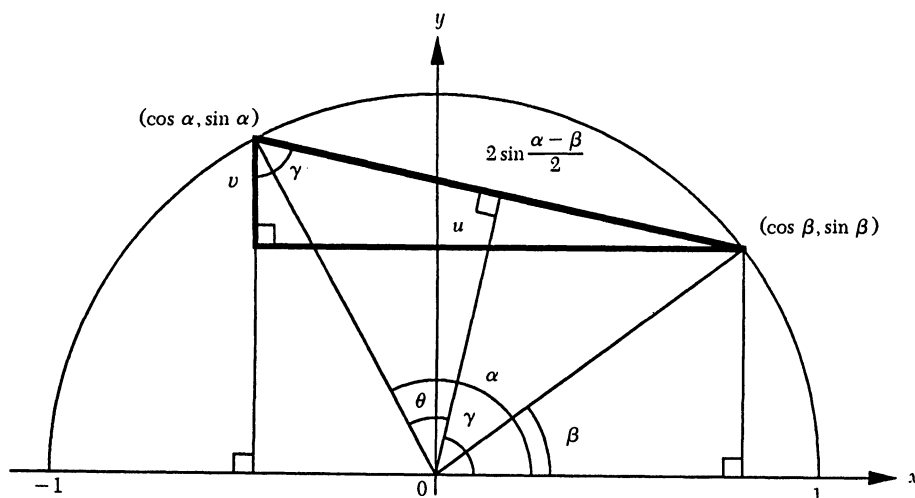*Proof.* The task is to minimize the function $f(x, y) = xy$ subject to the constraints, $x^y = k$, $x > 0$, $y > 0$. Solving the constraint equation $x^y = k$ for y yields $y = \ln k / \ln x$. Substituting that value for y into the formula for f yields a function of one variable, $g(x) = x \ln k / \ln x$. The function g is minimized where its (easily calculated) derivative is zero, namely at $x = e$. Thus f is minimized when $x = e$ and $y = \ln k$.

Conclusion In the first half of this paper we began with a problem intended for children, generalized it twice, and solved all three problems. In the second half we formulated three dual problems and, paralleling our strategies from the first half, solved the three new problems. For the first problems in the two families, the "integer-integer" problems, the results were a bit disappointing: (new) Theorem 1′ bore little resemblance to (old) Theorem 1. For the second problems, the "real-integer" problems, (new) Theorem 2′ paralleled (old) Theorem 2 closely. Some of the details of proof, however, were different and more difficult. For the third problems, the "real-real" problems, (new) Theorem 3′ and (old) Theorem 3 turned out to be almost identical, and even their proofs were "dual."

Proof Without Words:
The Difference-Product Identities

$$\theta = \frac{\alpha - \beta}{2} \qquad \gamma = \frac{\alpha + \beta}{2}$$

$$\sin \alpha - \sin \beta = v = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \beta - \cos \alpha = u = 2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$$

—Sidney H. Kung
Jacksonville University
Jacksonville, FL 32211