

available. In addition, steady bombardment of nuclei by free neutrons builds up atomic weights, and neutron decay (into a proton, an electron, and an antineutrino) contributes to the build up of atomic number. It would be very surprising, indeed, if these processes produced exactly the same amounts of U-235 and U-238.

Let's turn the problem around. Geologic evidence seems to suggest an age of about 5 billion years for the earth. If we accept that as accurate, and assume the time between the supernova and the formation of the earth to be negligible, then we can use the previous approach to calculate the original ratio,  $R$ , of U-235 to U-238, namely,

$$R \frac{\left(\frac{1}{2}\right)^{5/0.71}}{\left(\frac{1}{2}\right)^{5/4.5}} \doteq 0.00715.$$

So,  $R = 0.44$ . Instead of a 50-50 production, we deduce that 44 atoms of U-235 would have to have been produced for every 100 atoms of U-238.

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### Forward Homework — A Motivational Tool

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Since solutions manuals now accompany most calculus textbooks, it's not surprising that a recent survey found that 55% of all calculus students on semester schedules rarely or never have their homework collected and graded [Richard D. Anderson and Donald O. Loftsgaarden, A Special Calculus Survey, Preliminary Report, in *Calculus for a New Century*, MAA Notes 8, 1988]. For the past year I have used an incentive-based system that (a) allows students to have their mathematical progress evaluated throughout the semester; (b) gently forces students to be better prepared for class; and (c) gives students a chance to develop writing and library skills. For lack of a better title, I call the system "Forward Homework" (FH). I now end most classes with a written assignment to be handed in at the beginning of the next class. In most cases the problem is carefully chosen to highlight a central topic to be covered during that class. During the lecture, I use the problem handed in that day as a key example or as part of the discussion of a major topic. For example, early in a calculus course, before proving the power rule for derivatives, I ask students to expand  $(x + h)^2$ ,  $(x + h)^3$ , and  $(x + h)^4$ ; note any patterns in these expansions; and then write the first two terms of  $(x + h)^n$ . The problems are corrected and returned at the next class. (A correct answer or good attempt with a clear explanation earns 2 points; a correct answer with no written explanation or a half-hearted attempt earns 1 point; other papers are marked 0.) Late assignments are not accepted. To allow for legitimate absences, I generally drop the two lowest grades of the fifteen to twenty problems that are assigned during a fourteen week semester.

### Some Examples of FH:

Most FH exercises introduce the key topic of the next lecture. For example, just before I cover extreme points of a function, I assign a problem similar to the following:

1. Estimate the largest and smallest values of  $f(x) = -0.3x^2 + 1.4x + 5$  on the interval  $[0,3]$  by calculating  $f(x)$  for any ten  $x$ -values of your choice.

Having done this work, students are genuinely impressed during the next class to see how calculus can give an exact answer with far less effort.

I assign the following problem before a discussion of concavity:

- 2a. Draw the tangent line to  $y = x^2$  for  $x = 0, 1, 2$ , and  $3$ . Is the slope of the tangent line getting larger (increasing) as  $x$  moves from  $0$  to  $3$ , or is it getting smaller (decreasing)? Explain.
- b. Repeat, using  $y = -x^2 - 6x + 18$ .

At the start of the next class, I draw both graphs on the board and restate the problem. The answers from the class are generally correct, which gets the discussion of concavity and the second derivative off to a good start.

A final example of this first type of problem introduces students to the definite integral.

- 3a. Estimate the area under the curve  $f(x) = x^2$  from  $x = 0$  to  $x = 1$  using two rectangles, which may be inscribed, circumscribed, or neither.
- b. Repeat, using four rectangles.
- c. A final time: use eight rectangles.

Again, the results are generally excellent, and I find that I can move quickly, first to the general case of  $n$  rectangles, and then to the definite integral itself.

Another type of problem calls for students to read new material and either paraphrase it or use it to solve some simple problems. For example,

4. Read pages 138 and 139, especially noting line (8), which is called the chain rule. Paraphrase this material in a style so that your roommate, who is also in this class, could understand it. (This assignment will be graded for use of correct punctuation, syntax, and spelling, as well as content.)
5. Read how partial derivatives are computed (page 373) and use this material to compute the partial derivatives of functions found in problems 3 and 7 (page 378).

Sometimes FH is used strictly as review. Students often have not seen trigonometry in over a year, so before I begin a class on the limits and derivatives of trig functions, I assign the following problem:

6. Draw large accurate graphs of each of the six trig functions over the interval  $[0, 2\pi]$ .

A fourth type of problem asks students to research information not found in our text. Before covering Newton's method, for example, I point out the existence of formulas for solutions of third and fourth degree polynomial equations and assign the following problem:

- 7a. Write a general formula for solving cubic equations given in the *CRC Handbook* found in the Math Library.
- b. Use this formula to find all solutions of the equation  $x^3 + x = 0$ .

This exercise makes students vividly aware of a formula that solves all cubic equations (that they probably would never want to use again), and of the usefulness of the *CRC Handbook*.

FH can improve the mood of a mathematics class and students' ability to learn in a number of ways. First, *students* are better prepared for class. While they may not reach the ideal of studying an entire section or two beforehand, they do read and wrestle with some important concepts before they are presented.

Second, *I* am better prepared for class. When preparing a lesson for class  $N$ , I have to prepare a FH problem for class  $(N + 1)$  as well. This forces me to think about the central point of the next lesson and how to relate it to current material. Consequently, my presentations are more organized and continuous.

Finally, FH conveys an important subliminal message to a class; namely, that their teacher cares about their success. Students recognize that the teacher has thought not only about today's lesson, but the next lesson as well.

In fairness, FH does have its price.

Specifically, it takes time to prepare good problems and correct them. For a class of twenty-five students, I spend about an hour correcting and recording grades for one problem and preparing another to assign.

Good FH problems are not easily constructed. Care must be taken to sharpen each question so that it illustrates the point I want to make. In some cases, I have had to rephrase the same problem several times in successive semesters to make my meaning clear. For example, I first assigned the following problem to introduce the chain rule:

- 8a. Compute derivatives by expanding each expression and using the power rule.  
 b. Do you see any pattern that would lead to a general formula for the derivative of  $y = (f(x))^2$ ?

a)  $y = (2x + 1)^2$

b)  $y = (3x + 2)^2$

c)  $y = (4x - 6)^2$

d)  $y = (x^2 + 3)^2$

e)  $y = (x^3 + 1)^2$

The derivatives were correctly calculated, but no students saw a pattern because they left their answers in unfactored form ( $8x + 4$ ,  $18x + 12$ , and so forth.) The next time around I asked that each derivative be factored before looking for a pattern. This instruction still wasn't quite clear enough because students wrote  $4(2x + 1)$ ,  $6(3x + 2)$ , etc., which masked the form of the chain rule that I was looking for. Finally, I revised the problem as follows:

- 8a. Compute derivatives by expanding each expression and using the power rule.  
 b. Each function is of the form

$$y = (f(x))^2.$$

Write each of the derivatives in the form:  $y' = 2f(x)$ \_\_\_\_\_.

- c. Do you see any pattern that would lead to a general formula for the derivative of  $y = (f(x))^2$ ?

After I did an example in class, students were able to complete the assignment as I had originally intended. It was then an easy matter to move to the derivative of  $y = (f(x))^n$ , which, in turn, led to the most general form of the chain rule.

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