3. A. T. Benjamin and J. J. Quinn, The Fibonacci numbers—exposed more discretely, this MAGAZINE **76** (2003) 182–192.
4. A. T. Benjamin and J. J. Quinn, *Proofs That Really Count: The Art of Combinatorial Proof*, Mathematical Association of America, Washington, DC, 2003.
5. A. T. Benjamin and D. Walton, Combinatorially composing Chebyshev polynomials, to appear in *Journal of Statistical Planning and Inference, Proceedings of the 6th International Conference on Lattice Path Combinatorics*, 2008.
6. T. J. Rivlin, *Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory*, John Wiley, New York, 1990.
7. L. W. Shapiro, A combinatorial proof of a Chebyshev polynomial identity, *Discrete Math.* **34** (1981) 203–206.
8. D. Walton, A tiling approach to Chebyshev polynomials, senior thesis, Harvey Mudd College, Claremont, CA, 2007.

Math Bite: Sums of Sines and Cosines

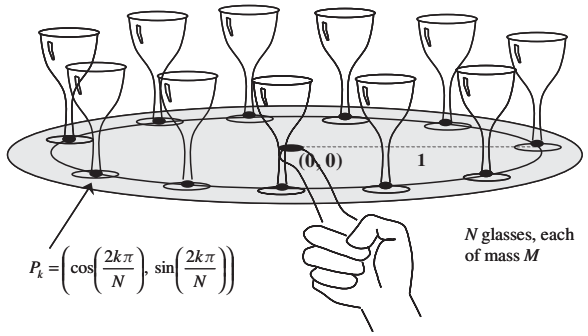
The sums of the title are, for N a natural number greater than 1,

$$\sum_{k=1}^{N} \cos\left(\frac{2\pi k}{N}\right) = 0 \quad \text{and} \quad \sum_{k=1}^{N} \sin\left(\frac{2\pi k}{N}\right) = 0.$$

The proofs of these two identities make a good exercise in complex algebra: Using DeMoivre's Theorem and Euler's formula, $e^{i(2\pi)} = (e^{i2\pi/N})^N = 1$, we derive the identities from the real and imaginary parts of the following equation:

$$\sum_{k=1}^{N} \left(\cos\left(\frac{2\pi k}{N}\right) + i \sin\left(\frac{2\pi k}{N}\right)\right) = \sum_{k=0}^{N-1} (e^{i2\pi/N})^k = \frac{(e^{i2\pi/N})^N - 1}{e^{i2\pi/N} - 1} = 0.$$

It seems that one must enter the realm of the complex numbers to prove this result, yet the validity of the identities becomes absolutely clear from the picture that follows.



$$\begin{aligned}(\overline{x}, \overline{y}) &= \left(\left(\textstyle\sum_{k=1}^{N} M\cos(2k\pi/N)\right)\Big/ NM, \left(\textstyle\sum_{k=1}^{N} M\sin(2k\pi/N)\right)\Big/ NM\right) \\ &= \left(\tfrac{1}{N}\textstyle\sum_{k=1}^{N}\cos(2k\pi/N), \tfrac{1}{N}\textstyle\sum_{k=1}^{N}\sin(2k\pi/N)\right) \\ &= (0, 0)\end{aligned}$$

—Judy A. Holdener
Kenyon College
Gambier, OH 43022