

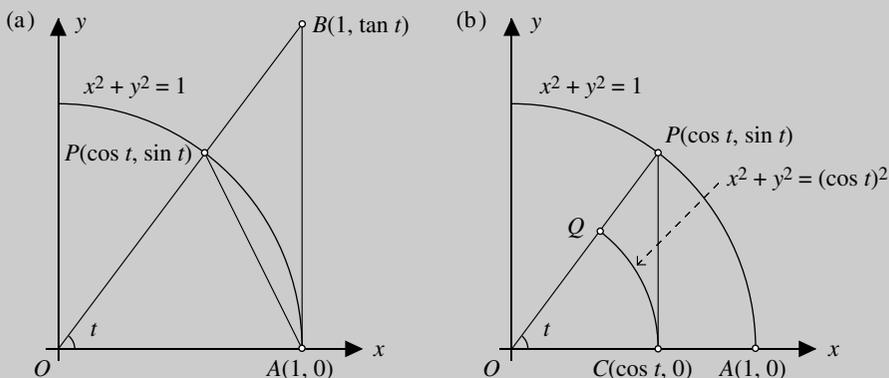
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Teaching Tip: The Limit of $(\sin t)/t$

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Almost all modern college calculus texts include a proof that $\lim_{t \rightarrow 0} (\sin t)/t = 1$. Most do so in the same way: for t in $(0, \pi/2)$, compare the areas of the circular sector and two triangles illustrated in part (a) of the figure. The area of $\triangle OAP$ is $\sin t/2$; the area of sector OAP is $t/2$; the area of $\triangle OAB$ is $\tan t/2$; and thus $(\sin t)/2 \leq t/2 \leq (\tan t)/2$. These inequalities are now cleverly manipulated to obtain $\cos t \leq (\sin t)/t \leq 1$, from which it follows that $\lim_{t \rightarrow 0^+} (\sin t)/t = 1$ (the left-hand limit is usually obtained by noting that $\sin(-t)/(-t) = \sin t/t$). In our experience, it is the manipulation of the inequalities that may cause some students difficulty.



The manipulation of the inequalities can be greatly simplified by using *one* triangle and *two* circular sectors, as illustrated in part (b) of the figure. The area of sector OCQ is $t(\cos t)^2/2$; the area of $\triangle OCP$ is $(\sin t \cos t)/2$; the area of sector OAP is $t/2$; and hence $t(\cos t)^2/2 \leq (\sin t \cos t)/2 \leq t/2$. Multiplication by $2/t \cos t$ immediately yields $\cos t \leq (\sin t)/t \leq 1/\cos t$, from which the limit follows.