Mathematics to DIE for: The Battle Between Counting and Matching

Positive sums count. Alternating sums match. So which is "easier" to consider mathematically? This talk is one part *performance art* and three parts combinatorics. The audience will judge a combinatorial competition between the competing techniques. Be prepared to explore a variety of positive and alternating sums involving binomial coefficients, Fibonacci numbers, and other beautiful combinatorial quantities. How are the terms in each sum concretely interpreted? What is being counted? What is being matched? Do alternating sums always give simpler results? You decide.

The Combinatorialization of Linear Recurrences

Binet's formula for the nth Fibonacci number,

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

is a classic example of a closed form solution for a homogenous linear recurrence with constant coefficients. Proofs range from matrix diagonalization to generating functions to strong induction. Could there possibility be a better way? A more visual approach? A combinatorial method?

This talk introduces a combinatorial model using weighted tiles. Coupled with a sign reversing involution, Binet's formula becomes a direct consequence of counting exceptions. But better still, the weightings generalize to find solutions for any homogeneous linear recurrences with constant coefficients.

Proofs That Really Count

Every proof in this talk reduces to a counting problem---typically enumerated in two different ways. Counting leads to beautiful, often elementary, and very concrete proofs. While not necessarily the simplest approach, it offers another method to gain understanding of mathematical truths. To a combinatorialist, this kind of proof is the only right one. I have selected some favorite identities using Fibonacci numbers, binomial coefficients, Stirling numbers, and more. Hopefully when you encounter identities in the future, the first question to pop into your mind will not be "Why is this true?" but "What does this count?"

This talk is a "Choose your own adventure"™ where the content is guided by the input and desires of the audience.

Fibonacci's Flower Garden

It has often been said that the Fibonacci numbers frequently occur in art, architecture, music, magic, and nature. This interactive investigation looks for evidence of this claim in the spiral patterns of plants. Is it synchronicity or divine intervention? Fate or dumb luck? We will explore a simple model to explain the occurrences and wonder whether other number sequences are equally likely to occur.

This talk is designed to be appreciated by mathematicians and nonmathematicians alike. So join us in a mathematical adventure through Fibonacci's garden.