The probability of the needle falling entirely within one of the rectangles is then

$$1 - P_{h\ or\ v} = \frac{\pi ab - 2n(a+b) + n^2}{\pi ab}$$

One can simulate the Buffon needle experiment with a computer, and there are many variations on the Buffon or Laplace needle problems that can be pursued as calculus exercises or with the assistance of a computer. For example, instead of parallel lines, one might try a collection of n lines through a single point with uniform angular spacing between the lines [3]. One might consider other tilings of the plane (see [6, 9]), say by hexagons rather than by rectangles. Another variation is to bend the needle and keep the parallel lines. Surprisingly, Barbier [12] gave an ingenious solution to the original Buffon needle problem by bending the needle into a circle and computing the probability that the circle crosses one of the parallel lines! Gnedenko [5] (also see [9]) showed that we obtain the same solution if the needle is bent into any convex curve. Buffon's Noodle Problem [7] is to find the probability of crossing one of the parallels when tossing a wet noodle of fixed length, but which randomly changes shape on each throw!

H. Solomon [11] also discusses higher dimensional analogues of these problems. For example, how can the problem be framed if our needle is to be positioned in euclidean m-space partitioned into parallel hyperplanes or into nonoverlappng m-dimensional rectangles? The problem is then to find the probability that a randomly placed vector with norm n lies entirely in one of the cells.

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Tangents to Conics, Eccentrically

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Geometrical notions are abundant in calculus, where one learns how problems involving them can be addressed via the derivative or integral. Interestingly, in the case of tangent lines (as I learned by chance, while reading [1]) one can make old-fashioned ruler and compass constructions as easily for conics as for circles. Constructions described in [2] and [3] are based upon properties that are somewhat unique to each of the three families of conic (parabola, ellipse, and hyperbola). My intent is to show how the eccentricity approach to conics offers a simple unification of this topic.

Figure 1 shows part of a conic section whose eccentricity is e. Let F be the focus, l the directrix, P the point of tangency, and Q the foot of the perpendicular from P to l, so that $PF = ePQ$. (For two points such as P and F, I will use the notation PF to mean either the line PF or the distance from P to F, depending upon context.) Figure 2 shows how the tangent line PT is obtained: Construct the perpendicular to PF at F, and let T be its intersection with l. (If PF is perpendicular to l, as when P is a conic's vertex, then T is a point at infinity, making PT parallel to l.) A proof that no other point of line PT lies on the conic is illustrated in Figure 3.

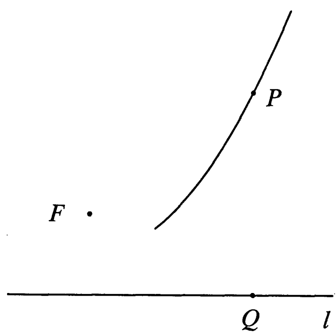

Figure 1

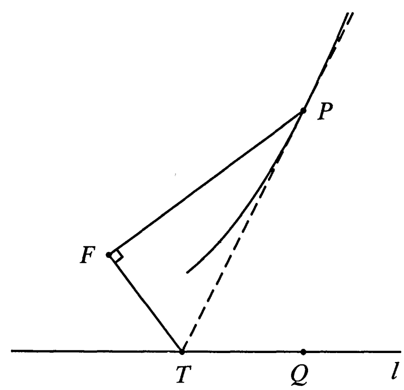

Figure 2

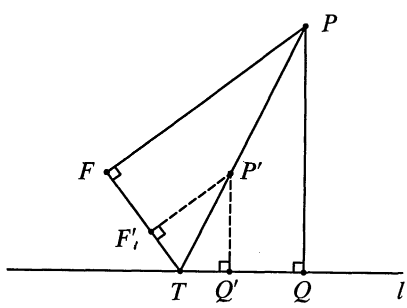

Figure 3

Let P' be a point on line PT different from P, and let F' and Q' be the feet of perpendiculars from P' to FT and l. Since both $PQ/P'Q'$ and $PF/P'F'$ are equal to $PT/P'T$ via similar triangles, they are equal to each other. It follows that $P'F'/P'Q' = PF/PQ$, which in turn is equal to e. Therefore $P'F' = eP'Q'$, which means that $P'F \neq eP'Q'$. So P' is not on the conic.

Figure 3 is also the scene of an interesting fact about the relationship between angles ∠TPF and ∠TPQ. Since $PF/PQ = e$, we can divide numerator and denominator by PT and conclude that $(\cos\angle TPF)/(\cos\angle TPQ) = e$.

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Lottery Drawings Often Have Consecutive Numbers

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There are few random processes more avidly watched than the state lottery drawings in which six numbered balls are chosen from a set of 44 (in Louisiana; other states vary.) People have been surprised to notice that the winning selection often contains two consecutive numbers. We can compute the probability of this happening, and see that it is actually greater than one half.

The probability can be computed using standard counting techniques found in any advanced combinatorics book; in fact, it appears implicitly as a problem in [**1**, p. 72] and [**4**, p. 59]. The purpose of this note is to make the solution accessible to the student who knows only the elementary principles of counting.

Let us begin by counting the number of ways that q identical objects can be distributed among p labelled boxes. Think of the objects lined up in a row. We can assign them to the boxes by inserting $p-1$ markers into the row: Box 1 will get the objects (if any) to the left of marker 1; box 2 will get those between marker 1 and marker 2; etc.

How many ways can this be done? What we have is a row of $q+p-1$ things: q objects and $p-1$ markers. The question is how many ways we can choose which $p-1$ of the things will be the markers: $C(q+p-1, p-1)$. We sum this up as:

Theorem. [**1**], [**4**] *The number of ways that q identical objects can be distributed among p labelled boxes is* $C(q+p-1, p-1)$.

This theorem is simple enough, but as in so many counting problems, the hard part is deciding for our problem what we should consider to be the objects and the boxes.

Consider the general case of choosing k numbered balls from a set of n. Think of the $n-k$ losing balls as objects and the k winning balls as dividing them into $k+1$ boxes: those before the first, those between the first and second, etc.

Using the theorem, the number of ways this can be done is

$$C(n-k+[k+1]-1, [k+1]-1) = C(n,k)$$

as we knew it must be.

Now, possibly some of these boxes will be empty; in fact, a box will be empty precisely when two winning numbers are consecutive. So to count the number of