CONFERENCE ON THE COMMITTEE ON THE UNDERGRADUATE PROGRAM

A conference on the work of the Association's Committee on the Undergraduate Program in Mathematics was held at the Burlington Hotel, Washington, D. C., on November 15 and 16, 1958.

The following forty-seven persons attended the conference:

Max Beberman	UICSM Mathematics Project, University of Illinois
E. G. Begle	Yale University, School Mathematics Study Group
R. H. Bing	University of Wisconsin
R. C. Buck	Stanford University
R. R. Bush	University of Pennsylvania. (American Psychological Association)
E. A. Cameron	University of North Carolina
J. W. Cell	North Carolina State College. (American Society for Engineering Education)
L. W. Cohen	University of Maryland
R. P. Dilworth	California Institute of Technology
W. L. Duren, Jr.	University of Virginia
F. A. Ficken	University of Tennessee
R. C. Fisher	Ohio State University
G. E. Forsythe	Stanford University
H. M. Gehman	University of Buffalo
A. M. Gleason	Harvard University
W. T. Guy, Jr.	University of Texas
P. G. Hoel	University of California, Los Angeles
R. D. James	University of British Columbia
J. L. Kelley	University of California, Berkeley
J. G. Kemeny	Dartmouth College
C. B. Lindquist	U. S. Office of Education
W. G. Madow	Stanford Research Institute
J. R. Mayor	American Association for the Advancement of Science
E. J. McShane	University of Virginia
L. J. Montzingo	University of Buffalo
Ivan Niven	University of Oregon
E. P. Northrop	University of Chicago
F. G. O'Brien	National Science Foundation
R. E. Paulson	National Science Foundation
G. B. Price	University of Kansas
A. L. Putnam	University of Chicago
Mina Rees	Hunter College
P. C. Rosenbloom	University of Minnesota
A. E. Ross	University of Notre Dame
R. E. K. Rourke	Commission on Mathematics
Patrick Suppes	Stanford University
H. W. Syer	Kent School
G. B. Thomas, Jr.	Massachusetts Institute of Technology
D. L. Thomsen, Jr.	Watson Scientific Laboratory, Columbia University. (Society for Industrial and Applied Mathematics)
A. W. Tucker	Princeton University
F. E. Ulrich	Rice Institute
R. J. Walker	Cornell University

A. D. Wallace	Tulane University
S. S. Wilks	Princeton University
C. R. Wylie, Jr.	University of Utah
J. W. T. Youngs	Indiana University
Mark Zemansky	City College of New York. (American Institute of Physics, and American Association of Physics Teachers)

Speakers were invited to discuss the various topics on the program, and in most cases outlines were prepared and distributed to the participants in advance of the Conference. Summaries of the prepared talks and of some of the discussion are given herewith.

Price: Purpose of the Conference. The purpose of the Conference was to assist in formulating plans and policies for the future work of the Committee on the Undergraduate Program. This Committee, formed early in 1953, proposed in 1958 that the time had come to reorganize and expand its efforts. Accordingly, at the Committee's request, it was discharged at the end of August, 1958. This Conference was called to re-examine the assignment of the Committee on the Undergraduate Program and to take steps to establish a new Committee with adequate funds, personnel, and program.

Duren: Report from the original Committee on the Undergraduate Program. The speaker reviewed the goals and activities of the original CUP since its appointment in January 1953. The primary purpose of the CUP was to act as a bridge between research and curriculum. The effort of the CUP was focused on the first year of college. No attempt was made to write text books, but source books were written. Among the primary goals of the CUP were:

1. To interest young mathematicians in teaching problems, and to encourage senior mathematicians to become more aware of the needs of teaching; to point out the importance of finding a balance between critical mathematics, and the desires and motivations of 18-year olds;
2. To contribute to institute and lecture programs;
3. To establish contact with other groups with purposes similar to those of the CUP.

No specific curriculum suggestions were made until it was known that the original CUP was to be discharged.*

The speaker suggested that in the reorganization of the CUP some thought be given to reorganizing into groups, or one large group subdivided into subgroups by locality.

