SOME OBSERVATIONS ON UNDERGRADUATE MATHEMATICS IN AMERICAN COLLEGES AND UNIVERSITIES

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1. Introduction. During the academic year 1951–1952 it was my privilege, under a grant from the Ford Foundation, to visit the mathematics departments of thirty-three American colleges and universities. These institutions, located in the East, the Midwest, and the Far West, extended from Boston to Los Angeles, from Philadelphia to Seattle. Among them were privately endowed universities, state universities, private colleges, and city colleges. They were selected on the basis of academic excellence, and it is believed that they represent institutions of the highest scholastic standing in the regions mentioned. My primary mission was to study undergraduate mathematics programs. The members of the departments visited were most helpful and generous with their time in supplying information about their own institutions and in participating in discussions on various aspects of undergraduate mathematics instruction. This experience has resulted in certain impressions concerning the present state of undergraduate mathematical education in this country, at least as reflected by the institutions visited, which may be worth passing on to others interested in this matter.

2. Mathematics in general education. Relative to the status of mathematics as a required subject, in only four of the institutions visited is it required of all students for graduation. In three others, mathematics or philosophy is required. The most usual general requirement is one which specifies that a certain number of courses be selected from a group consisting of the natural sciences and mathematics. A question of interest here is the extent to which mathematics is regarded as an essential subject in a liberal education. Concerning this point, I made the following observations. In the institutions which have inaugurated "general education" courses in the humanities, the social sciences, or the natural sciences, with but a single exception mathematics has no place in any of these courses. Indeed, in some institutions which have had the longest experience with such courses, and also in other institutions, the mathematical needs of only those students who follow certain special curricula are seriously considered. This suggests that some mathematics departments are not especially concerned with the contribution their subject might make to a liberal education or else feel that this contribution is automatically provided by the usual introductory courses. On the other hand, one finds individual members of these departments vitally interested in this question, but who, for various reasons, are unable at present to implement their convictions with changes in course content or organization. Another interesting observation is that, generally speaking, in the West one finds fewer instances of mathematics considered as a basic part of a liberal arts program than in the East.

3. Types of freshman courses. The traditional college algebra and trigonometry are still the usual freshman courses in most institutions. However, eighteen of the institutions visited provide courses of a non-traditional character which at least some students can elect or can take to satisfy requirements involving mathematics. These non-traditional courses are usually designed for students not planning to pursue work in which mathematics is needed as a tool. Their content frequently includes elements of analytic geometry and calculus as well as some algebra and trigonometry, and occasionally topics from even more advanced subjects such as number theory and topology. Recently, in some places probability and statistics have come to be regarded as appropriate subjects for inclusion in such a course. With the increasing application of mathematical statistics to more and more areas of human knowledge, there seem to be cogent reasons for teaching as many students as possible something about the nature of statistical inference. In some courses considerable attention is devoted to logic and the character of mathematics as a logical structure. Most of the people with whom I talked agree that much of the trigonometry and some of the algebra in the traditional freshman course can well be replaced by mathematics that is more interesting and of greater significance. Especially is this true for the student who will not take any more mathematics.

The type of course which offers most promise of substantial contribution to a general education is not adequately described by merely listing the topics covered. The spirit in which the subject is treated is of the greatest importance. An understanding of the nature and significance of mathematics is sought through an emphasis on basic concepts, the logical processes used in developing the subject, and the relation of the discipline to other fields through a consideration of its origins and its applications. Techniques, of course, are necessary, but there is plenty of evidence that many students pass their freshman courses by memorizing techniques without obtaining the slightest insight into the true nature of mathematics. Such a procedure could hardly contribute much to a liberal education. These courses, whose nature is so sketchily suggested here, have as their primary objective the fuller realization of the educational values long believed to be inherent in the discipline of mathematics. It is to be emphasized that it is not thought that this can be accomplished by use of descriptive material *about* the subject. Serious mathematics must be the backbone of the course. But exactly what topics constitute the most suitable content for such a course is by no means fully decided. Much experimentation remains to be done. Increasing dissatisfaction with the inadequate contributions of traditional courses to a general education is impelling some institutions to undertake serious investigation in this area. The number of good textbooks suitable for a course of this character is extremely small. This fact has undoubtedly discouraged some institutions from instituting such a course. It seems likely that in time this deterrent will be eliminated.

It would appear that one of the characteristics of many mathematics teachers is their reluctance to try something new. As a result they frequently get into a rut—it is easy to do in elementary courses in this field—and the listlessness of their students is a reflection of their own boredom with a subject grown stale from endless repetition. The institutions in which the quality of instruction impresses one most favorably are those in which the courses are constantly studied for possible improvement. A stabilized course tends to become a stagnant course.

The freshman course designed for the non-specialist, which we have been discussing, is thought by some also to be the best type of introductory course for students who will take further courses in the field. It is recognized that in the transition from courses of this type to advanced work provision must be made to supply certain knowledge and skills not included in a course planned as terminal.

