

2011 USAMO/USAJMO Indices and Selection

April 11, 2011

1 Statistics for 2011 AIME I and 2011 AIME II

Below are statistics summarizing selected outcomes for the two contests.

1.1 Basic Descriptive Statistics

For all students taking the 2011 AMC I and 2011 AMC II, we have the following descriptive statistics:

	2011 AIME I	2011 AIME II
N	3586	2704
Mean	2.836	5.245562
StdDev	2.594	3.161
Skewness	1.198	0.66879
Kurtosis	4.297	2.914
Q1	0.9167	2.41667
Med	1.5	4.5
Q3	3.0833	6.58333
AMC Avg	111.96	115.51

(The AMC Average is a mixed average over all scores, both AMC 10 and AMC 12, of students taking each respective contest, so these are not a perfect indicator.)

For students in the US and Canada taking the 2011 AIME I and 2011 AIME II, we have the following descriptive statistics:

	2011 AIME I	2011 AIME II
N	3400	1140
Mean	2.79	4.81
StdDev	2.37198	2.83
Q1	1	3
Median	2	4
Q3	4	6

For students in the US and Canada qualifying from the AMC 12 and then taking the 2011 AIME I and 2012 AIME II, we have the following descriptive statistics:

North America AMC 12 Takers only:

	2011 AIME I	2011 AIME II
N	2320	838
Mean	3.4	5.32
StdDev	2.74	3.32
Q1	1	3
Median	3	4
Q3	4	6

1.2 The proportions of students taking various contests

The total number of AIME takers in North America is $3400 + 1174 = 4574$. The ratio of North America AIME I to North America AIME II is: $3400/1174 = 2.896$. The fraction of North America AIME I takers in the total is: $3400/4574 = 0.743$. The fraction of North America AIME II takers in the total is: $1174/4574 = 0.257$.

Note the total of North American students who took both an A-date test and a B-date test was 1010.

The number of North American students who took the same test (10A/10B or 12A/12B) is 787, the number who mixed a 10 and 12 on the A and B dates is 223.

	2011 AIME I	2011 AIME II
N Universal	3586	2704
N USA	3281	1045
N Canada	119	95
N North America	3400	1140

1.3 Comparison of Difficulty Vectors

From the table of Correctly Answered Problems I took the number of students who answered each problem correct, out of the total of 3586. Then out of 3586, I calculated the percentage correct. Recall, that these are universal, out of *all* participating students, (not just in North America.) The difficulty chart is:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2666	1612	914	282	979	1145	314	266	1036	134	396	77	25	58	331
74.34%	44.95%	25.49%	7.86%	27.30%	31.93%	8.76%	7.42%	28.89%	3.74%	11.04%	2.15%	0.70%	1.62%	9.23%

The Difficulty Order Ranking is:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
74.34%	44.95%	31.93%	28.89%	27.30%	25.49%	11.04%	9.23%	8.76%	7.86%	7.42%	3.74%	2.15%	1.62%	0.70%

From the table of Correctly Answered Problems I took the number of students out of 3586 that got each problem correct.

Then out of 2704, I calculated the percentage correct. Recall, that these are universal, out of ALL participating students, (not just in North America.) The difficulty chart is:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
957	2331	1689	1432	1921	1574	723	607	221	578	559	225	699	293	3
35.39%	86.21%	62.46%	52.96%	71.04%	58.21%	26.74%	22.45%	8.17%	21.38%	20.67%	8.32%	25.85%	10.84%	1

The Difficulty Order Ranking is:

2	5	3	6	4	1	7	13	8	10	11	15	14	12
86.21%	71.04%	62.46%	58.21%	52.96%	35.39%	26.74%	25.85%	22.45%	21.38%	20.67%	13.87%	10.84%	8.32%

1.3.1 Some Comparison Vectors of Difficulty

2009 Combined AIME I and AIME II: (That's the only data I could find, so this is not perfect comparison.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5521	5802	6835	4768	2017	2227	3047	2938	1970	1624	1550	351	821	778	14
55.05%	57.85%	68.15%	47.54%	20.11%	22.21%	30.38%	29.30%	19.64%	16.19%	15.46%	3.50%	8.19%	7.76%	1.15%
3	2	1	4	7	8	6	5	9	10	11	13	14	12	15
68.15%	57.85%	55.05%	47.54%	30.38%	29.30%	22.21%	20.11%	19.64%	16.19%	15.46%	8.19%	7.76%	3.50%	1.15%

