The authors have indicated on the cover of the book that it is "a book of experimental text materials." Trial and experiment will determine how much of the material can be effectively incorporated into courses designed for a large number of beginning students. The formal exposition, which is in a fairly concise mathematical style, will probably require, for the most part, more intellectual effort and innate mathematical ability than can reasonably be expected of most beginning students. But under the guidance of a experienced teacher the more able and ambitious students can study much of this material profitably. A larger number will be able to profit from the more discursive intuitive treatment upon which most of the class work is supposed to be based. This treatment makes use of diagrams, figures, and good illustrative examples wherever these aids are helpful. Both the formal and intuitive treatments include adequate lists of well chosen exercises. In the opinion of the reviewer the book would be easier to read if the formal parts were placed in appendices at the ends of the various chapters instead of being interspersed with the intuitive parts.

In spite of the general excellence of the exposition the reviewer believes that even the intuitive treatment will be difficult for most students to follow and that the instructor will not always find it easy to present the material effectively. But, as Professor Alfred Whitehead once wrote: "Whenever a textbook is written of real educational worth, you may be quite certain that some reviewer will say that it will be too difficult to teach from it. Of course it will be difficult to teach from it. If it were easy the book ought to be burned; for it cannot be educational."

Whether or not he expects to experiment with a course organized along the lines of this book the teacher of mathematics will do well to examine the book carefully. Its treatment of classical subject matter from a fresh and modern viewpoint will give him a deeper insight into his subject which cannot fail to have an impact upon his teaching.

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TULANE EXPERIENCE WITH UNIVERSAL MATHEMATICS, PART I

The University of Kansas and the Social Science Research Council supported in 1954 a Kansas Summer Writing Group which wrote a book of experimental text materials called: *Universal Mathematics, Part I*, which was an effort to put into concrete form some of the results of the studies of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. This hastily written preliminary edition was given a mass trial at Tulane University as a universal mathematics course for all first year students who do not have to take intermediate algebra. Some 750 students in engineering, liberal arts, and business administration were involved. About 28 instructors, ranging from graduate teaching assistants to full professors, took part in the trial. Recently, a staff meeting was held to discuss the results.

The book is not yet suitable as a textbook and caused considerable difficulty to students and instructors. The main trouble is that students cannot read it. However, the chairman's proposal to replace it with one of the existing mathematical analysis texts met with unanimous opposition. Most of the staff wanted radical revisions of the book. Moreover, the failure rates using this text were higher than normal, although the conclusion that this was due entirely to the text is unwarranted since the quality of freshman classes was poorer than usual.

One thing is clear; and that is the fact that *Universal Mathematics* is not adaptable to students whose high school background in mathematics is scant. It was not intended to be "universal" in the sense that everybody should take this course at the beginning of the first college year, but universal in the sense that is for everybody who continues mathematics after acquiring competence in intermediate algebra and geometry. Thus some students are ready for *Universal Mathematics* while still in school and others are not ready for it when they enter college. It was written to what the Committee judged to be realistic prerequisites for a beginning first year college student whose high school training properly qualifies him for the traditional first college courses in mathematics.

In judging the value of *Universal Mathematics, Part I*, it should be emphasized that it was not intended to be a textbook in the proper sense. For one thing, it includes entirely too much material, too many different ideas, for a textbook. A proper textbook should be much simpler. This book was intended as a kind of mother of textbooks. Another aspect of it, which cannot be overlooked, is the unusual method by which it was uncompromisingly written. The book was written with two relatively complete and independent presentations. First a formal account was written to establish the mathematical structure of the work. After the formal account, with its postulates, definitions, theorems, and proofs was complete, an intuitive presentation was written to follow it closely in the same order. The two accounts appear side by side in the book. The intuitive account is used in the classroom, with the formal one as a reference. This method has several advantages and disadvantages. With the complete mathematical theory established and made dominant by principle, the textbook writer's usual freedom to choose his material and to choose the order of presentation of his topics is severely limited. It requires much more imagination to find appropriate motivations and interpretations for young students if mathematical theory determines the formulations of concepts and their order of introduction, while serious teaching purpose dictates that the concepts shall be well motivated and supported by meaningful exercises so as to become part of the student's working mental equipment. However, it turns out that if the really general and abstractly simple form of the theoretical ideas is chosen then the interpretation becomes surprisingly easy and rich.