4. Analytic geometry and calculus. In some institutions with high entrance requirements the usual freshman course consists of analytic geometry and calculus. Whether taught in the freshman year or later, there is an increasing tendency to teach these two subjects together. A goodly number of mathematicians are convinced that from a purely mathematical viewpoint there is much to be gained by teaching them together. Also, from the standpoint of the student's whole program an earlier introduction to the calculus has many advantages; one obvious one is the availability of this tool for use in elementary physics courses. There are topics in analytic geometry which, while interesting mathematics in themselves, are not a necessary part in the mainstream development, and consequently can be and frequently are omitted in these combination courses. Several institutions are experimenting with a more rigorous type of calculus course for their better students. There is considerable difference of opinion as to the degree of rigor feasible in a first course in calculus. Here the importance of variation in ability among students becomes very evident. It appears that only the best ones are able at this stage to assimilate the type of rigor proposed. Another important factor may be the nature of previous work in mathematics, whether a certain maturity in understanding mathematical concepts and proofs has been developed.

5. Upper college courses. In the work of the junior and senior years, a fairly recent innovation is the offering at the majority of the institutions visited of one or more courses in modern algebra. The proper content and level of abstractness for these courses are by no means universally agreed upon. There are those who believe that the first course should be fairly concrete, perhaps devoted mostly to matrices and vector spaces, while others think an introduction to various abstract algebraic systems is the most valuable type of first course. In any event, in the near future instructors will have a considerably larger number of textbooks from which to choose than has been the case previously. In many institutions courses in classical theory of equations are being replaced by some form of modern algebra.

In analysis there is considerable thought being given to courses which make the transition from elementary calculus to graduate courses in function theory. The nature of these courses varies rather widely among different institutions.

One also finds great variation among institutions in interest and course offering in geometry. Occasionally, mention is made of the need for the reformulation of courses in this field. One possible future trend is the teaching of certain parts of geometry and algebra together—such as introductory projective geometry and linear algebra.

Most departments now offer one or several courses in mathematical statistics. A few institutions provide sequences of special courses designed to give the necessary mathematical training for social scientists. The increasing interest in applications of mathematics in the social sciences may influence courses in the calculus, matrix theory, and other topics in algebra. It is rather surprising to learn that a knowledge of the structure of some of the abstract algebraic systems is proving useful in certain types of investigations in the social science field.

In about half of the institutions visited, something besides formal courses is provided for undergraduates specializing in mathematics. These extra activities take the form of reading courses, honors work, seminars, tutorials, *etc.* The main purpose is to have the student do some independent work under appropriate supervision. The chief deterrent to a more extensive occurrence of these practices is the cost in terms of faculty time. In six of the institutions comprehensive examinations in mathematics are given to all seniors specializing in the subject, and in three others these examinations are given students in the honors program. At several places, theses in the major subject are required of seniors.

6. Teacher training. In the realm of teacher training, courses in algebra and geometry aimed at meeting the special needs of secondary school teachers are frequently offered. It would appear that the character of these courses in some institutions should be scrutinized for their relevance to their purported purpose. Occasionally, courses in fundamental concepts and the history of mathematics are recommended for prospective teachers. It must be admitted that university mathematics departments do not always fully discharge their obligations in the training of teachers. Too often mathematicians are content to complain of the poor preparation students receive in high schools without taking any steps to determine how departments of mathematics might help to remedy the situation. As a result, too large a part of the training of high school teachers is frequently left to less competent agencies.

7. Universities versus colleges. It is generally conceded that undergraduate education in the large universities is by and large not up to the high standards set by the good small colleges. There are, of course, many reasons for this: differences in admission requirements, extensive use in universities of graduate assistants to teach elementary courses, the presence or absence of an atmosphere conducive to good academic work, *etc.* One of the most important factors is that in universities the men with the imagination, the energy, and the enthusiasm to raise teaching to truly inspirational levels are frequently so heavily engaged in research, training graduate students, and other activities that they simply do not have the time to devote to elementary teaching. Thus, in many of our great universities the quality of undergraduate instruction has lagged behind that of research and graduate training. There appears to be in some places a genuine need for more mathematicians with the qualities mentioned above, drawing on their learning and inventiveness, to contribute ideas, constructive suggestions, and a part of their time to the essential task of undergraduate education.

State universities have their own peculiar problems. Many of them are due to a heterogeneous student body, representing an incredibly wide range of ability, preparation, and interest. In several states all graduates of accredited high schools must be admitted to the state university. In most of the state institutions some provision for individual differences is made in the freshman year by offering algebra courses at various levels. It is quite usual to find "intermediate algebra"—second-year high school algebra—taught in universities, and frequently college credit is given for the course. In state universities with relatively unselected freshman classes, differentiated programs of study appear to be the only feasible way of providing the type of education appropriate to various levels of ability. The present practice of setting standards and adjusting levels of teaching for the median student frequently results in something which is beyond the grasp of the poorer students and at the same time fails to challenge the better ones. This problem is recognized at many institutions but it is far from being solved.

8. Exchange of information. It became obvious to me that there is a genuine need for a freer exchange of information between institutions, particularly information regarding innovations and experiments. Perhaps the Association, through its meetings and through this MONTHLY, can make a greater contribution to this end. Also, personal correspondence and private conversation, where opportunity permits, would certainly be of great mutual benefit to everyone engaged in this process of trying to improve undergraduate mathematical education.