the dot product with **c** and using (2) yields

$$\alpha \mathbf{c} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{n} \times (\mathbf{c} \times \mathbf{n}) = (\mathbf{c} \times \mathbf{n}) \cdot (\mathbf{c} \times \mathbf{n}) = |\mathbf{c} \times \mathbf{n}|^2 = |\mathbf{c}|^2|\mathbf{n}|^2;$$

so $\alpha = |\mathbf{n}|^2$, proving (3). This shows that (3), and thus (1), would remain valid if the cross product were defined by a left-hand rule instead of a right-hand rule.

———o———

A Nonstandard Solution to a Standard Problem

Florence S. Gordon, New York Institute of Technology, Old Westbury, NY

Sometimes, the usual solution to a standard problem becomes so routine that one never thinks to look beyond it for an alternate solution. A case in point is the old standby:

Find the equation of the circle, given the coordinates $P_1(a_1, b_1)$ *and* $P_2(a_2, b_2)$ *of the endpoints of a diameter.*

The standard approach leads us to locate the center, calculate the radius, and then substitute into the formula for the equation of a circle to obtain

$$\left(x - \frac{a_1 + a_2}{2}\right)^2 + \left(y - \frac{b_1 + b_2}{2}\right)^2 = \frac{(a_1 - a_2)^2 + (b_1 - b_2)^2}{4}.$$

Expanding and simplifying this, we obtain

$$x^2 - (a_1 + a_2)x + y^2 - (b_1 + b_2)y + a_1a_2 + b_1b_2 = 0. \qquad (1)$$

A simpler and more elegant approach can be based on geometric principles. If $Q(x, y)$ is any point on the circle different from P_1 and P_2, then the angle P_1QP_2 is a right angle. Thus, the slopes of P_1Q and P_2Q are negative reciprocals, and we have

$$\frac{y - b_1}{x - a_1} = -\frac{x - a_2}{y - b_2}.$$

This, upon cross-multiplying, immediately leads to the same solution as in (1) above. This method also provides a different geometric insight to this standard problem. This approach can be extended to handle the comparable problem in three dimensions: *find the equation of the sphere, given the endpoints of any diameter*. The standard approach is to parallel the initial argument mentioned above, using analytic geometry. A much simpler approach is to utilize some elementary vector analysis. If $P_1(a_1, b_1, c_1)$ and $P_2(a_2, b_2, c_2)$ are the two given points and $Q(x, y, z)$ is any other point on the surface of the sphere, then the vectors P_1Q and P_2Q must be perpendicular, and so their dot product will be zero. This leads directly to the solution

$$x^2 - (a_1 + a_2)x + a_1a_2 + y^2 - (b_1 + b_2)y + b_1b_2 + z^2 - (c_1 + c_2)z + c_1c_2 = 0.$$

Editor's Note: The same type of argument [see, for example, page 112 of C. H. Lehmann's *Analytic Geometry*, John Wiley & Sons, 1942] can be used to prove that any angle P_1QP_2 inscribed in a circle is a right angle. $P_1(-r, 0)$ and $P_2(r, 0)$ determine the circle $x^2 + y^2 = r^2$. For any point $Q(x, y)$ on this circle, P_1Q has slope $m_1 = y/(x + r)$ and P_2Q has slope $m_2 = y/(x - r)$. Hence, $m_1m_2 = y^2/(x - r)^2 = -1$.

———o———