

## The Age of the Solar System

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It is commonly believed that supernovas spectacularly consummate the deaths of certain stars, and the simultaneous births of some exotic remnants. The Crab Nebula is commonly believed to mark just such an event. According to one theory, supernovas are also agents of creation. Two aspects of this theory are of interest to us here:

- a. Elements with atomic number greater than 26 (iron) arise either from supernova explosions or as by-products of radioactive decay.
- b. The shock waves from supernovas cause interstellar dust and gas to “condense” into star systems.

Let us suppose that some single supernova caused the precipitation of our solar system and originated all the heavy metals found in it. Since uranium is not a by-product of radioactive decay, all of the uranium presently found in the earth's crust would have been produced in the explosion. Uranium is naturally found chemically bound to oxygen in uranous uranate, or pitchblende. The fissionable isotope, U-235, is found in concentrations of approximately 0.71%, while U-238 comprises about 99.28%. (The remaining isotopes of uranium account for the difference of 0.01%. Physical data in this capsule come from the Chemical Rubber Company's *Handbook of Chemistry and Physics*.) Suppose we assume, for the moment, that these two isotopes of uranium were created in roughly equal amounts by the supernova. Then we can date the explosion, and deduce an upper bound for the age of the solar system!

The half-life of U-238 is about 4.5 billion years, while the half-life of U-235 is 0.71 billion years. The amount of radioactive substance present at time  $t$  is

$$A(t) = A_0 \left(\frac{1}{2}\right)^{t/t_h},$$

where  $A_0$  is the initial amount, and  $t_h$  is the half-life. Given our assumptions, it only remains to write down the ratio of the amount of U-235 to U-238 in two different ways and solve for  $t$ :

$$\frac{\left(\frac{1}{2}\right)^{t/0.71}}{\left(\frac{1}{2}\right)^{t/4.5}} = \frac{0.71}{99.28} \doteq 0.00715,$$

or

$$t \left( \frac{1}{4.5} - \frac{1}{0.71} \right) = \log_2(0.00715) \doteq -7.13.$$

So, our upper bound is  $t \doteq 6$  billion years.

What if U-235 and U-238 were not created equal? In fact, there is no justification for the assumption that they were. As most high school students know, the fusion of two hydrogen atoms into one helium atom results in the release of energy. This is the primary process fueling our sun. In addition to the fusion of two hydrogens, it may occasionally happen that a hydrogen nucleus fuses with a helium nucleus to produce lithium, or two heliums combine giving birth to beryllium. All of these nuclear fusions release energy, up to and including the creation of iron. To produce heavier elements, energy must be *added*. In a supernova, that energy is

available. In addition, steady bombardment of nuclei by free neutrons builds up atomic weights, and neutron decay (into a proton, an electron, and an antineutrino) contributes to the build up of atomic number. It would be very surprising, indeed, if these processes produced exactly the same amounts of U-235 and U-238.

Let's turn the problem around. Geologic evidence seems to suggest an age of about 5 billion years for the earth. If we accept that as accurate, and assume the time between the supernova and the formation of the earth to be negligible, then we can use the previous approach to calculate the original ratio,  $R$ , of U-235 to U-238, namely,

$$R \frac{\left(\frac{1}{2}\right)^{5/0.71}}{\left(\frac{1}{2}\right)^{5/4.5}} \doteq 0.00715.$$

So,  $R = 0.44$ . Instead of a 50-50 production, we deduce that 44 atoms of U-235 would have to have been produced for every 100 atoms of U-238.

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### Forward Homework — A Motivational Tool

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Since solutions manuals now accompany most calculus textbooks, it's not surprising that a recent survey found that 55% of all calculus students on semester schedules rarely or never have their homework collected and graded [Richard D. Anderson and Donald O. Loftsgaarden, A Special Calculus Survey, Preliminary Report, in *Calculus for a New Century*, MAA Notes 8, 1988]. For the past year I have used an incentive-based system that (a) allows students to have their mathematical progress evaluated throughout the semester; (b) gently forces students to be better prepared for class; and (c) gives students a chance to develop writing and library skills. For lack of a better title, I call the system "Forward Homework" (FH). I now end most classes with a written assignment to be handed in at the beginning of the next class. In most cases the problem is carefully chosen to highlight a central topic to be covered during that class. During the lecture, I use the problem handed in that day as a key example or as part of the discussion of a major topic. For example, early in a calculus course, before proving the power rule for derivatives, I ask students to expand  $(x+h)^2$ ,  $(x+h)^3$ , and  $(x+h)^4$ ; note any patterns in these expansions; and then write the first two terms of  $(x+h)^n$ . The problems are corrected and returned at the next class. (A correct answer or good attempt with a clear explanation earns 2 points; a correct answer with no written explanation or a half-hearted attempt earns 1 point; other papers are marked 0.) Late assignments are not accepted. To allow for legitimate absences, I generally drop the two lowest grades of the fifteen to twenty problems that are assigned during a fourteen week semester.

### Some Examples of FH:

Most FH exercises introduce the key topic of the next lecture. For example, just before I cover extreme points of a function, I assign a problem similar to the following:

1. Estimate the largest and smallest values of  $f(x) = -0.3x^2 + 1.4x + 5$  on the interval  $[0,3]$  by calculating  $f(x)$  for any ten  $x$ -values of your choice.