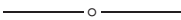We call a number *n* an *all-digit* number if it contains all ten digits. The last series we consider avoids the reciprocals of these numbers.

Corollary 3.   *The series obtained from the harmonic series by removing all terms whose denominator is an all-digit number converges and its sum is less than* 810.

———— ∘ ————

A New and Improved Method for Finding the Center of Gravity of a Quadrilateral

Behzad Khorshidi (b_khorshidi@yahoo.com), Tabriz, Iran

The standard method for finding the center of gravity of a quadrilateral is to use its diagonals to split it into two pairs of triangles and then find the intersection of the lines connecting the centroids of the pairs of triangles that share a common diagonal. In this note we present a much simpler approach, one that involves finding the centroid of just one triangle instead of four.

Let $\square ABCD$ be a quadrilateral with diagonals $\overline{BD}$ and $\overline{AC}$ intersecting at E, and take F on the line $\overleftrightarrow{AC}$ with $\overline{CF}$ congruent to $\overline{AE}$ as shown in Figure 1. (Note that the quadrilateral in (a) is convex, while the one in (b) is not.)
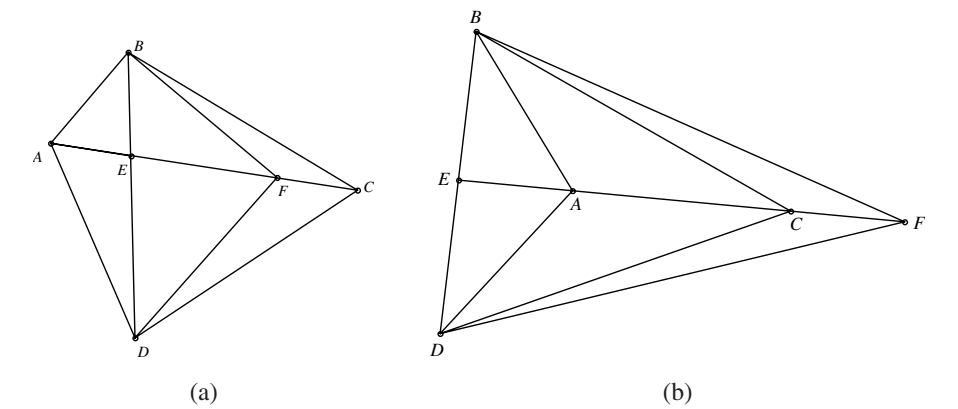

(a)   (b)

Figure 1.

Theorem.   *The center of gravity of $\square ABCD$ is the centroid of $\triangle DBF$.*

Proof. We prove the result in the convex case (see Figure 2); the non-convex case is similar. By construction, $\overline{AC}$ and $\overline{EF}$ have the same midpoint. It follows that $\triangle ABC$ and $\triangle EBF$ have a common centroid G and $\triangle ADC$ and $\triangle EDF$ have a common centroid H. If we view $\triangle DBF$ as the degenerate quadrilateral $\square DEBF$, we see that the centers of gravity of both $\triangle DBF$ and $\square ABCD$ lie on the segment joining G and H.
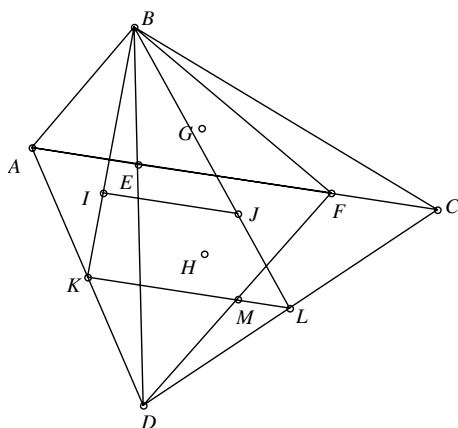
Figure 2.

Let K, L, and M be the respective midpoints of $\overline{AD}$, $\overline{DC}$, and $\overline{DF}$, and let I and J denote the respective centroids of $\triangle ABD$ and $\triangle DBC$. Since I and J are centroids, $\overline{IJ}$ cuts the segments $\overline{BK}$ and $\overline{BL}$ in a 2-to-1 ratio. It follows that $\overline{IJ}$ is parallel to the base $\overline{KL}$ of $\triangle KBL$, so it also cuts $\overline{BM}$ in a 2-to-1 ratio, and hence it contains the centroid of $\triangle DBF$. By the same argument, $\overline{GH}$ also contains the centroid of $\triangle DBF$, so $\overline{IJ}$ and $\overline{GH}$ intersect at that centroid, which is then also the center of gravity of the quadrilateral.

To appear in *Mathematics Magazine*, June 2007

ARTICLES

Solving the Ladder Problem on the Back of an Envelope, *by Dan Kalman*

The Recreational Gambler: Paying the Price for More Time at the Table, *by Joseph Bak*

The Probability of Relatively Prime Polynomials, *by Arthur T. Benjamin and Curtis D. Bennett*

NOTES

Fitting One Right Triangle in Another, *by Charles H. Jepsen and Valeria Vulpe*

Determinants of Matrices over the Integers Modulo m, *by Jody M. Lockhart and William P. Wardlaw*

The Classification of Similarities: A New Approach, *by Aad Goddijn and Wim Pijls*

Perfect Matchings, Catalan Numbers, and Pascal's Triangle, *by Tomislav Došlić*

On Candido's Identity, *by Claudi Alsina and Roger B. Nelsen*

Monotonic Convergence to *e* via the Arithmetic-Geometric Mean, *by József Sándor*