for Income Averaging and whose taxable incomes are less than $50,000, approximately 20% of the time the tax computed by Income Averaging exceeds the tax from the Tax Tables. So this is not an infrequent occurrence, and the warning by the Government in Publication 17 should be heeded.

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Medical Cozenage on Fermat's Last Theorem

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Recently a new form of the disease craniosis has surfaced which may have a profound impact on a famous problem in mathematics due to Fermat. Craniosis is a disease whereby the stricken individual is unable to distinguish between left and right—this being due to a neurochemical malfunction in the medulla oblongata, the organ connecting the left and right hemispheres of the brain. The new form of the disease may be identified by the brain's inability to distinguish between up and down. Although the causes are not well understood, medical researchers are making inroads and several outstanding articles in medical journals have already appeared.

There is now strong evidence to support the belief that Fermat suffered from up/down craniosis. A thorough study of his writings, particularly his earlier writings, indicates a rather frequent transposition of mathematical expressions. For instance, there are at least twelve cases where 2^x was written when x^2 was really intended. Several transposition errors in fractions also occurred, though many of these types of errors were probably caught and corrected by Fermat's printer and publisher, Henri von Blatt, a knowledgeable mathematician in his own right. In total, there are no less than 45 up/down transpositions in Fermat's earlier writings.

With the medical evidence available, it seems highly probable that Fermat's "marginal proof' of his Last Theorem was based on a transposition of the famous equation

$$x^n + y^n = z^n.$$

It is likely that Fermat actually had in mind a proof of the theorem: *There are positive integer solutions to*

$$n^x + n^y = n^z \tag{1}$$

only when $n = 2$. The proof is as follows. Since $z \geqq \max(x, y)$, we see from (1) that $n^x \mid n^y$ and $n^y \mid n^x$. Therefore, $x = y$ and it follows that $z = x + 1$ and $n = 2$. An obvious generalization of (1) is to ask for which positive integers n there exist positive integers x, y, u, z satisfying

$$n^x + n^y + n^u = n^z. \tag{2}$$

The only possible solutions of (2) occur for $n = 2, 3$. To see this, first assume $u \geqq \max(x, y)$. Then $n^x \mid n^u$. And since $n^x \mid n^z$, we also have $n^x \mid n^y$. By symmetry, $n^y \mid n^x$. So $x = y$ and equation (2) reduces to

$$2n^x + n^u = n^z. \tag{3}$$

Thus, $n^u \mid 2n^x$ and $2n^{x-u}$ is integral for $x \leqq u$. If $n > 2$, then $x = u$ and equation (3) becomes

$$3n^x = n^z,$$

from which it follows that $n = 3$ and the solution of (2) is $x = y = u = z - 1$. For

$n = 2$, the solution is $x = y = u - 1 = z - 2$. The generalization of equations (1) and (2) can continue, but as Fermat so aptly put it, the margin is too small.

Other cases of Fermat's craniosis have recently come to public attention. It is alarming that a preponderance of these documented cases have occurred within the mathematical community. In some cases, the infection has been traced back 400 years to an association with Fermat himself. The disease is eventually fatal, and the most debilitating effect is the patient's zealous desire to study point-set topology.

The good news is that only number theorists are susceptible to infection.

Behold! The Midpoint Rule is Better Than the Trapezoidal Rule for Concave Functions

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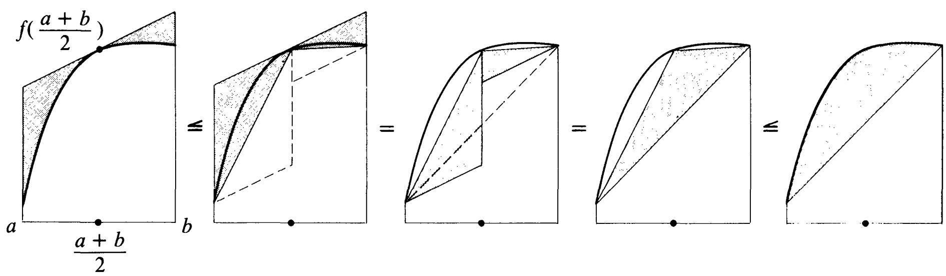

The Bisection Algorithm is Not Linearly Convergent

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The bisection algorithm is a method for finding a root of the equation $f(x) = 0$, where f is continuous on $[a,b]$ and $f(a)f(b) < 0$. Let $a(1) = a$, $b(1) = b$ and let $p(1)$ be the midpoint of $[a(1), b(1)]$. Now let $[a(2), b(2)]$ be one of the halves of this interval for which $f[a(1)]f[b(2)] < 0$, and let $p(2)$ be the midpoint of $[a(2), b(2)]$. Proceeding in this manner, the bisection algorithm generates a sequence of bracketing intervals $[a(i), b(i)]$ and a sequence of midpoints $p(i)$ of these intervals. It is well known that $p(i)$ converges to a root p of f. Some authors go even further and assert that $p(i)$ converges to p linearly—that is, there is some integer N and some constant Q such that $|p(i+1) - p| \leqslant Q|p(i) - p|$ for all $i \geqslant N$. This is not true in general. Although other authors have not made such an assertion, it is surprising that they give no counterexample. Here is a counterexample.

Let $p = 2^{-1} + 2^{-4} + \cdots + 2^{-n^2} + \cdots$ and let $f(x) = x - p$. Let the first braketing interval of the root p be $[2^{-1}, 1]$. Then it is easily verified that the second and the third bracketing intervals are $[2^{-1}, 2^{-1} + 2^{-2}]$ and $[2^{-1}, 2^{-1} + 2^{-3}]$. In general, let $S(n) = \{n^2, n^2 + 1, \ldots, (n+1)^2 - 1\}$ for $n = 1, 2, \ldots$. Then these sets form a partition of the set of positive integers. We assert that for $i \in S(n)$, the ith bracketing interval is

$$\left[a(i), b(i)\right] = \left[2^{-1} + 2^{-4} + \cdots + 2^{-n^2}, 2^{-1} + 2^{-4} + \cdots + 2^{-n^2} + 2^{-i}\right]. \quad (*)$$

The proof is by induction. We have seen above that $(*)$ holds for each $i \in S(1)$.