

ELEMENTARY AND SECONDARY SCHOOL TRAINING IN MATHEMATICS*

S. S. CAIRNS, University of Illinois

1. Introduction. The general problem to which these comments pertain is that of adapting our public schools to the needs of society and the nation. Attention is restricted, however, primarily to the question of mathematical instruction, the field in which the writer is best qualified to offer advice. This is also a phase of the general problem to which a peculiar importance attaches as a result of the national emergency and the associated critical shortage of scientifically trained manpower. The shortage is expected to become worse before it gets better, since industry, the government, and the military services are making increasingly heavy demands with no signs of a corresponding increase in the supply. A substantial improvement in the situation could be effected by remedying some of the serious defects in elementary and secondary school mathematical training. It is the present object to support this assertion by discussing such defects and suggesting remedial measures.

2. The principal questions. Recent discussions of educational problems have raised subsidiary questions which becloud some of the main issues and present a danger to effective progress. These troublesome and almost irrelevant problems include (1) whether certain subjects are better taught now than at some previous time (2) who is to blame for some of the recognized shortcomings of our schools and (3) whether we are preparing most students to meet their expected needs in later life. We should rather concentrate on the magnitude and nature of our national needs, on the obstacles to meeting them and on methods for overcoming these obstacles.

3. Mathematical shortcomings of our schools. To commence with generalizations, our high schools are sadly deficient both in preparing students for college and in offering adequate education to those not bound for college. Our elementary schools, in turn, are deficient in preparing students for high school.

Children of average to superior abilities, in the earliest grades, are frequently (perhaps generally) offered no encouragement to proceed at their natural pace in learning those aspects of arithmetic which appeal to them. At a slightly later stage, they are introduced to the fundamental operations of addition, subtraction, multiplication and division, but they are generally not drilled in such basic necessities as the multiplication tables. As a consequence they enter high school severely handicapped, save for that small proportion who learn so readily that they need no drill. In high school, the mathematics courses hit a slow pace, partly because the students have inadequate backgrounds, partly because there is no genuine incentive for the schools to provide suitable courses

* This is a revision of a statement presented to the School Problems Commission of the State of Illinois, meeting at Carbondale, March 6, 1952.

or for the students to take them. As a consequence, the colleges lose a year or two of mathematical training by having to teach to the freshmen, and frequently to the sophomores, courses which properly belong in the high schools. For students in the humanities, this situation is less serious than for those in the sciences and engineering. The latter are delayed to such an extent in commencing their essential studies that (1) a net loss of at least a year is difficult to avoid before their training is complete and (2) their programs are so crowded as to preclude many of the broadening studies which should form part of their general education. The delay carries over into the graduate schools, where we find ourselves teaching large proportions of our students material which belongs in a good undergraduate program.

Before substantiating the foregoing remarks, let it be noted that exceptional schools exist, generally in certain urban areas, and that in other schools exceptional teachers can be found who somehow partially counteract the general difficulties.

While a wealth of data could be offered in support of the preceding statement of shortcomings, a selection will be made, for convenience and brevity, from experience with two categories of students at the University of Illinois: (1) students who enter the College of Liberal Arts and Sciences with deficiencies in mathematics and (2) students in the elementary schools program of the College of Education. There were 234 students in the first group in the period 1949 to 1951 and 268 in the second between 1947 and 1952. Deficiencies in mathematics imply only failure to have taken one full year each of high school algebra and plane geometry. The lack may be due to lack of opportunity or to a deliberate avoidance of the subject for one reason or another. There is evidence that a good proportion of the students with such deficiencies (constituting about 10% of the L. A. S. freshmen) are suitable, though poorly prepared, college material. Identical standardized arithmetic tests have been administered to both groups. The results reveal a shocking inability to handle elementary arithmetic. To mention a few examples from data supplied by Mr. Clarence Phillips of the Mathematics Department, only 41% of the first group and 59% of the second correctly figured one year's interest at 6% on \$175; the percentages of success were 34 and 55 in computing $7-6+2-4$, and 30% and 53% in arranging the numbers .40, 2.5 and .875 in order of magnitude. The difference between the two groups is due to the fact that the second group (1) had more mathematics in high school, (2) is more selective as to admission and (3) contains 77% seniors and graduate students, while the first group is almost all freshmen.

Of the students entering with mathematical deficiencies 50% fail to attain sophomore standing. This percentage is far out of line with the native abilities within the group and reveals the handicap of a student who is so poorly prepared by his high school.

The College of Education students just mentioned are required to take an arithmetic course in the Mathematics Department, intended to deepen their

understanding of what they will soon be teaching. Many of them are deplorably weak on the fundamentals, will hesitate over such things as eight times seven or seven plus six and will frankly express their easily understood fear of teaching arithmetic. This often takes place in the second semester of the senior year, after they have done practice teaching and a few months before they will be on the job, perhaps unconsciously transmitting their own aversions and lack of confidence to their students.

Turning to the College of Engineering, suffice it to remark that the inadequate mathematical preparation of the entering freshmen recently led to the establishment of a joint committee from that College, the College of Education and the Department of Mathematics. The work of this committee culminated in a pamphlet entitled *Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois*, which has been widely circulated among Illinois high schools. At present, a similarly composed committee, under the chairmanship of one of our University High School mathematics teachers, is studying means of adjusting the high school program to meet these needs. This cooperative effort is encouraging. It is to be hoped that the work of the committee will lead to widespread improvements in the teaching of mathematics and will serve as a model for cooperation elsewhere.