Rourke: Report from the Commission on Mathematics. The speaker drew attention to the books and pamphlets that have been written and distributed by the Commission, and to the conferences that have been arranged to discuss the work of the Commission. Out of these conferences there has emerged a clarification of the proposals of the Commission, especially on the following particular points:

(1) The role of set theory in the high school curriculum;
(2) the continuing role of skills;
(3) the increasing emphasis on structure as contrasted with manipulation in algebra;
(4) the use of the word "modern."

The speaker outlined the following nine point program of the Commission for the class of college-capable students in the secondary schools:

1. Strong preparation both in concepts *and* in skills, for college mathematics at the level of calculus and analytic geometry.

* Anyone interested in the reports issued by this committee may obtain the following by writing to the Buffalo office of the Association: (1) Collected Reports of the CUP (1957) and (2) Outline of Recommended Courses (1958).

2. Understanding of the nature and role of deductive reasoning—in algebra, as well as in geometry.
3. Appreciation of mathematical structure ("patterns")—for example, properties of natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas—set, variable, function, and relation.
5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some coordinate geometry, and also essentials of solid geometry and space perception.
7. Introduction in grade eleven of fundamental trigonometry—centered on coordinates, vectors, and complex numbers.
8. Emphasis in grade twelve on elementary functions (polynomial, exponential, circular).
9. Recommendation of additional alternative units for grade twelve: *either* introductory probability with statistical applications *or* an introduction to modern algebra.

Finally, the speaker emphasized the need of appropriate teacher training programs, for prospective teachers as well as those in service. He cited cases of teachers attending university summer sessions in the hope of building up their knowledge of newer viewpoints in mathematics, only to get traditional courses of a sort that have almost disappeared from many university programs.

Discussion: The question was raised as to the desirability of teaching polynomial calculus in secondary school, and Mr. Rourke pointed to the critical shortage of qualified teachers. The problem of the certification of teachers was discussed; it was pointed out that the Commission has no authority here, but can only make recommendations. There was considerable feeling that the Mathematical Association of America should likewise formulate a set of strong recommendations.

Beberman: Report on the University of Illinois Committee on School Mathematics. The speaker stated that the aim of the UICSM is to develop a four-year program of high school mathematics that develops understanding as well as manipulative skills. The program is aimed primarily at the college-capable student. At present there are about 55 schools in 20 states using the UICSM materials as pilot schools.

The units and grade level which comprise the UICSM program are:

Unit	*Grade Level*	*Descriptive Title*
1	9th Grade	Arithmetic of real numbers.
2	9th Grade	Pronumerals, generalizations, manipulation.
3	9th Grade	Equations, inequations, applications.
4	9th Grade	Ordered pairs, graphs.
5	10th Grade	Relations, functions.
6	10th Grade	Geometry.
7	11th Grade	Real number system, induction.
8	11th Grade	Exponents, logarithms.
9	11th Grade	Complex numbers, systems of quadratics.
10	11th Grade	Polynomial functions, theory of equations.
11	12th Grade	Circular functions, trigonometry.
12	12th Grade	Postulational systems.
13	12th Grade	Analytic Geometry.

The discussion that followed centered around the question of finding a middle ground between a whole deductive system and none, "rigor" versus "intuitively obvious."

Begle: Report from the School Mathematics Study Group. The aim of the School Mathematics Study Group (SMSG) is to improve the amount and quality of mathematical

training in the secondary and elementary schools. It is attempting to make mathematics courses in the schools more interesting so that more students are attracted, to improve the curriculum by working for better mathematics, and to help teachers prepare themselves to instruct these improved courses. SMSG is financed by the National Science Foundation. It hopes to work toward its goals by a joint effort of representatives from all parts of the mathematical profession.

There are three projects in progress. The first is the production of sample textbooks for grades 9 through 12. The second is the production and testing of several experimental units of instruction for the 7th and 8th grades; these units are being tested in a large number of classrooms in all parts of the country. Both these projects will involve writing sessions in the summer of 1959. The third project under way is the production of a series of monographs designed for the better students in high schools. These monographs are intended to supplement the high school program by showing something of the scope and interest of mathematics in our culture.

