$$S + \frac{1}{2a} \ln \left(\frac{v}{1-v} \right).$$

The details of the proof are a bit more tedious than those presented here, but the general approach should be clear after reading the two relevant papers.

6. For pedagogical purposes, the series

$$1 - 1 + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \cdots$$

illustrates the rearrangement theorem very well. The rearrangement of this series with p positive terms followed by q negative terms is easily shown to converge to $\ln(p/q)$.

References

1. C. C. Cowen, K. R. Davidson, and R. P. Kaufman, Rearranging the alternating harmonic series, *Amer. Math. Monthly*, **87** (1980), 817–819.
2. G. Klambauer, *Problems and propositions in analysis*, Marcel Dekker, 1979.
3. W. Rudin, *Principles of mathematical analysis*, 3rd ed., McGraw-Hill, 1976.

Generating Functions and the Electoral College

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What is the probability of a tie in the Electoral College? That is, if there are only two candidates, what is the chance that they each earn 269 of the 538 electoral votes? The solution given here involves generating functions. I think students in a combinatorics class would enjoy it.

We need some data. The following table indicates the number of electoral votes a state has, and the number of states that have that many votes. The values are those determined by the 2000 U.S. census.

Votes	3	4	5	6	7	8	9	10	11	12	13	15	17	20	21	27	31	34	55
States	8	5	5	3	4	2	3	4	4	1	1	3	1	1	2	1	1	1	1

This information can be stored in a generating function that will very elegantly calculate the number of ways any number of votes can be obtained. Let

$$\begin{aligned} p(x) = {} & (1+x^3)^8(1+x^4)^5(1+x^6)^3(1+x^7)^4(1+x^8)^2(1+x^9)^3(1+x^{10})^4(1+x^{11})^4 \\ & \times (1+x^{12})(1+x^{13})(1+x^{15})^3(1+x^{17})(1+x^{20})(1+x^{21})^2(1+x^{27}) \\ & \times (1+x^{31})(1+x^{34})(1+x^{55}). \end{aligned}$$

The coefficients of the various powers of x when this is multiplied out will give the number of ways to obtain a given number of votes. For example, using Mathematica, we find the coefficient of x^{269} is 17,057,441,245,652. The probability of getting exactly 269 votes is this number divided by $p(1)$, which is 0.00758. Of course, this is assuming that all possible combinations of votes are equally likely, which may not be realistic.