or

$$\frac{f(x)-f(0)-f'(0)x}{x^2}=c_2+O(x).$$

Therefore,

$$c_2=\lim_{x\to 0}\frac{f(x)-f(0)-f'(0)x}{x^2}.$$

Using L'Hôspital's rule on the left-hand side of the above identity yields

$$c_2=\lim_{x\to 0}\frac{f'(x)-f'(0)}{2x}$$

which is equivalent to

$$c_2=\tfrac{1}{2}f''(0).$$

Continuing in this fashion gives $c_n=(1/n!)f^{(n)}(0)$ for $n=0,1,2,3,\ldots$.

For most functions encountered in elementary calculus, there are easier ways to find Maclaurin expansions. The technique of using L'Hôspital's rule can be viewed as supplementing, rather than replacing, the standard techniques.

Reference

1. M. R. Spiegel, L'Hôspital's rule and expansion of functions in power series, *American Mathematical Monthly* 62 (1955) 358–360, reprinted in T. M. Apostol *et al.*, *Selected Papers in Calculus*, Mathematical Association of America, Washington, DC, 1969, 211–213.

An Exponential Rule

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Students of elementary calculus who are quite comfortable with the power rule

$$\frac{d}{dx}[f(x)]^p=p[f(x)]^{p-1}f'(x)$$

become discouraged to discover that it fails when $(d/dx)p^{g(x)}=p^{g(x)}g'(x)\ln p$, $p>0$. They become further chagrined to see that a derivative of the form $(d/dx)f(x)^{g(x)}$ requires a different technique. They ask why, when these are structurally similar, are they so different? Insightful students will note that the above constant p may be considered a constant function, so some connection must exist. To reduce the mystery and prompt discussion, one might introduce a general exponential rule

$$\frac{d}{dx}f(x)^{g(x)}=g(x)[f(x)]^{g(x)-1}f'(x)+f(x)^{g(x)}g'(x)\ln f(x),\qquad f(x)>0$$

Ostrowski, *Differential and Integral Calculus*, Scott, Foresman, 1968, p. 276].

The first term appears as the power rule with $g(x)$ treated as a constant and the second term the rule for functional exponents with $f(x)$ treated as a constant.

While not a substitute for the valuable technique of logarithmic differentiation (the rule is a good exercise in it), it allows students to see the harmony in the various rules. Its similarity to the product rule, "take the derivative of f holding g constant plus the derivative of g holding f constant," adds to its appeal for students.

See FFF 47 in *CMJ* 22 (1991) 404.

A Useful Notation for Rules of Differentiation

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The freshman calculus instructor is often faced with the task of deciphering the freshman calculus student's versions of the product rule, quotient rule and chain rule. Even when correctly computed, it can be difficult to follow the student's steps, especially when the differentiated function involves several trigonometric functions. I propose a notational convention to deal with this problem.

Several popular calculus books (e.g. Swokowski, Larson et al., etc.) represent the differential operator as $D[\cdot]$ or $d/dx[\cdot]$. With this as motivation, I have used the following notation quite successfully in the classroom. Whenever a function is differentiated using the product rule, quotient rule, or chain rule, put the differentiated parts in square brackets. This means that the product rule would be written as

$$D[(f)(g)] = [f'](g) + (f)[g'].$$

In fact, if the instructor introduces the convention of "always differentiate f (the first function in the product) first," then the product rule can be presented pictorially as

$$D[(\ )(\ )] = [\ ](\ ) + (\ )[\ ].$$

This reduces exercises involving the product rule to nothing more than fill-in-the-blank problems.

Similarly, the quotient rule can be represented as

$$D\left[\frac{(\ )}{(\ )}\right] = \frac{[\ ](\ ) - (\ )[\ ]}{(\ )^2}.$$

The habit of "differentiating f first" developed in the product rule, must be carried over to the quotient rule.

The chain rule is a bit harder to "draw"; however the special case known as the power rule can be illustrated in this manner:

$$D\left[(\ )^n\right] = n(\ )^{n-1}[\ ].$$

My experience has shown that, not only is the students' work easier to follow, but the material is easier to present clearly. Students have reacted quite positively to this notation. In fact, when encouraged to adopt the notation, but not required to, I have found that practically all students choose to use the notation. This has been the case even when the notation was introduced some time after the rules of differentiation, i.e., Calculus 2 and Calculus 3 students quickly "pick up the habit."