

MATHEMATICAL EDUCATION NOTES

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CUPM REPORT ON THE TRAINING OF TEACHERS OF ELEMENTARY SCHOOL MATHEMATICS

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The Committee on the Undergraduate Program in Mathematics (CUPM) is a committee of the Mathematical Association of America. Operating under a grant from the National Science Foundation, it is charged with making studies of and recommending improvements in undergraduate mathematics curricula, thereby reflecting the basic concern of the Association for improving the content and teaching of college mathematics. Moreover, the Committee devotes its energies to reasonable efforts in attempting to effect the realization of its recommendations. A major portion of the work of CUPM is carried on by four Panels, one of which is the Panel on Teacher Training.

The Panel on Teacher Training is concerned with the improvement of the mathematical education of future teachers of mathematics. It is an action group which recommends revisions of mathematics teacher education curricula and works toward the realization of such revisions. Further, it attempts to assist in the development of improved programs for the college mathematics education of those who plan to teach mathematics at any level.

Thus far, the Panel on Teacher Training has produced Recommendations on the Training of Teachers of Mathematics. These Recommendations suggest for each of several levels the type and amount of mathematics which should be required of any teacher of the subject. At the same time, the Recommendations outline briefly the courses which are suggested. The courses recommended are described more fully, with appropriate bibliographical details, in the Panel's "Course Guides for the Training of Teachers of Junior High and High School Mathematics," and in "Course Guides for the Training of Teachers of Elementary School Mathematics."

In cooperation with the National Association of State Directors of Teacher Education and Certification (NASDTEC) the Panel on Teacher Training held five regional conferences during the spring of 1961, for the purpose of discussing the Panel's Recommendations for the mathematical portion of secondary education programs and problems associated with the realization of their Recommendations. A report of these conferences is available.

For the purposes of discussing the problems associated with establishing the curriculum suggested for elementary school teachers (Level I), the Panel held a series of ten conferences during the autumn of 1962. These conferences covered

the following states: Oklahoma, Florida, Michigan, Maine, Vermont, New Hampshire, Illinois, Minnesota, New York, Tennessee, Texas, and California.

The purpose of this article is to outline the recommendations for elementary school teachers (Level I), report upon the current requirements in mathematics for prospective elementary school teachers in colleges and universities, and to present a summary and conclusions of the ten Level I Conferences of 1962.

CUPM Recommendations for Level I. (Teachers of Elementary School Mathematics): This level consists of teachers confronted with the problem of presenting the elements of arithmetic and the associated material now commonly taught in grades K through 6. The Panel recognizes that special pedagogical problems may be connected with teaching in grades K through 6, and that a special program may be appropriate for the teachers of these grades.

As a prerequisite for the college training of elementary school teachers, CUPM recommends at least two years of college preparatory mathematics, consisting of a year of algebra and a year of geometry, or the same material in integrated courses, and that these teachers be competent in the basic techniques of arithmetic. The exact length of the training program will depend on the strength of their high school preparation. For their college training, the equivalent of the following courses is recommended:

(A) A two-course sequence devoted to the structure of the real number system and its subsystems.

(B) A course devoted to the basic concepts of algebra.

(C) A course in informal geometry.

The material in these courses might duplicate topics studied in high school by the prospective teacher, but it is urged that these topics be included again, this time from a more sophisticated college-level point of view.

Whether the material suggested in (A) above can be covered in one or two courses will clearly depend upon the previous preparation of the student.

It is strongly recommended that at least 20 per cent of the Level I teachers in each school have stronger preparation in mathematics, comparable to Level II preparation but not necessarily including calculus. Such teachers would clearly strengthen the programs of elementary schools by their very presence on the school faculty. This additional preparation is certainly necessary for elementary school teachers who teach an introduction to algebra or geometry.

Current requirements. During the academic year 1962–63, the Central Office of CUPM conducted a study of requirements and offerings of mathematics in the preservice education programs for teachers in the elementary schools. Programs in elementary education are offered in 906 colleges and universities. Of the 906 schools studied, 762 or 84.1% submitted usable responses. The results indicate that 22.4% of the respondents require no mathematics of prospective elementary school teachers, and 68.9% require the equivalent of four or fewer semester hours of mathematics. A table of the requirements reported, in terms of semester hours, is given as follows:

Requirements in Preservice Programs for Elementary Teachers

Number of hours required	Number of colleges requiring mathematics	Number of Colleges requiring methods
0	171	194
1-2	46	272
3-4	308	271
5-6	200	21
7-8	17	0
9-10	10	0
11-12	7	2
13+	3	2
Total	762	762

Especially significant is the fact that of the colleges reporting 55.6% offer no mathematics courses specifically designed for prospective elementary school teachers. In many of the institutions, courses in elementary algebra or elementary trigonometry are the only college mathematics courses available to those students.

