Since the outlines of the characters in modern computer fonts are actually described by parametric equations, the alphabet is a natural source of exercises for this topic.

First the class is partitioned into an even number of groups with three or four students apiece. Each group receives a copy of the following instructions.

Spelling out a four-letter word:
An exercise in parametric equations

In the first part of today's lab, your group is being asked first to agree on a four-letter word and then to write a set of parametric equations which when graphed will spell out your word. (ANY four-letter word is fine as long as your whole lab group agrees to it.)

In the second part of the lab, you will receive the parametric equations from one of the other groups, and you will be asked to graph them. NOTE: You should graph the equations that you are given and not try to draw the letters that you *think* the group was trying to encode.

Your grade will be based equally on both parts of the lab. Half of your grade will come from your ability to write the parametric equations of the letters in your four-letter word. The other half of your grade will be based on your ability to graph the other group's parametric equations for their word.

As soon as any two groups finish writing parametric equations for their words, their work is exchanged and they start on the second part of the lab. Typically, each part of the lab takes less than one hour.

We can control the level of difficulty by explicit instructions regarding the words and letters used. For instance, we might require that no letter be repeated in the word, or that the first letter be capitalized and the rest be lower case; that the letters appear on the same Cartesian plane, and in correct order; that the domains of the parameter variable for each letter fit together into a closed interval. . . . Clearly, variants abound.

Even without prodding, many students have gone beyond straight line segments and circular arcs in writing their words—choosing parts of conic sections, for example, such as a parabola to draw a C and a hyperbola for an X. Some of the outcome, of course, depends on the students' preparation and creative spirit, and on promised rewards.

This lab is always quite "vocal," as groups decide on their four-letter word, discuss the best way to write certain letters, and begin to figure out exactly what word the other group was trying to spell. The ways in which students benefit from this lab are numerous. Not only do they learn well how to write and to graph parametric equations, but they get to discuss mathematics, to actively engage in it, to exercise creativity, to be mathematically inquisitive, and—depending on their choice of four-letter word—to be downright irreverent. Our students have enjoyed the two hours spent on this lab exercise, and we hope that yours will too.

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A Novel Approach to Geometric Series

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Here is my informal way of explaining the geometric series formula,

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + \cdots = \frac{a}{1-r}, \qquad \text{where} \quad |r| < 1$$

(of course, you can present it to your students with specific numerical values, if you wish):

> Imagine you are to receive a gift of \$$a$. Unfortunately, gifts are subject to a flat tax rate of r, with $0 < r < 1$. Your benefactor wishes you to have \$$a$, not fewer. How much should your benefactor give so that your after-tax net, not your gross, is \$$a$?

We can solve this problem in two ways. On the one hand, since your benefactor knows that giving only a dollars will require you to pay ar dollars in tax, he decides instead to give $(a + ar)$ dollars. Has he thereby compensated for your tax bill? Alas, not enough; for now you need to cover the tax on the extra ar dollars. Clearly he must give ar^2 dollars more. That is, your benefactor must now give you $(a+ar+ar^2)$ dollars. You see where this is going, don't you? You will now need to pay an extra ar^3 dollars tax on the added ar^2 dollars. This continues, in theory, ad infinitum. So, for you to end up with exactly a dollars net, your benefactor must hand over the gross amount

$$S = a + ar + ar^2 + \cdots.$$

On the other hand, suppose we look at this problem more directly. Consider the formula *net = gross minus tax*, or $a = S - rS = S(1 - r)$. That is, $S = a/(1 - r)$. Comparing our two solutions leads to the geometric series formula, $a+ar+ar^2+\cdots = a/(1 - r)$, for $0 < r < 1$.

Finally, here is another twist. If $-1 < r < 0$, we can interpret the geometric series formula in terms of "immediate interest" instead of taxation. This is more of a stretch for the students; the details are left to imaginative readers.

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The Ergonomic Office

Steinmetz's retreat was an isolated camp on a tributary of the Mohawk River. He had the small stream dammed to form a placid pond, and more than one writer commented upon Steinmetz's use of it. A journalist who visited wrote, "Probably there is no other office like it in all the world—a battered twelve-foot tippy canoe with a cushion in the bottom and four boards laid together from gunwale to gunwale, thwartwise to serve as a desk. When he goes down to the river to work he carries his papers under his arm, with them Hutchinson's volume of four-place tables, and a little Nabisco box wherein he keeps his pencils." The tables of logarithms often remained unopened, and Steinmetz was said to have taken them along only in case his memory failed him.

Henry Petroski, *Remaking the World: Adventures in Engineering,* Knopf, 1997, page 5.
Contributed by Lawrence Braden, St. Paul's School, Concord, NH.