2nd United States of America Junior Mathematical Olympiad

Day II 12:30 PM – 5 PM EDT

April 28, 2011

JMO 4. A *word* is defined as any finite string of letters. A word is a *palindrome* if it reads the same backwards as forwards. Let a sequence of words $W_0, W_1, W_2, \ldots$ be defined as follows: $W_0 = a$, $W_1 = b$, and for $n \geq 2$, W_n is the word formed by writing W_{n-2} followed by W_{n-1}. Prove that for any $n \geq 1$, the word formed by writing $W_1, W_2, \ldots, W_n$ in succession is a palindrome.

JMO 5. Points A, B, C, D, E lie on circle ω and point P lies outside the circle. The given points are such that (i) lines PB and PD are tangent to ω, (ii) P, A, C are collinear, and (iii) $\overline{DE} \parallel \overline{AC}$. Prove that $\overline{BE}$ bisects $\overline{AC}$.

JMO 6. Consider the assertion that for each positive integer $n \geq 2$, the remainder upon dividing 2^{2^n} by $2^n - 1$ is a power of 4. Either prove the assertion or find (with proof) a counterexample.