We do not advocate such a complete presentation for every example of L'Hôpital's Rule, but we do suggest that students be made aware of the importance of the existence hypothesis. Although the existence of $\lim_{x\to a}\dfrac{f'(x)}{g'(x)}$ is a sufficient condition for the existence of $\lim_{x\to a}\dfrac{f(x)}{g(x)}$, very little is usually said about whether or not this is a necessary condition for the existence of $\lim_{x\to a}\dfrac{f(x)}{g(x)}$. Thus, it may be enticing for students to believe that when a chain of "L'Hôpital equalities" leads to a limit that does *not* exist, then the original limit also does not exist. The following examples show that such a conclusion may or may not hold.

Example 1 ($\frac{0}{0}$ indeterminate forms). If $f(x) = x^2\sin(x^{-1})$ and $g(x) = \sin x$, then $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$ does not exist, whereas $\lim_{x\to 0}\dfrac{f(x)}{g(x)} = 0$. On the other hand, if $f(x) = x\sin(x^{-1})$ and $g(x) = \sin x$, then neither $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$ nor $\lim_{x\to 0}\dfrac{f(x)}{g(x)}$ exists.

Example 2 ($\frac{\infty}{\infty}$ indeterminate forms). If $f(x) = x(2 + \sin x)$ and $g(x) = x^2 + 1$, then $\lim_{x\to\infty}\dfrac{f'(x)}{g'(x)}$ does not exist, whereas $\lim_{x\to\infty}\dfrac{f(x)}{g(x)} = 0$. On the other hand, if $f(x) = x(2 + \sin x)$ and $g(x) = x + 1$, then neither $\lim_{x\to\infty}\dfrac{f'(x)}{g'(x)}$ nor $\lim_{x\to\infty}\dfrac{f(x)}{g(x)}$ exists.

Editor's Note: For a variation on this theme, see J. P. King's Classroom Capsule "L'Hôpital's Rule and the Continuity of the Derivative," TYCMJ 10 (June 1979), 197–198.

An Analytic Approach to the Euler Line

Jonathan W. Lewin, Kennesaw College, Marietta, GA

In the November 1982 Classroom Capsules Column, Norman Schaumberger presented a demonstration of the Euler line using vector methods. It may be interesting to supplement Schaumberger's argument, which has a distinct geometric flavor, with the following analytic approach.

For any triangle ABC, let $G = \frac{1}{3}(A + B + C)$. Then G must lie on all three of the triangle's medians—that is, G is the centroid of triangle ABC.

Let D be the midpoint of BC. Then since $\overrightarrow{BD} = \overrightarrow{DC}$, we have $D - B = C - D$ and so $D = \frac{1}{2}(B + C)$. Now we let P be the point on AD such that $AP = 2PD$. Since $\overrightarrow{AP} = 2\overrightarrow{PD}$, we have $P - A = 2(D - P)$ and we deduce at once that $P = \frac{1}{3}(A + B + C) = G$.

Let a triangle ABC be placed with the origin 0 *at its circumcentre and let the point H be defined by* $H = A + B + C$. *Then H lies on all three of the altitudes of triangle ABC—that is, H is the orthocentre of the triangle.*

Since $OA = OB = OC$, we have $|A| = |B| = |C|$ from which it follows that

$$(A + B) \cdot (A - B) = 0, \quad (B + C) \cdot (B - C) = 0, \quad \text{and} \quad (A + C) \cdot (A - C) = 0.$$

Since $\overrightarrow{AH} = H - A = B + C$ and $\overrightarrow{BC} = C - B$, it follows that AH is perpendicular to BC. Similarly, BH is perpendicular to AC, and CH is perpendicular to AB.

For any triangle ABC with circumcentre 0, *centroid G and orthocentre H, the points* 0, *G and H must be collinear and* $GH = 2OG$.

This follows at once for if the axes are chosen with the origin at 0, then from the above results we have $H = 3G$.

———o———

Digital Roots of Mersenne Primes and Even Perfect Numbers

Syed Asadulla, St. Francis Xavier University, Antigonish, Nova Scotia, Canada

This note offers students a way to link Thomas P. Dence's "The Digital Root Function" with G. W. Leavitt's "Mersenne Primes and the Lucas–Lehmer Test"—two articles that appear in *Two-Year College Mathematics Readings* [The Mathematical Association of America (Warren Page, editor), 1981].

The *digital root of a natural number N*, denoted DR(N), is the single digit obtained by successively adding the digits of N. Thus, DR(13) = 4 and DR(93) = DR(12) = 3. Readers can verify or refer to Dence's Proof (using induction on the number of digits N has) that

$$N \equiv \{\mathrm{DR}(N)\} (\mathrm{mod}\, 9). \tag{1}$$

Prime numbers of the form $M_p = 2^p - 1$ are called *Mersenne Primes*. Although p must be prime when $2^p - 1$ is prime, the converse is false. (For instance, $M_{11} = (23) \cdot (89)$.) A number is said to be *perfect* if it equals the sum of its divisors (not including itself). The first two perfect numbers are 6 ($= 2M_2$) and 28 ($= 2^2 M_3$). Euclid seemed aware of the fact (proved by Euler) that an even number is perfect if and only if it is of the form $2^{p-1} \cdot M_p$. (It is not known if there are odd perfect numbers.)

Since most Mersenne primes and perfect numbers are enormously large (M_{44497} has 13,395 digits and $2^{44496} \cdot M_{44497}$ has 26,790 digits!), ordinary calculations are not always practical when investigating their properties. Thus, it may be interesting to illustrate how the digital roots of the 27 known Mersenne primes are obtained.