Two hundred and seventy-six schools reported that they were presently in the process of considering a revision of requirements, and 456 schools reported that it might be possible to increase the requirements within the framework of their local programs. CUPM intends to restudy the colleges in those states in which Level I Conferences were held in order to ascertain the possible effect of the ten state conferences of 1962.

The Ten Level I Conferences. In general, the program in each of the conferences included these activities: get-acquainted luncheon; general session with ideas presented on the mathematics education of elementary school teachers and problems within a specific state; small group discussion of "Course Guides"; open meeting with reports of group discussions followed by general discussion; small group discussion on problems of implementation of "CUPM Recommendations"; and a general meeting at which group reports were followed by an open discussion.

The following people took part in these programs (some in several): E. G. Begle, Marguerite Brydegaard, Louise Combs, J. A. Cooley, George Cunningham, Roy Dubisch, Mary Folsom, Helen Garstens, W. T. Guy, Jr., Clarence Hardgrove, Bernard Jacobson, P. S. Jones, Donovan Johnson, J. T. Kelley, J. G. Kemeny, Dorothea Meagher, Bruce Meserve, E. E. Moise, W. W. Osborn, W. P. Robinson, Jr., Robert Sloan, Rothwell Stephens, Joseph Stipanowich, W. P. Viall, and R. J. Wisner.

The participants at the conferences represented many areas which are concerned with the education of elementary school teachers. The table below shows the number of participants at each conference and their area of concern.

<i>State</i>	<i>Mathematics Department</i>	<i>School of Education</i>	<i>State Dept. of Education</i>	<i>Teachers and Principals</i>	<i>Total</i>
Oklahoma	24	12	4	14	54
Florida	23	17	4	18	62
Michigan	14	13	6	19	52
Maine, New Hampshire, Vermont	20	12	9	11	52
Illinois	31	19	7	10	67
Minnesota	20	8	6	18	52
New York	23	18	9	32	82
Tennessee	29	13	4	10	56
Texas	41	15	6	23	85
California	49	28	8	36	121
	<hr/> 274	<hr/> 155	<hr/> 63	<hr/> 191	<hr/> 683

Results of Conferences. In general, the CUPM Recommendations seemed to meet with the approval of the majority of the participants at the conferences. The need for greater competence in mathematics through an expanded program of preservice training was recognized, and the "spirit" of the CUPM Course Guides was generally approved.

The participants from elementary schools enunciated clearly and strikingly the need for implementation of CUPM Recommendations. Teachers need to feel secure in the subject matter they teach. Several reports mentioned unfortunate experiences of teachers who were unprepared to teach new mathematics material, and principals reported that most elementary school teachers are less well-prepared to teach the mathematics of the elementary school than any other subject. Some superintendents of schools reported that they were forced to go out of their states to recruit elementary school teachers capable of teaching some of the new programs in mathematics in their elementary schools. The introduction of new programs of study into the elementary school has made the widespread implementation of CUPM Recommendations not only desirable but necessary.

The widespread opinion that some prospective elementary school teachers have inadequacies in mathematics led in several of the conferences to the suggestion of utilizing mathematics specialists in the schools (as opposed to the self-contained classroom). This proposal always gave rise to heated and lengthy debate with little consensus. Proponents of the self-contained classroom favored adequate training in mathematics for *all* prospective elementary school teachers in order to maintain the one-teacher classroom, and it was repeatedly pointed out that the self-contained classroom will be continued. Some participants felt that, should it prove to be impossible for each teacher to be adequately prepared, the specialist may become a necessity in the future.

Variability in preparation of college freshmen and the problems posed by these variations were recognized. Many state institutions must accept and enroll

all high school graduates, regardless of background and ability. Most institutions, however, have their own regulations for acceptance into particular programs within the institution. Therefore, it is not unreasonable for schools and departments of education to require that students entering a program of elementary education have two years of college preparatory mathematics.

