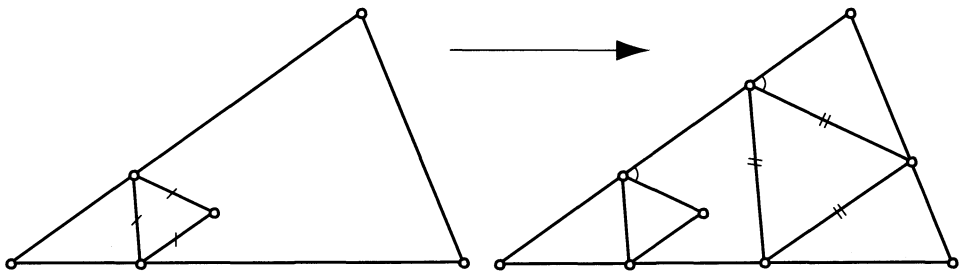
Proof Without Words: Every Triangle Has Infinitely Many Inscribed Equilateral Triangles



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Perfect Cyclic Quadrilaterals

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Are there any quadrilaterals with integer sides having perimeter P equal to area A? A square of side length 4 might come to mind. Are there any more? More generally, what is the number $N(k)$ of integer-sided quadrilaterals whose ratio of perimeter to area is a fixed value, say $P/A = k$? This question is interesting mainly for cyclic quadrilaterals (that is, those that can be inscribed in a circle) since there are, for example, an infinite number of parallelograms satisfying $P = kA$ for a given positive number k (as the reader can check). In addition, the cyclic case generalizes the question for triangles, which has been treated successfully.

In 1971, M. V. Subbarao [**6**] considered the problem of finding the number $N(\lambda)$ of triangles for which the sum of the integer sides a, b, and c is equal to λ times the triangle's area, where λ is a given positive real number. These are called *perfect triangles*. He showed that $N(\lambda)$ is finite with $N(\lambda) = 0$ for $\lambda > \sqrt{8}$, $\lambda \neq 2\sqrt{3}$, and $N(\lambda) = 1$ for $\lambda = 2\sqrt{3}$, the triangle being equilateral with edge 2. Although Subbarao

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