The Root Mean Square-Arithmetic Mean-Geometric Mean-Harmonic Mean Inequality

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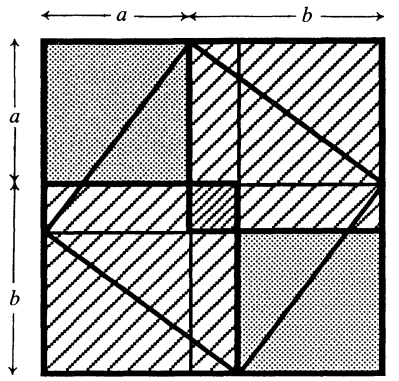
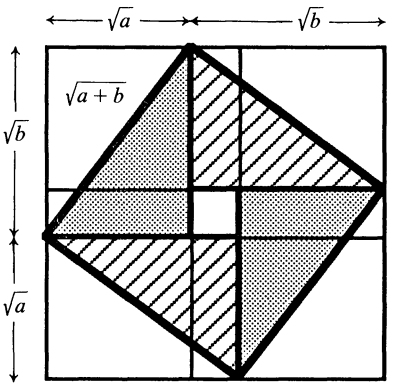
$$a, b > 0 \Rightarrow \sqrt{\frac{a^2 + b^2}{2}} \geq \frac{a + b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a + b}$$



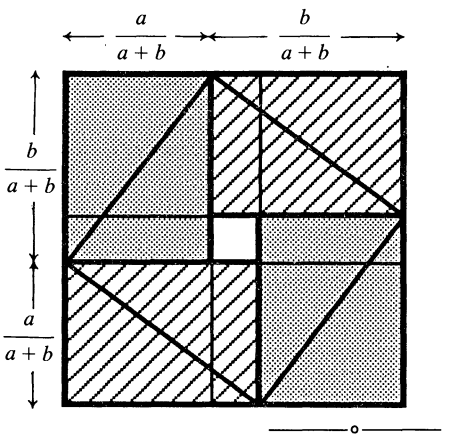
$$2a^2 + 2b^2 \geq (a + b)^2$$

$$\sqrt{\frac{a^2 + b^2}{2}} \geq \frac{a + b}{2}$$



$$\left(\sqrt{a + b}\right)^2 \geq 4 \cdot \tfrac{1}{2}\sqrt{a}\sqrt{b}$$

$$\frac{a + b}{2} \geq \sqrt{ab}$$



$$1 \geq 4\frac{a}{a + b} \cdot \frac{b}{a + b}$$

$$\sqrt{ab} \geq \frac{2ab}{a + b}$$