4. Underlying causes for shortcomings. This partly speculative section could run to great lengths. To avoid that, a few false principles will be listed, with brief comments and with no effort to estimate how widely these principles are accepted by those responsible for administering our schools.

- a. *The theory that drill and deliberate memorizing must be avoided, especially in the lower grades.* This clearly works to the detriment of (1) learning the multiplication tables, (2) learning the alphabet at the proper time, (3) learning to spell, (4) learning the essentials of English grammar, (5) at a somewhat later stage developing a vocabulary when studying a foreign language and (6) acquiring the study habits demanded by effective college work. This theory is generally associated with the unwarranted belief that techniques will be incidentally acquired.
- b. *The theory that local needs and desires should dominate in determining curricula, to the practical exclusion of needs on a national scale.*
- c. *The theory that high schools should limit their programs to those skills and manipulations that some group of individuals finds necessary to the average adult.*
- d. *The belief that the less successful students should be kept in the same classes with the more successful.*
- e. *The aversion to competition among students.* This, and the previous item have a deadening effect on those who should be stimulated and encouraged.

5. A proposed guiding principle. To quote from a letter by Professor J. W.

Peters of the Department of Mathematics at the University of Illinois, "The American public high school has the responsibility to develop and administer a sound educational program which will provide for the education at the high school level of every youth to the full extent of his capacity and ability." This is a lofty ideal, but one which we can take as a guide, even though its full realization may be in the distant future.

6. Possible remedial measures. The following suggestions were offered for the consideration of the School Problems Commission.

a. The establishment of standards on a state-wide basis for the grade schools and high schools. The enforcement of such standards would require some sort of testing procedures. The merits of the Regents Examinations of the State of New York might be considered in this connection. In the establishment of standards and indeed in all phases of studying school problems, it is essential that due consideration be given to the views of scholars and scientists as well as to those of Departments and Colleges of Education, parents, industrial employers and educational administrators.

b. The establishment of adequate college entrance requirements. This would involve upward revisions, which could be introduced only gradually, as elementary and secondary schools adapt themselves thereto. In this connection, certain quotations may be in order from Bulletin Number 9 of the Illinois Curriculum Program series, entitled *New College Admission Requirements Recommended*, issued through the office of the State Superintendent of Public Instruction.

"The specification by the colleges of certain high school courses to be taken by all students seeking college entrance sets definite limitations to curriculum revision. If a considerable block of courses must be retained in the high school to provide for the preparation of students who hope to go to college, the opportunity to re-examine the total high school curriculum and to replan the program in terms of the needs of all high school youth is hereby curtailed." (p. 5.)

This disturbing quotation suggests that no essential college preparatory courses are recognized on the basis of all our educational experience to date.

"The committee recognizes that small high schools will not always be able to provide a sufficient variety of specialized courses to meet the need for the special programs of all its graduates. In such cases, the colleges are urged to make provision for the basic specialized work with as little handicap to the student as possible." (p. 14.)

This implies that the colleges are to continue, as a matter of policy, and perhaps even to expand, their offerings of high school types of instruction.

"With limited resources, the high school's first responsibility is to provide education of general value to all its students, rather than to provide for the

specialized needs of parts of the student body when the latter effort is taken at the expense of a good program of general education." (p. 13.)

This is at variance with the guiding principle suggested above. It implies that, although talent is uniformly distributed throughout the population, students in certain communities are to be denied the opportunity for their full development.

ON SEQUENCES OF OPERATIONS IN COMPLETE VECTOR SPACES

I. S. GÁL, Institute for Advanced Study

1. Introduction. The first purpose of this paper is to describe how various results of classical analysis can be summarized in two general theorems concerning linear operations from a Banach space into a normed vector space. The two theorems are known as the *principle of uniform boundedness* and the *principle of condensation of singularities*. Our second object is to show how one can generalize these principles in the case of certain sequences of non-linear operations. In order to formulate our principles we start from simple examples, namely the summation of infinite sequences and the divergence problem of Fourier series of continuous functions.

2. The consistency of summation methods. As is well known, a real matrix-summation correlates with any sequence of real numbers $\{x_n\}$ a new sequence $\{u_m\}$ by means of a fixed infinite square matrix (a_{mn}) ($m, n = 1, 2, \dots$); more precisely

$$(1) \quad u_m = a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + \dots \quad (m = 1, 2, \dots),$$

where the infinite series on the right hand side must be considered as a formal expression. It may diverge if the sequence $\{x_n\}$ is not sufficiently regular. However, it is natural to require of a summation method at least that it be efficient in the case of any convergent sequence, *i.e.* if $x_n \rightarrow \xi$ then all formal series (1) must converge and also $u_m \rightarrow \xi$ as $m \rightarrow \infty$. If a matrix-summation method (a_{mn}) has this basic property it is called a *consistent* or *regular* summation process. Perhaps the simplest non-trivial example of such consistent methods is the summation by arithmetical means.

There is a simple necessary and sufficient condition in order that a matrix-summation method should be consistent. Namely, according to a theorem of H. Steinhaus and O. Toeplitz [1], a summation process (a_{mn}) is regular if and only if