exist exactly 9 pairs $q(a)=r(a)=b$ where $1 \leqq b \leqq 9$. This is illustrated in the table given below.

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$q(a)$	*5*	2	*6*	*3*	1	5	7	8	*9*	4	*7*	3	6	*8*	*4*	*2*	*1*	0
$r(a)$	*5*	12	*6*	*3*	11	15	17	18	*9*	14	*7*	13	16	*8*	*4*	*2*	*1*	10

MATHEMATICAL TEACHING IN UNIVERSITIES

ANDRÉ WEIL, University of Chicago

The following is the outline of a lecture once given by the author at a joint meeting of the Nancago Mathematical Society and of the Poldavian Mathematical Association. It is printed here at the editor's request, as the principles stated there seem to be of general application.

1. Improvements in the mathematical teaching in Poldavian Universities depend largely upon general improvements in the educational system in Poldavia. Mathematicians should devote themselves to the task of making such improvements as lie within their power at present, and thus contributing their share towards general reforms, which in turn will enable them to make further progress.

2. No satisfactory results can be achieved unless reforms are made both in school-teaching and in University teaching. So far as school-teaching is concerned, the efforts of mathematicians in the country should be mainly directed towards necessary changes in the curricula and towards the training of better teachers.

3. University teaching in mathematics should: (a) answer the requirements of all those who need mathematics for practical purposes; (b) train specialists in the subject; (c) give to all students that intellectual and moral training which any University, worthy of the name, has the duty to impart.

These objects are not contradictory but complementary to each other. Thus, a training for practical purposes can be made to play the same part in mathematics as experiments play in physics or chemistry. Thus again, personal and independent thinking cannot be encouraged without at the same time fostering the spirit of research.

4. The study of mathematics, as well as of any other science, consists in the acquisition of useful reflexes and in that of independent habits of thought. The acquisition of useful reflexes should never be separated from the perception of their usefulness.

It follows that problem-solving should never be practised for its own sake; and particularly tricky problems must be excluded altogether. The purpose of problems is twofold; either to drill the student in the application of some method of special importance, or to develop his originality by guiding him along some new path. Drill is essentially a school-method, and ought to become unnecessary at the final stages of University teaching.

5. Rigor is to the mathematician what morality is to man. It does not consist in proving everything, but in maintaining a sharp distinction between what is assumed and what is proved, and in endeavoring to assume as little as possible at every stage.

The student should therefore be gradually accustomed, by means of startling examples, to question the truth of every unproved proposition, until at last he is able to deduce from the ordinary axioms everything that he has learned.

6. Knowledge of a proof means the understanding of its machinery and the ability to reconstruct it. This implies: (a) perfect correctness in the definitions; (b) a faculty of connecting a given question with the general ideas underlying it; (c) a perception of the logical nature of any proof.

The teacher should therefore always follow, not the quickest nor even the most elegant method, but the method which is related to the most general principles. He should also point out everywhere the relation between the various elements of the hypothesis and the conclusion; students must be accustomed to draw a sharp distinction between premises and conclusion, between necessary and sufficient conditions, between a theorem and its converse.

7. The teaching of mathematics must be a source of intellectual excitement. This can be achieved, at the higher stages, by taking the student to the brink of the unknown; at earlier stages, by making him solve for himself questions of theoretical or practical importance.

This is the method followed in the "seminars" of the German Universities, first organized by Jacobi a century ago, and even now the most prominent feature of the German system; division of labor between students in the study of a given group of questions is a common practice in these seminars, and proves to be a powerful incentive to work.

8. Theoretical lectures should neither be a reproduction of nor a comment upon any text-book, however satisfactory. The student's notebook should be his principal text-book.

In fact, taking down notes intelligently (not under dictation) and working them out carefully at home should be considered as an essential part of the student's work; and experience shows that it is not the least useful part of it.

9. The right of any topic to form part of any curriculum is to be tested according to: (a) its importance for modern mathematics or for the applications of mathematics to modern science or technique; (b) its relations with other branches of the curriculum; (c) the intrinsic difficulty of the ideas underlying it.

This involves a revision of the present curriculum. For instance, the idea of function, the process of differentiation and integration, should appear at an early stage, because of their enormous importance both for the theory and for the most ordinary practice. Because of its practical importance, numerical calculation, and all the devices connected with it, would seem to deserve a far more prominent place in elementary teaching than they receive at present.

MATHEMATICAL NOTES

EDITED BY F. A. FICKEN, University of Tennessee

Material for this department should be sent to F. A. Ficken, University of Tennessee, Knoxville 16, Tenn.

AN EXPRESSION FOR THE EULER ϕ-FUNCTION

N. C. SCHOLOMITI, University of Illinois, Chicago

Let a and x be non-negative integers, $x>0$. If we divide a by x we have:

$$a = x\cdot Q + r$$

where $Q=[a/x]$ is the integral quotient and r is the remainder in the division. Then:

$$r = a - x\cdot [a/x].$$

(The symbol [ ] will represent the bracket function throughout this article and not a parenthesis. By definition:

$[u]=$ the greatest integer not greater than u.)

For r we have the inequalities:

$$0 \leqq r = a - x\cdot [a/x] < x.$$

1. We construct the function:

$$G(a, x) = \left[\frac{1}{a - x\cdot [a/x] + 1}\right]. \tag{1}$$

Obviously $G(a, x)=1$ if, and only if, x divides a exactly; otherwise $G(a, x)=0$; a, x are non-negative integers, $x>0$.

2. Next we construct the function:

$$G(a, b, x) = \left[\frac{1}{a - a\cdot [a/x] + b - b\cdot [b/x] + 1}\right]; \tag{2}$$

a, b, x are non-negative integers, $x>0$. Comparing (2) with (1) it is easily seen that $G(a, b, x)=1$ if, and only if, x divides exactly both a and b. Otherwise