It was generally agreed that students who have had a strong program in high school mathematics and enroll in elementary education programs should complete a program of study equivalent to the Level II Recommendations of CUPM. These students would then be prepared for a leadership role in the elementary school.

The feasibility of fitting twelve semester hours of mathematics into elementary education curricula was discussed. The professional education participants seemed to be evenly split on this matter—about half believing that twelve semester hours of mathematics could be included in the general curriculum with little or no difficulty, the others maintaining that major adjustments would be necessary. In this connection, the five-year program for elementary school teachers was mentioned repeatedly as a means of helping elementary teachers to become competent in all of the subject matter areas which they are called upon to teach. Everyone agreed that a program of studies consistent with the CUPM Recommendations should at least be offered as electives to students in elementary education.

Representatives from state departments of education at eight of the ten conferences were among the most avid supporters of the CUPM Recommendations. Most certification officers were influenced by the NASDTEC-AAAS Study which parallels the CUPM Recommendations for future elementary school teachers. Five states were undergoing re-evaluations of certification codes at the time of the conferences, and it may be hoped that results of these conferences will influence improvements in certification standards. In every state visited, certification requirements are couched in terms of minimum numbers of semester hours, and thus *in no state do certification requirements bar or hinder the improvement of the mathematical education of elementary teachers by individual colleges and universities.*

Several participants from colleges of education feared that raised standards would discourage students from entering a field in which a shortage exists. This usually led to a discussion of ways of attracting better students to the teaching profession. This problem transcends the powers and aims of CUPM. Representatives from a few schools requiring the suggested courses reported, however, that enrollments in elementary education have risen rather than declined.

It was agreed at all of the conferences that the nature of the content of the required courses is as important as the number of hours required. The need for specialized training in mathematics for future elementary teachers was generally recognized. It was emphasized that courses in high school algebra or trigonometry do not meet the needs of the prospective elementary school teacher. The mere inclusion of certain topics and exclusion of others does not

automatically make a standard catalog course appropriate for elementary school teachers. The spirit of the course and the manner in which it was presented was of the utmost importance. The use of the "discovery method" by the student is essential, and the development of proper attitudes towards mathematics is imperative.

A few participants were concerned about the advisability of giving college credit for courses which include high school material. It was pointed out emphatically that most of the topics contained in the Guides could be presented at various levels of sophistication, and that the type of credit given should depend upon the level of presentation and the maturity of the students.

The possibility of integrated courses instead of separate courses in number, algebra, and geometry was discussed at a few of the conferences. It was generally agreed that there are various acceptable patterns of organizing the recommended material and that each institution must work out a plan which is best for it.

The acute shortage of competent faculty, already felt in many colleges, would be greatly accentuated if new multiple-section courses were added in the implementation of CUPM Recommendations. It was agreed, however, that these courses should not be left in the hands of inexperienced graduate assistants. In many institutions, courses in mathematics for teachers are now taught by people with minimal or nonexistent mathematical background. In the past, elementary courses in mathematics for future teachers have evoked very little interest or attention on the part of bona fide mathematicians. Consequently, these courses were often among the most poorly taught at the undergraduate level. A reversal of this situation is imperative if programs for teachers are to be improved in the future, and mathematics staff must be convinced of the importance of these courses. To assure success, they must be staffed by able people who are willing to devote some time to the organization and presentation of the material, who are interested in the problems of elementary education, and who possess empathy for the students.

The urgent need for appropriate textbooks which embrace the Recommendations of CUPM was voiced repeatedly. It was felt that good textbooks could prove to be a partial guard against the problems which will arise when courses are taught by instructors with inadequate background in mathematics. Participants at two of the conferences suggested that teacher commentaries be written to accompany any textbooks.

CUPM was encouraged to expedite the commissioning of authors to write suitable textbooks since a much better picture of the spirit and intent of the Recommendations can be gained from textbooks than from the bare outlines in the Course Guides.

Participants from both departments of mathematics and departments of education expressed the need for greater liaison between their departments. In particular in those institutions where the mathematics courses for elementary education majors will be taught in the department of education, there is a need

for very close cooperation. It was also emphasized that the methods course should be closely correlated with the material of the content courses.