There are two projects being organized, one on films and television as teaching aids, and another on the production of teacher training materials. In addition there are various projects under consideration, such as a study of the mathematics of elementary schools, and a study of such topics as concept formation and attitudes towards mathematics. This latter study would enlist the aid of psychologists and other social scientists.

Kemeny: Courses for Teacher Training. The speaker outlined a proposed mathematics requirement for high school mathematics teacher training programs that consisted of 36 semester hours of undergraduate mathematics courses. During the first two years the student would carry one course each semester and complete the work that is roughly equivalent to Universal Mathematics I and II, and a course in the calculus of $y=f(x)$. During the junior and senior years the student would carry two courses each semester in order to complete four one-year units specially designed for teachers. These four units would consist of:

(a) One unit in modern algebra: half of it in groups, *etc.*; and half in linear algebra.
(b) One unit in geometry: substantial work in analytic geometry with introductions to projective and non-Euclidean geometrics.
(c) One unit in probability and statistics: half of it a course in probability theory, half of it a course in statistical inference.
(d) One unit in the history of mathematics: half of it to cover the period from Euclid to non-Euclidean geometry, half to cover the last 120 years, with special emphasis on topics not covered in other courses.

The speaker went on record as being in favor of a statement by the MAA concerning minimum requirements for training of high school mathematics teachers.

The discussion that followed centered about three questions:

1. Are the standards set forth by the speaker realistic?
2. Can we give students better training in mathematics and expect them to remain in the high school teaching profession?
3. Should we not be more vitally concerned with the teachers in the high school teacher training programs?

Zemansky: The Relation of Mathematics to Physics Instruction. The speaker felt that it would be of considerable value to get a short article into one of the physics journals on the trends in mathematical instruction which had been presented by previous speakers. He urged that calculus be taught early in the college program so that students would have the subject available for use in their study of physics. The speaker questioned the emphasis on understanding and insight, not for the physics students, but for the pre-engineering students, on the grounds that there was a great deal of manipulative technique that the students needed.

Discussion: Several persons present responded to this last point of the speaker by giving evidence of the need for understanding in mathematics by engineering students.

Madow: The Mathematical Training of Social Scientists. The speaker reported an increased interest among social scientists concerning the mathematical training of their students, but that many departments in the social science areas could increase the use of mathematics in their own courses. It was pointed out that the ordinary use of mathematics in the social sciences is to construct theories rather than to work out problems. The speaker also noted that the major use of mathematics is the use of probability and statistics and hence it is not so important that the average social science student get his calculus early. Since it is important that physical scientists get calculus early, the speaker asked that the CUP reconsider the proposal of the original CUP concerning a common first year mathematics course for all students.

Hoel: Undergraduate Statistics in a Mathematics Department. The speaker addressed his remarks to those situations where a college or university has neither a department of statistics nor a collection of statisticians in a mathematics department. First, an elementary service course in statistics should be given in a mathematics department only if the department has someone trained in statistics and the department is much better equipped than other departments to give it. A year course in statistics without mathematical prerequisites is more useful than a one semester course requiring a semester of mathematics first.

Next, in the case of a statistics course with a calculus prerequisite, a knowledge of integration should be insisted upon since it is inefficient to teach such a course with lower prerequisites. Integral calculus is the major tool in beginning statistical theory.

The speaker also commented on universal mathematics courses, and urged that statistics be not included as a topic in such programs. The reasons for this are that it would be a duplication for those who will be taking a statistics course, and for the others the time available is inadequate to do justice to a good explanation of statistical theory.

Regarding the undergraduate program in algebra in its relation to statistics, the speaker urged the replacement of the traditional theory of equations course by an elementary course in linear algebra with a strong orientation towards geometry.