Thus the advantages of the dual presentation, mathematical and intuitive, lie in the fact that every intuitive or incomplete exposition will definitely permit expansion and elaboration in a correct formal theory which has been set down in axioms. Moreover the "hard" theory is out of the way so that the "easier" intuitive account can be read by itself, with the student and teacher, instead of the textbook writer, determining when they shall turn to the more austere formal theory; yet the formal theory fits with the intuitive account both in notation and structure so that this desirable transition is facilitated. The Tulane experience indicates that this was realized in practice. A disadvantage of the method is that the first stage of intuitive simplification here presented is still too formal. It needs at least two more stages of simplification and many more exercises. Another aspect of the same difficulty is concerned with the compromise between a correct formal mathematical language and the common classroom jargon of elementary mathematics. This method of writing does not facilitate the making of such a compromise and results in difficult reading even on the intuitive side.

The excess of ideas had interesting effects. The early introduction of relations and vector concepts which was a kind of side issue seemed to appeal to many instructors and students, though others did not like it. In general, one class would take hold of one set of ideas, and another class a different one. Some instructors found the Introduction very stimulating and used it as a part of the text material. The radical method of presenting limits was well liked by some and disliked by other instructors. One set of ideas seems to have been almost universally disliked, and that was the treatment of the divided difference as an interval of numbers. The chapter on scientific measurement was omitted to shorten the course. The fact that the book presents more ideas than is considered desirable caused unfavorable first reactions on teachers. Mathematics textbooks are required by convention to be as empty as possible of ideas, certainly empty in comparison with a textbook in chemistry, such as that of Pauling, or a first college textbook in physics. The Tulane trial seems to indicate that, although an idea-packed book requires that the teacher help the students to read the book, it turns out to be less boring to him in the end and also more interesting to the better students, without causing excessive trouble for the slow student. Why don't we use textbooks in mathematics which have a high density of ideas? What could it hurt?

Some of the defects of the book as a textbook could be obviously remedied by some worked-out examples, some answers to the problems, and more exercises. The method of paging two parallel accounts could be much improved, though it is more inconvenient to a casual reader than to a student who studies the book.

One of the Committee's general efforts seems to have been fairly well realized. This was an effort to rogue out unessential prerequisites to the study of calculus. An important one of these was the reexamination of the relation of the Euclidean analytic geometry to the calculus. The *Universal Mathematics, Part I* was written with a consistent "graphs" geometry, which is the geometry of the Cartesian product of the real line and the real line without the imposition of the invariant Euclidean metric in the plane. Perhaps, teachers will feel that this is a doctrinaire position, but it seems in trial to make things actually easier. The modified Menger notation scheme seems to cause no particular difficulty if one does not attempt to explain it, and it may make teachers feel more honest.

There is still a serious difficulty in getting through the foundation of the elementary calculus. The method of writing so far has resulted in an excessive dwelling upon the real numbers system in the beginning although it was the intention of the authors to find a correct but brief foundation for calculus. A simpler start must be found. But despite the over-formality of this writing, the calculus theory presented does represent a real simplification.

At Tulane, as everywhere else, when the students actually get to elementary calculus they find it interesting and comparatively easy. In short, the Tulane experience indicates that elementary calculus is more appropriate for beginning college students than college algebra, or trigonometry, or analytic geometry. The morale of the students is better, and the technical difficulties with it are less than in the conventional subjects. Moreover, there is still reason to hope that students will learn more algebra by studying a well simplified calculus than they will in a course in algebra.

The treatment of the exponential and logarithmic functions before the trigonometric function worked out well. The fact that the calculus was imbedded in the "graphs" geometry dictates a postponement of the study of the trigonometric functions until one returns to Euclidean geometry. This analytic trigonometry can be postponed to the beginning of the second year calculus without causing any difficulty in most programs.

Thus *Universal Mathematics, Part I*, belongs to a growing trend in first year college mathematics to start with calculus as soon as possible and on as simple a foundation as possible. Whether one continues with more calculus and geometry in the second semester or whether one turns to sets and integers, as *Universal Mathematics, Part II*, proposes to do, is a matter to be determined by what is the most valuable in actual experiences, rather than by opinion. The Committee is preparing *Part II* which is formally sets and integers and intuitively "choice and chance" to make a comparison by experience possible.

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Differential and Integral Calculus. By H. M. Bacon. Second Edition. McGraw-Hill Book Company, New York, 1955. vii+547 pages. $6.00.

Calculus. By William L. Hart. D. C. Heath & Company, Boston, 1955. xiii+626 pages. $5.50.

The first edition of Bacon's book was published in 1942 but apparently was not reviewed in this Monthly at that time. The author states that several sections have been rewritten for the second edition but he does not claim to have added new material or to have changed the order of topics. This book is typical