

INTRODUCTION AND CRITIQUE

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1. Sponsorship and Organization. The papers appearing here under a single cover were presented in the course of a somewhat novel type of symposium on special topics in applied mathematics which was organized by the Mathematics Division of the National Research Council and accommodated by the American Mathematical Society as an integral part of its regular meeting at Northwestern University, Evanston, Illinois, on November 27–28, 1953. The character of this symposium was largely set by the fact that its organization constituted one of the phases of a Survey of Training and Research in Applied Mathematics which the National Research Council has been conducting during the past year under provisions made by the National Science Foundation jointly with the three military Offices of Research in response to a recognized need for a comprehensive stock-taking of this country's resources in the way of mathematics and mathematicians likely to play a significant part in the future advances of our science and technology.

The arrangement of one or several conferences on the various phases of training and research in applied mathematics formed part of the plans for the Survey from the beginning. As subsequent work brought the picture better into focus, it became clear that the modern range and depth of the subject matter of applied mathematics was as yet insufficiently understood by many mathematicians. Today applied mathematics is emphatically no longer just the art of solving by dull methods repulsive differential equations whose coefficients carry physical or engineering names! Rather than merely pointing out the necessity of a systematic effort to dispel this deadly myth, the NRC Committee on Training and Research in Applied Mathematics which was in charge of the Survey decided to take at once an active step in this direction. Hence one of the planned conferences was set aside for this particular purpose and developed into the Symposium on Special Topics in Applied Mathematics here reported.

2. Aim and Impact. The Symposium was designed to present characteristic current research to a large audience of mathematicians, illustrating particularly active sectors of the front along which mathematics interacts today with other scientific disciplines. Its philosophy differed therefore markedly from the Applied Mathematics Symposia convened annually by the Mathematical Society or the younger series held under Office of Ordnance Research sponsorship. No attempt was made, as in the latter, to aim the program at the workers and specialists in some unified area, however broadly defined, and the interchange of information among them. Instead, the emphasis was on diversity of topics, on highlighting some exciting developments of the day and doing it in a manner to induce broad understanding among the mathematically literate. What we wanted was a collection of significant examples, not brought as yet too often or

accessibly to the attention of the mathematicians, which would show just how varied and substantial the contributions of the genuinely secular mathematician (as distinct from the monastic one) can be in this interaction of his modes of thought and analysis with scientific endeavors in other fields.

The Symposium was therefore planned in three 90-minute sessions of three half-hour papers each. The first of these sessions was to be concerned with new mathematical theories and techniques which are currently being fashioned for applied purposes; the second one was to be dedicated to the characteristic mixture of combinatorial analysis and probability theory now being developed for the sake of constructing analytic models and the numerical simulation of control and communication systems; the third one, finally, was to deal with situations where the interplay between physical ideas and mathematical construction is at present the critical element. This structure of the sessions is still discernible in the Symposium as it took place in the end, although an unusually large number of last minute changes beyond the planners' control compromised it to some extent. It proved helpful, however, in assuring the diversity and range which was in fact achieved.

The theory of partial differential equations, and especially of the physically relevant non-linear types, shows no signs of exhaustion as a primary source of important and stimulating problems which fascinate the mathematicians of applied interests. There are the basic questions revolving around the idea of what constitutes a well-formulated problem in this area which J. Leray touches in his paper and which have naturally led to the viewpoint, pioneered by Leray and others, of looking at solutions not individually but as families with topological and algebraic structure, capable of being supplemented by ideal elements, *etc.* There are the powerful function theoretical arguments which are being developed for the construction and analysis of important particular solutions, here represented by the methods with which P. Garabedian and his co-workers have recently attacked hydrodynamic flows possessing free surfaces. There are finally the fascinating problems posed by the nature of the singularities which can arise in the solution of these non-linear systems and the structure of the loci on which they can occur: A. H. Taub discusses such a problem using methods of differential geometry to analyze the configuration of interacting shock fronts in compressible flow. All of these are vital areas of substantial mathematical research today.