Forsythe: The Role of Numerical Analysis in an Undergraduate Program. The speaker pointed out that most of our mathematics students major in other fields and are primarily interested in the applications of mathematics, and not in its structure. He also pointed out that about half of those who attain a Ph.D. in mathematics go into industry and apply mathematics in various areas, and that almost all of them will be connected in some degree with automatic computation. The speaker noted that there is now a strong demand for the A.B. mathematician, but that his training has not made him a good practitioner of mathematics. The speaker suggested that the aims of an undergraduate mathematics education are the following:

a. To learn as much as possible about the structure of mathematics.
b. To learn to read independently the mathematical literature at his level.
c. To know the tools of mathematics, books and machines, and how and where to find them.
d. To cultivate and practice solution of mathematical problems new to him.
e. To go fairly deeply into some other field of knowledge where mathematics is used.
f. To learn to enjoy mathematical study.

The speaker criticized much of mathematical education as not intuitive enough nor well enough illustrated, and stated his agreement with the words of Felix Klein who said, "The living thing in mathematics, its most important stimulus, its effectiveness in all directions, depends entirely upon the applications."

The speaker stated his belief that an increased role must be given to numerical

analysis in the undergraduate program, and that for the most part this work should be mixed into undergraduate courses rather than to have a great many separate courses in numerical analysis. The speaker also favored a special coding course for all students, to be taken early in their studies.

The discussion that followed brought out the need to distinguish carefully between a vocational point of view and the point of view of illustrating the fundamental ideas in mathematics by computing techniques. It was noted that even a coding course gives training in precise thought since it forces a student to think in terms of exactly what he is doing, since he must write a complete set of directions.

Cohen: Report on Films and Television for Mathematical Instruction. The speaker pointed out that films and television as a means of mathematical instruction have some, but not all, of the properties of books and teachers; consequently their use is not to be undertaken with the object of replacing either of the older aids to learning. At the October 18 meeting of the MAA's Committee on Production of Films, it was decided

1. To produce motion picture films of three hour lectures.
2. To aim the instruction at highly competent students.
3. To make one film spanning the competence of high school seniors and college freshmen.

The topics and level under consideration are:

1. Mathematical Induction (H. S. Senior—College Freshmen)
2. Theory of Limits or Integration (Sophomore or Junior Undergraduates)
3. Topology (Undergraduate Math. Club)

Correspondence is under way to determine a lecturer for each topic. Questions relating to distribution, royalties and property rights in the films to be produced were recognized as requiring answers. These answers will be sought in due course. The cost of the program will be met through a grant which the Association has obtained from the National Science Foundation.

1. The preliminary script should be a draft of the lecture prepared by the lecturer.
2. The director and lecturer then collaborate in preparation of the script based on the draft lecture.
3. The director, lecturer and committee meet to review the script and production before the making of the film.
4. The committee should be present at the making of the first film.
5. There should be a mathematician present during the making of each film to catch slips overlooked by the lecturer and director.
6. Short, inexpensive tests for image and voice should be made as an aid to the choice of a lecturer.
7. Animation may be introduced into the films as an aid to clarity in blackboard writing and diagramming.

In the long discussions that followed the report many representatives reported experimental use of television and of the large lecture system. There seemed to be general agreement that there is student acceptance of the large lecture system, and it is preferred at this time to pure television courses. A question was raised regarding the merits of such instruction for the gifted student. It was generally agreed that special instruction should be provided for the "top 1%," but there was not general agreement as to whether or not the new CUP should concern itself with such special instructional programs.

Begle: Discussion of writing sessions. The speaker described the different kinds of writing programs which had been operated: small writing groups as contrasted with large ones, and both of these compared with scattered efforts by individuals.

Kemeny: Financial Arrangements for Authors. The speaker cited the experience of the Dartmouth writing group, which did not receive royalties from its published works. It was felt that in the future such limitations should be removed.

Discussion: There was a considerable response to this matter of financial arrangements. The basic problem is the granting of royalty rights to writers who have been paid for their writing efforts. It was felt that if an author was paid to write a book and in addition received full royalties, there might be criticism from the free-lance mathematical writers. This criticism could be met by subtracting the initial payment to the writer from subsequent royalty payments. Many preferred to avoid any such subtraction, in order to provide adequate economic motivation for high calibre persons in writing projects.

