anthony Ralston's classic text [*A First Course in Numerical Analysis*, McGraw-Hill, 1965], we find:

> If y is any approximation to a true value x, then the k-th decimal place of y is said to be *significant* if
>
> $$|x-y| < 0.5 \times 10^{-k}.$$
>
> Therefore, every digit of a correctly rounded number is significant.

This definition works equally well for digits to the left or right of the decimal point if we don't require k to be positive; that is, if we allow the units place to correspond to $k=0$, the tens place to $k=-1$, and so on. However, there is no mention of "leading zeros," and the author follows the definition with a paragraph to explain how this leads to difficulty in determining when numbers are "equally significant." The paragraph ends, "We shall therefore avoid the use of the notion of the number of significant digits in a number."