Ultimately destined, perhaps, to be of comparable importance in its mutually stimulating interaction with modern analysis is the area of signal and noise problems, illustrated by those presented in the paper of M. Kac. Here again the quest for analytic and measure-theoretical information on certain families of functions, containing individually observed time series as members, leads naturally to problems of substantial interest also to current front-line thinking in pure analysis. Another fertile field for mathematical ingenuity in applied contexts is furnished by what, in effect, are elementary problems but which are posed with

such a massive complexity of detail as to render the solution totally inaccessible to the traditional class-room approach. S. Chandrasekhar's discussion of eigenvalue problems of high order furnishes a good example how such an obstacle can be overcome by an ingenious mathematical trick, perhaps even, as in this case, without knowing in a rigorous way what accounts for the success of the trick.

Probably the least familiar aspect of the "new look" in applied mathematics is the steadily growing interaction between the combinatorial and algebraic mathematics of discrete structures on the one hand, and the use and operation of large-scale digital computers on the other. The use of Boolean algebra in the analysis of switching circuits is of course no longer new, but the idea of carrying forward basic research in Boolean algebras, as reported in the sequel by D. E. Muller, with the hope of anticipating the future needs of switching circuit design rather than being merely concerned with supplying the present ones, and to make use, moreover, of high-speed computing equipment in doing it—this gives evidence of the substantial development of a major new province of applied mathematics. In the same general area lies the work of Project SCAMP at the Institute for Numerical Analysis, whose purpose is that of exploring the use of high-speed automatic computers in the analysis of discrete structures. Some of it has been reported for the first time in any completeness at the Symposium by S. S. Cairns. Since summer after summer a sizable group of our ablest mathematicians participate in this work, it cannot fail in the long run to have a noticeable impact on American mathematical research.

Another important development, once more in the traditional vein of applied mathematics, is illustrated by the investigations, summarized by E. Montroll, concerning the energy distribution over the frequencies at which a molecular crystal lattice is capable of oscillating. It turns out that this analysis depends in an essential way on the relations known to exist between the topological structure of a closed manifold, here a certain kind of phase space, and the analytic properties of functions and integrals defined thereon. These problems are among the most intensively investigated ones in modern analysis, and if they should furnish the appropriate terms for analyzing significant aspects of current theories of molecular physics, we might be faced with their proliferous growth similar to the development of Hilbert space methods under the impact of quantum mechanics.

Everyone who scans this list will miss certain topics which he would consider as particularly interesting or important for the development of modern applied mathematics. Let him be assured that but few, if any, of these omissions were inadvertent. It is perhaps indicative of how much needs doing along these lines, that a first attempt could achieve as much and still leave so much untouched. Probably most glaring is the absence of a report, aimed at mathematicians, on the current problems and difficulties faced by quantum mechanical theories of fields. All endeavors of the Committee to provide such a report were unsuccessful.

ful, and in hindsight this may have been all to the good, for the matter is serious enough to warrant a special effort.

3. Experimental Features. In arranging for a novel type of meeting, some innovations as regards organization are inevitable. The Symposium on Special Topics in Applied Mathematics introduced two such departures from standard practice. For one, the sessions of the Symposium were interspersed among the regular sessions of the Mathematical Society meeting, rather than grouped into a separate conference preceding or following the former and thus too easily avoided by the non-specialist. The example which was followed here is that of the American Physical Society whose meetings provide for a sizable number of invited half-hour papers, mixed in among the contributed papers either singly or in groups. Thus, the speakers were permitted some expository breadth in order to bring those of their colleagues not immediately concerned with the subject on hand up to date. The second innovation consisted in inviting representatives of other scientific fields to speak to mathematicians. The ones selected for this purpose naturally have extensive contact with mathematics in their daily scientific lives, yet they do not think of themselves as mathematicians. It is from such persons that a mathematician will generally get his most stimulating insight into the mathematical problems which arise in other scientific disciplines. Both of these experiments proved successful at the Symposium under discussion and their adoption, also in other than applied mathematical contexts, is recommended.