

2nd United States of America Junior Mathematical Olympiad

Day I 12:30 PM – 5 PM EDT

April 27, 2011

JMO 1. Find, with proof, all positive integers n for which $2^n + 12^n + 2011^n$ is a perfect square.

JMO 2. Let a, b, c be positive real numbers such that $a^2 + b^2 + c^2 + (a + b + c)^2 \leq 4$. Prove that

$$\frac{ab + 1}{(a + b)^2} + \frac{bc + 1}{(b + c)^2} + \frac{ca + 1}{(c + a)^2} \geq 3.$$

JMO 3. For a point $P = (a, a^2)$ in the coordinate plane, let $\ell(P)$ denote the line passing through P with slope $2a$. Consider the set of triangles with vertices of the form $P_1 = (a_1, a_1^2)$, $P_2 = (a_2, a_2^2)$, $P_3 = (a_3, a_3^2)$, such that the intersections of the lines $\ell(P_1), \ell(P_2), \ell(P_3)$ form an equilateral triangle Δ . Find the locus of the center of Δ as $P_1P_2P_3$ ranges over all such triangles.