We have determined $\tau(u_i)$ for only four values of p: 2, 1, 1/2, 2/3. Students may enjoy some of the following projects in analysis and numerical methods:

(a) plotting τ versus u_i for these four values of p;

(b) finding other values of p for which the integral equation (3) for u_f is reducible to a transcendental equation, and plotting $\tau(u_i)$ for these;

(c) a class exercise in which different values of $p < 1$ are assigned to students or student groups, and each is asked to find the u_i for which $\tau = 1$.

This problem originated from a first-year physics question set by Tim Shirtcliffe. I am grateful to him and to John Harper and Graeme Wake for helpful comments.

Reference

[1] G. K. Batchelor, An Introduction to Fluid Dynamics, Cambridge, 1967, p. 341.

A Method of Duplicating the Cube

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The old problem of duplicating the cube—that is, of constructing a cube with volume twice that of a given cube—was solved geometrically in several ways by the ancient Greek mathematicians (see Eves [**1**] for a summary). It is the purpose of this note to show how analytic geometry can be used to construct two curves which will give one more solution to the problem.

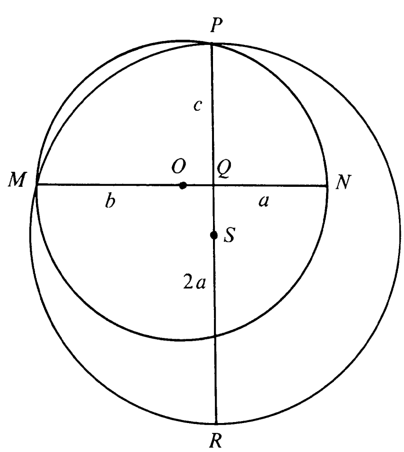

Figure 1

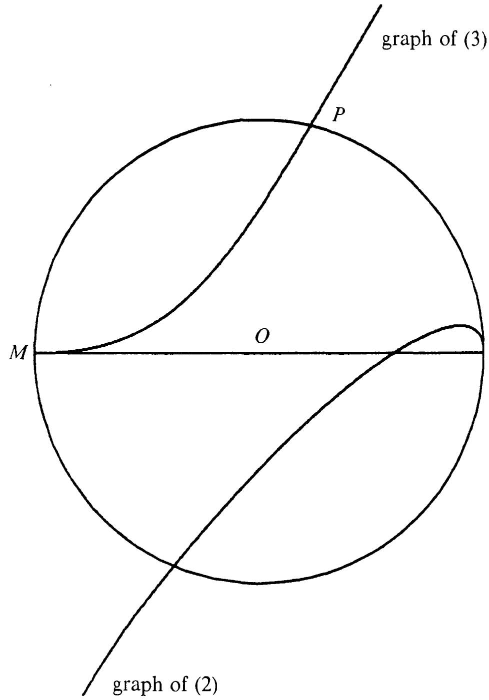

FIGURE 2

Given a cube whose edge has length c, if numbers a and b can be found such that

$$\frac{2a}{b}=\frac{b}{c}=\frac{c}{a}, \tag{1}$$

then the problem is solved, because $b^2=2ac$ and $c^2=ab$ imply $b^3=2abc$ and $c^3=abc$, so $b^3=2c^3$. The relations (1) can be thought of as relations between segments of two perpendicular chords of intersecting circles (see FIGURE 1). The problem is, given c and circle MPN with center at the origin O and radius r, to determine circle MPR with center $S(x,y)$ and radius $a+(c/2)$.

First we obtain the locus of the centers of all circles such that $|QR|=2|QN|$, without requiring that they also pass through M. The y-coordinate of R is $-2a=-2(r-x)$, and the y-coordinate of P is $(r^2-x^2)^{1/2}$, so the y-coordinate of S, their midpoint, is

$$y=\frac{1}{2}\left((r^2-x^2)^{1/2}-2(r-x)\right). \tag{2}$$

Next we take circles with centers on the graph of (2) which in addition pass through $M(-r,0)$ and obtain the locus of the upper endpoints of their vertical diameters. If $T(x,Y)$ is a point on that locus then, since $|ST|=|SM|$,

$$Y-y=\left((r+x)^2+y^2\right)^{1/2}, \tag{3}$$

where y is as in (2). FIGURE 2 shows the graphs of equations (2) and (3). Where the graph of (3) intersects the circle—that is, where $Y=(r^2-x^2)^{1/2}$— determines the point P and hence S. This duplicates the cube, because (3) reduces to $(r+x)^3=2((r^2-x^2)^{1/2})^3$, or $b^3=2c^3$.

The authors wish to thank the referee for helpful suggestions.

Reference

[1] Howard Eves, An Introduction to the History of Mathematics, Holt, Reinhart and Winston, New York, 1964.