$y_1, y_2, \ldots, y_k\}$ and this set then forms a basis for R^n. A dimension count yields $r + k = n$, giving the rank and nullity theorem without use of the echelon form.

Note that the reliance on orthogonality in the arguments above is elementary and does not require the Gram-Schmidt process. The ideas used here can be readily applied to complex matrices with minor modifications. The hermitian inner product is used instead in the complex vector space C^n, as is the hermitian transpose. Observation 2) would note then that vectors in the null space of A are orthogonal to those in the row space $\bar{A}$.

The row and column rank theorem is a well-known result that is valid for matrices over arbitrary fields. The notion of orthogonal complement can be generalized using linear functionals and dual spaces, and the general structure of the arguments here can then be carried over to arbitrary fields. The text [**1**, pp. 97 ff.] contains such an approach.

Many authors base their discussion of rank on the echelon form. The fact that the non-zero rows of the echelon form are a basis for the row space, or that columns in the echelon form containing lead ones can be used to identify a basis for the column space in the original matrix A, are central to such a development of rank. Results such as these follow easily if it is established independently that row rank and column rank must be equal.

The author would like to thank the referee for several valuable suggestions.

REFERENCE

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A One-Sentence Proof That $\sqrt{2}$ Is Irrational

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If $\sqrt{2}$ were rational, say $\sqrt{2} = m/n$ in lowest terms, then also $\sqrt{2} = (2n - m)/(m - n)$ in *lower* terms, giving a contradiction.

(The three verifications needed—that the second denominator is less than the first and still positive, and that the two fractions are equal—are straightforward.)

The argument is not original; it's the algebraic version of a geometric argument given in [**1**, p. 84], and it was presented (in slightly different form) by Ivan Niven at a lecture in 1985. Note, though, that the algebraic argument, unlike the geometric, easily adapts to an arbitrary $\sqrt{k}$ where k is any positive integer that is not a perfect square. Indeed, let j be the integer such that $j < \sqrt{k} < j + 1$. If we had $\sqrt{k} = m/n$ in lowest terms $(m, n \in \mathbf{Z}^+)$, then also $\sqrt{k} = (kn - jm)/(m - jn)$ in lower terms, a contradiction.

REFERENCE

1. H. Eves, *An Introduction to the History of Mathematics*, 6th edition, W. B. Saunders Co., Philadelphia, 1990.