The conferences led to several suggestions of activities which CUPM could undertake in order to foster the implementation of the Recommendations. CUPM was encouraged to present the Recommendations along with reports of the ten conferences to other organizations such as the Associations of Elementary Principals, State Associations of Superintendents, and NCTEPS. It was pointed out that more publicity is needed in order to impress the general public with the need for improved training for elementary school teachers. Many people felt that it would be very helpful if CUPM would publish a list and an evaluation of available mathematics text material for the undergraduate preparation of elementary school teachers.

The participants generally felt that considerable progress toward the realization of the CUPM Recommendations could be made in the near future. Resolutions were passed at almost all of the Conferences approving the spirit of the CUPM Recommendations and urging their adoption by colleges and state departments of education.

One of the resolutions adopted is included here because it is representative of others and because it is brief. It is the resolution passed at the Level I Conference in Texas.

This group approves the spirit of the topical outline of CUPM covering the subject matter competencies in mathematics needed by elementary school teachers. It feels further that, in order to train these teachers adequately, our colleges should include twelve semester hours of course work in mathematics in the curriculum for elementary teachers.

Some resolutions were more specific and detailed and a few less specific as to hours. (This MONTHLY, Vol. 70, No. 4, pp. 433-34.) The resolutions are available in "Ten Conferences on the Training of Teachers of Elementary School Mathematics."

Present action and future plans. The Panel on Teacher Training is presently engaged in the following. First, it is in the process of revising the Course Guides for the Training of Elementary School Teachers as a result of the 1963 Level I Conferences. The revised edition should be available by September, 1963. Second, the Panel is seeking and commissioning teams of authors to write textbooks for the suggested courses. Some of the textbooks will be used at a few experimental centers during the 1963-64 academic year. These and other books should be available for general use by September, 1964. Third, CUPM has had a survey prepared of existing mathematical materials suggested by publishers for use in elementary teacher training programs, and the results are now available in "CUPM Study No. 2." Fourth, the Panel is planning Level I conferences in the following states for the autumn of 1963: Indiana, Kansas, Kentucky, Maryland, New Mexico, Ohio, Pennsylvania, Virginia, Washington, and Wisconsin.

The Panel, members of the Central Office, and members of the CUPM Consultants Bureau will continue to cooperate and consult with state depart-

ments of education and faculties of colleges and universities in order to facilitate expanded and improved programs of teacher training.

Free copies of the Recommendations, Course Guides, CUPM Study No. 2, and Reports of the Five Regional and the Ten Level I Conferences are available upon request from the CUPM Central Office: Committee on the Undergraduate Program in Mathematics, Post Office Box 1024, Berkeley, California.

AN EXPERIMENTAL COURSE IN ANALYSIS FOR COLLEGE FRESHMEN

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Introduction. The usual college program in calculus has two pressures on it, one to make it more rigorous and the other to make it more down to earth. The writer believes that the calculus course that is now generally given in colleges will be split, as a result of these pressures, into two courses, one elementary and computational, and the other advanced and conceptual, both mathematically rigorous. He offers the course discussed in this article as a candidate for the first of these anticipated successors to the calculus sequence.

This course covers material distributed throughout the entire traditional calculus sequence, including differential equations. It is concerned only with functions that can be represented by power series. The writer believes that this restriction yields a body of knowledge which is easily learned and yet which is broad enough so that it can be the terminal course in calculus for a great many students, including certain categories of engineering students, who now have the choice of having to attempt either too much or too little.

Content of the proposed course. The course is devoted to the study of real functions which can be represented by power series, with special emphasis on the elementary functions—polynomials, rational functions, algebraic functions, exponentials, logarithms, trigonometric functions and their inverses. Two basic problems recur. The first is to deduce properties of a function from its representation. The second is to find a representation for a function given certain of its properties. Differential and other functional equations are used frequently as a means of expressing the properties in question.

A rudimentary text has been prepared for the course, entitled *Power Series and the Elementary Functions*. Its contents are:

Chapter I—Polynomial algebra: Definitions and Notations. Operations with polynomials. Division of polynomials. Infinite series arising from division of polynomials. Binomial theorem.

Chapter II—Polynomials as functions: Substitution in polynomials. Polynomials having prescribed values. The factor theorem and its applications. Graphs. Graphs of polynomial functions; continuity. Derivatives of polynomial functions. Derivatives and tangents. Higher derivatives; Taylor's theorem for polynomials.