2010 Combined AIME I and AIME II:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2823	4391	3794	3977	3188	3010	1358	2453	1563	1192	935	1018	801	1484	1484
43.24%	67.26%	58.12%	60.92%	48.84%	46.11%	20.80%	37.58%	23.94%	18.26%	14.32%	15.59%	12.27%	22.73%	22.73%
2	4	3	5	6	1	8	9	7	10	12	11	13	15	15
67.26%	60.92%	58.12%	48.84%	46.11%	43.24%	37.58%	23.94%	20.80%	18.26%	15.59%	14.32%	12.27%	1.85%	1.85%

2 USAMO Selection

Using the means, there is an approximately 2-problem difference in scores. This 2-problem difference is the same regardless of the population over which the mean is computed. Using the medians and quartiles there is a 2-problem difference over the population of North American students. (The difference is larger over the universal populations.) Using the rank ordering of problem difficulties over the universe of takers for each contest, there is an approximately 3-problem difference between the AIME I and the AIME II. This is taken into account in the USAMO selection.

From AMC 12 takers who then took AIME I with an AMC12-based index of 188.0 and higher, there were 199 students.

From AMC 12 takers who then took AIME II with an AMC12-based index of 215.5 and higher, there were 78 students.

The difference in indices based on AIME I and AIME II is a difference of 27.5 points which is roughly consistent with the difference in descriptive statistics for the two contests. The ratio of AIME I selectees to AIME II selectees is 2.55.

This makes a total of 277 students invited to the USAMO. Given that there is essentially a fixed number (about 260) of students invited to the USAMO (and we are already over that number by 17) an increase in the number of selectees from the AIME I means a corresponding decrease in the number of selectees from AIME II. In turn that would mean an increase in the difference between the indices, already approaching the limit of what it should be. A decrease in the number of selectees from AIME I would mean a corresponding increase in the number of selectees from the AIME II and a decrease in the ratio of selectees, which is also not desirable.

There were also 4 students added from states not already represented in the USAMO and USAJMO, see the section on the USAMO and USAJMO. This makes a grand total of 281 students invited to the USAMO.

The mean AMC 12 score for USAMO selectees is 125.2 and the median AMC 12 score for USAMO selectees is 124.5. Of these 281 students, 150 took an A-date contest, and 130 took a B-date contest. Of these 281 students, 158 took both an A-date contest and a B-date contest. Of these 281 students, 138

indicated they have taken the USAMO before, 134 have not taken the USAMO before and 9 were unknown. There are 268 from the US and 13 from Canada. The grade levels are:

≤ 7	3
8	9
9	25
10	49
11	102
12	91

3 USAJMO Selection

From the group of USAMO selectees, I removed 35 students who had also had AMC 10 based indices but had already qualified for the USAMO.

From AMC 10 takers who then took AIME I with an AMC 10-based index of 179.0 and higher, there were 162 students.

From AMC 12 takers who then took AIME II with an AMC12-based index of 196.5 and higher, there were 54 students.

The difference in indices based on AIME I and AIME II is a difference of 17.5 points which is roughly consistent with the difference in descriptive statistics for the two contests. The ratio of AIME I selectees to AIME II selectees is 3.00.

This makes a total of 216 students invited to the USAMO. Given that there is essentially a fixed number (about 220) of students invited to the USAJMO an increase in the number of selectees from the AIME I means a corresponding decrease in the number of selectees from AIME II. In turn that would mean an increase in the difference between the indices, already approaching the limit of what it should be. A decrease in the number of selectees from AIME I would mean a corresponding increase in the number of selectees from the AIME II and a decrease in the ratio of selectees, which is also not desirable.

From the union of all selectees for the USAMO and USAJMO there were 11 states not already represented. These students were added, 7 added to the USAJMO, and 4 added to the USAMO. This makes a total of 223 students invited to the USAJMO.

The mean AMC 10 score for USAJMO selectees is 135.6 and the median AMC 12 score for USAMO selectees is 135. Of these 227 students, 149 took an A-date contest, and 72 took a B-date contest. Of these 223 students, 143 took both an A-date contest and a B-date contest. Of these 223 students, 31 indicated they have taken the USAJMO or USAMO before, 182 have not taken the USAMO or USAJMO before and 9 were unknown. There are 219 from the US and 4 from Canada. The grade levels of the students are:

≤ 7	15
8	39
9	72
10	96