Discussion: The Nature of Books to be Written. The question discussed was the desirability of source-books vs. textbooks. Since source-books are written too compactly for classroom use, their role is to serve as materials for use by writers of texts. The experience of some writers of source-books is to say "never again!," but it was urged that some compromise stage between source-books and textbooks be sought.

RESOLUTIONS ADOPTED BY THE CUP CONFERENCE

Since the Committee on the Undergraduate Program is charged with responsibility for advising the Mathematical Association of America on all matters connected with the undergraduate program in mathematics, this Conference hereby adopts the following recommendations concerning certain special aspects of the work of the Committee:

1. This Conference recommends that the Committee on the Undergraduate Program in Mathematics consist of from ten to fifteen persons and that the Committee have power to delegate its various activities to subcommittees whose members need not be members of the Committee.

2. This Conference recommends that the Committee on the Undergraduate Program be provided with adequate facilities and staff, free of other responsibilities, sufficient to carry out the tasks assigned to it.

3. This Conference recommends that the Committee on the Undergraduate Program continue to be concerned with the development of undergraduate courses in mathematics and with the production of adequate materials and methods for these courses, keeping in mind the varying needs of students destined for the many careers which make use of mathematics, and particularly the needs of prospective high school mathematics teachers.

4. This Conference recommends that the Committee on the Undergraduate Program continue to plan its activities with due regard for other groups concerned with similar problems.

5. Since the encouragement of writing of adequate mathematical textbooks is a primary problem facing the Committee on the Undergraduate Program, this Conference recommends to the Board of Governors of the Association and to the Committee that the Committee develop a program to subsidize the preparation of textbooks and monographs without prejudice to the royalty rights of the authors.

6. This Conference recommends that, upon the recommendation of the Committee on the Undergraduate Program, the Mathematical Association of America publish a statement of minimal standards for teachers of mathematics in high schools, junior colleges, and colleges. The review and revision of such standards should be a continuing responsibility of the Committee and the Committee should recommend to the Association future changes in such standards. The existence of such statements of minimal

standards should be given wide publicity through the Sections of the Association to all interested state and local groups.

7. The Conference recommends that the Committee on the Undergraduate Program make recommendations to the Association regarding the desirable mathematical preparation of students expecting to pursue graduate work in mathematics or expecting to pursue mathematical careers in industry.

8. Since the vastly increased demands upon mathematicians and upon teachers of mathematics in the undergraduate colleges of this country have put upon the Mathematical Association of America the obligation of providing professional leadership in the design of appropriate courses to satisfy these demands, and since the obligations of the Association in this field have been delegated to the Committee on the Undergraduate Program, and since this delegation implies a vastly expanded program of activity as described in these resolutions, therefore this Conference recommends that the officers and the Board of Governors of the Association communicate to the appropriate agencies the compelling need for grants of funds commensurate with the magnitude of the task and with its importance for the development of mathematics and science in this country.

Concluding note. A Committee on the Undergraduate Program in Mathematics is now being appointed. Members of this committee met in New York City on December 29 and 30, 1958 to plan its future activities in view of the recommendations of the Conference.

R. C. Fisher and Ivan Niven, *Recorders*
Harry M. Gehman, *Secretary-Treasurer*

MATHEMATICAL NOTES

Edited by Roy Dubisch, Fresno State College

Material for this department should be sent to Roy Dubisch, Department of Mathematics, Fresno State College, Fresno 26, California.

NOTE ON HYPERGEOMETRIC POLYNOMIALS

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1. In a recent paper [1], V. R. Thiruvenkatachar and T. S. Nanjundiah have proved the following nonlinear recurrence relation for ultraspherical polynomials:

$$(1.1)\quad (1-x^2)D_n^{\lambda}(x) = n(n+2\lambda)\left[P_n^{\lambda}(x)\right]^2 - (n+1)(n+2\lambda-1)P_{n-1}^{\lambda}(x)P_{n+1}^{\lambda}(x),$$

where

$$D_n^{\lambda}(x) \equiv \left[DP_n^{\lambda}(x)\right]^2 - \left[DP_{n-1}^{\lambda}(x)\right]\left[DP_{n+1}^{\lambda}(x)\right].$$