CLASSROOM CAPSULES

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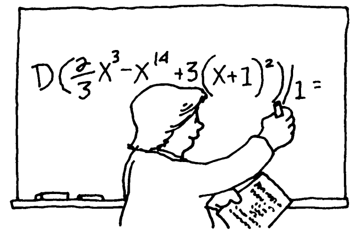


A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Frank Flanigan.

Hanging a Bird Feeder: Food for Thought

John W. Dawson, Jr., Penn State, York, PA

Calculus instructors who have grown weary of the usual maximum/minimum problems may find the following example of interest. Its context is one familiar to students, yet unlike most geometric optimization problems they will have encountered, the optimal configuration depends in an unexpected way on the numerical values chosen for the parameters. It is a thinly veiled variant of Steiner's problem, a classic problem in geometry which has been unduly neglected by authors of calculus texts.

The Problem. In the autumn, many people put up feeders for wild birds—and thereby initiate the annual round of "squirrel wars." Seasoned veterans of the combat have learned to thwart the acrobatic rodents by suspending the feeders from wires, which raises the question: What configuration will minimize the length of wire needed?

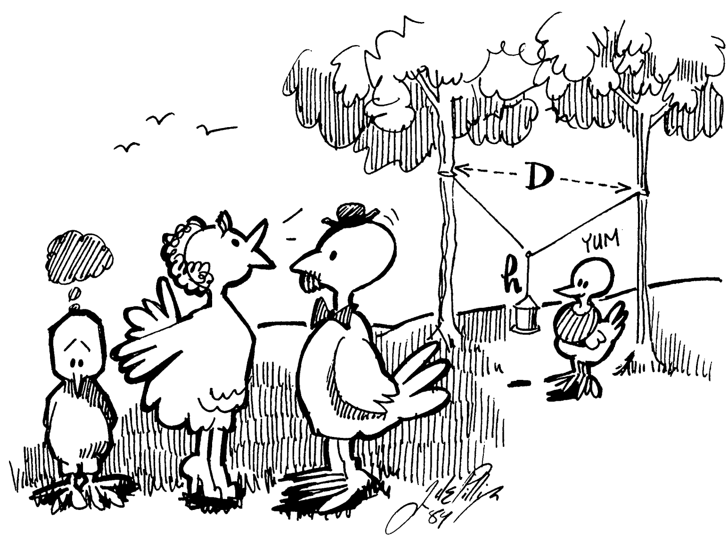
The wire is strung between two trees a distance D apart and is attached to each of them at a common height above the ground—high enough so a person can walk under the wire near the trees. The feeder is suspended midway between the trees but must be a distance d below the height at which the wires are attached to the trees so a person can reach the feeder easily. There are three configurations to consider, whose shapes resemble the letters T, V, and Y. The first two are special cases of the third. Indeed, if we take the length h of the "tail" of the Y as the independent variable, then $0 \le h \le d$ and the V and T configurations correspond to the endpoints of this interval. Letting L_C denote the length of wire required for configuration C, we have $L_T = D + d$, $L_V = (D^2 + 4d^2)^{1/2}$, and a straightforward calculation shows that $L_Y = h + (D^2 + 4(d-h)^2)^{1/2}$ has but one critical value, namely $h = d - D/2\sqrt{3}$. In order for this quantity to be positive, we must have $D < 2\sqrt{3}\,d$; if so, then we find that $(L_Y)_{\min} = (\sqrt{3}/2)D + d$. Hence $(L_Y)_{\min} < L_T$, and a bit more calculation shows that $(L_Y)_{\min} < L_V$. So, if $D < 2\sqrt{3}\,d$, the Y

configuration is best. On the other hand, $L_V < L_T$ whenever $D > \frac{3}{2}d$, so if $D > 2\sqrt{3}\,d$, the V configuration is best.

It is instructive to have students draw the optimal configurations to scale for several different values of the parameters. Better yet, to obtain a physical solution to the minimization problem, wedge three thin pegs between two transparent plates and then dip the apparatus into soapy water. The film makes a configuration that minimizes the total distance connecting the three pegs [Richard Courant and Herbert Robbins, *What is Mathematics*?, Oxford University Press, New York, 1941, p. 392]. By projecting the image of the soap film onto a screen with the aid of an overhead projector, students may then notice, as Steiner did, that the angles between the pegs in the Y configuration are equal. Having made that observation, the students can verify it by computing

$$\frac{D/2}{d-h} = \sqrt{3} = \tan 60°.$$

For an overview of Steiner's problem, see the article by H. W. Kuhn in G. B. Dantzig and B. C. Eaves, *Studies in Optimization*, MAA, Washington, D.C., 1974.



"I don't understand it. Rodney feels he can't use the feeder since he flunked calculus!"

Determinants of Sums

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A determinant formula. In an issue of the *College Mathematics Journal* [Evaluating "uniformly filled" determinants, *CMJ* 19 (1988) 343–345], S. M. Goberstein exhibits a formula for computing the determinant of a matrix V obtained from a matrix U by adding the scalar v to every entry of U. The author then evaluates several determinants of this type and mentions that freshmen mathematics majors in the Soviet Union have used the method for several decades.