CLASSROOM CAPSULES

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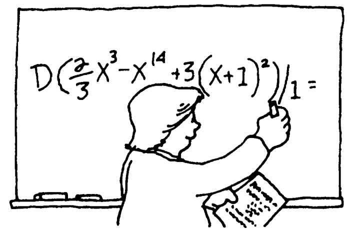

A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Tom Farmer.

Group Operation Tables and Normalizers

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In a study of certain concepts in linear algebra, a common practice is to select a vector space basis that best showcases a concept. Operation tables can play a similar role in finite group theory. A group G of order n has $(n!)^2$ different operation tables because there are $n!$ ways to list the column headings and $n!$ ways to list the row headings. For a subgroup H of G, it is possible to pick an operation table for G showing properties of the normalizer, $N_G(H)$, the largest subgroup of G having H as a normal subgroup. This we do in this note.

Suppose G is a group of order n, H is a subgroup of order k, the distinct right cosets are $Ha_1, \dots, Ha_s$, and the distinct left cosets are $a_1^{-1}H, \dots, a_s^{-1}H$. For our purpose, we choose an operation table for G having column headings chosen first from Ha_1 and then from Ha_2 and so on until the last k elements are chosen from Ha_s. Similarly, the row headings are chosen first from $a_1^{-1}H$ and last from $a_s^{-1}H$. Note that, for any $a \in G$, $(a^{-1}H)(Ha) = a^{-1}Ha$. Thus, the entries in the operation table (Figure 1) in each "diagonal block" corresponding to the products of elements of $a_i^{-1}H$ times elements of Ha_i are all the elements of the conjugate subgroup $a_i^{-1}Ha_i$.

$$\begin{array}{ccccc}
 & Ha_1 & Ha_2 & \cdots & Ha_s \\
a_1^{-1}H & a_1^{-1}Ha_1 & * & \cdots & * \\
a_2^{-1}H & * & a_2^{-1}Ha_2 & \cdots & * \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_s^{-1}H & * & * & \cdots & a_s^{-1}Ha_s
\end{array}$$

Figure 1

The following observations can be made:

1. Every conjugate subgroup of H is exhibited as the collection of elements in some diagonal block. For if $a \in G$ then $a \in Ha_i$, for some $i = 1, \dots, s$, and then $a^{-1}Ha = (a^{-1}H)(Ha) = (a_i^{-1}H)(Ha_i)$.

2. The normalizer, $N_G(H)$, is the union of all the right cosets Ha_i such that the table reveals $a_i^{-1}Ha_i = H$ (the entries in the ith diagonal block are exactly the elements of H).
3. Let m be the number of diagonal blocks consisting of the elements of H, then mk is the order of $N_G(H)$, m is the index of H in $N_G(H)$, and $s - m$ is the index of $N_G(H)$ in G.

Getting Normal Probability Approximations without Using Normal Tables

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Introduction

Using the normal distribution to approximate probabilities involving sums of discrete random variables is standard fare for beginning calculus-based statistics courses. These approximations are typically made with the aid of a normal probability table. In this paper, we discuss an alternative approach using the probability density function (pdf) for the normal distribution.

Exposing students to the alternative approach is well worth the minimal class time it requires. The technique is not only easier to apply in certain approximation situations than the standard method, but also gives students a different slant on random variables, probabilities, and areas.

The Normal PDF Approximation

Suppose that we have a sum of n independent discrete random variables, denoted by S, and that S takes consecutive integer values. Let $\mu = \mathrm{E}(S)$ and $\sigma^2 = \mathrm{Var}(S)$. Appealing to the Central Limit Theorem we can argue that for a sufficiently large value of n, S has an approximately normal distribution. Using the standard continuity correction, we get

$$\mathrm{P}(S = s) \approx \mathrm{P}\big(s - 0.5 < \mathrm{normal}\big(\mu, \sigma^2\big) < s + 0.5\big).$$

Viewing this approximating probability as an integral, we calculate a midpoint approximation using a single subinterval and get

$$\mathrm{P}(S = s) \approx \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(s-\mu)^2/2\sigma^2}.$$

We call this the *normal pdf approximation.*

It should be noted that this approximation is far from new. Similar approximations preceded not only the Central Limit Theorem, but also the normal distribution itself. De Moivre derived an approximation of this type for binomial probabilities directly, using a limit argument. His derivation is credited by some to be the origin of the normal distribution (see Stigler [**2**]).

The normal pdf and the normal table both give essentially the same approximation. The benefits of using the normal pdf instead of the table are that it is faster and only a calculator is needed (no table). It can often even claim an accuracy advantage in calculating normal probabilities, especially if interpolation is not used