Alternatively, one may establish the corollary directly from Lemmas 1 and 2 and then proceed with the proof of the fundamental theorem in the usual fashion. To establish the corollary directly, suppose that $p|ac$ but $p\nmid a$. Then $c(a/p)$ is an integer, but a/p is not. By Lemma 2, p is the least positive integer b such that $b(a/p)$ is an integer and hence, by Lemma 1, $p|c$.

MATHEMATICAL EDUCATION NOTES

EDITED BY JOHN R. MAYOR, AAAS and University of Maryland
COLLABORATING EDITOR: JOHN A. BROWN, University of Delaware

All material for this department should be sent to John R. Mayor, 1515 Massachusetts Avenue, N.W., Washington, D. C. 20005.

PREGRADUATE TRAINING IN MATHEMATICS—A REPORT OF A CUPM PANEL

A. B. WILLCOX, Executive Director of CUPM

1. The mathematical theory of curriculum construction.[1] Every mathematician and increasingly many college sophomores know of the intimate connections between differential equations $y'=f(x, y)$ and direction fields. In this section we introduce an application of a differential equation (of sorts), describing, first, the space in which the associated direction field and integral curves are embedded. To our knowledge, this particular application of the notion of a differential equation cannot be found in print at present. We are confident, moreover, that it will not be found in print in the future, outside of this article, the entire purpose of the theory having been accomplished when the reader reaches section 2.

Let us denote by A a "space"[2] each of whose points represents a *mathematical activity appropriate for a college student*. For example, one point might denote "studying the calculus integrated with material from linear algebra"; others, "studying the calculus with some differential equations but with no use of concepts from linear algebra," or, "studying linear algebra and calculus as separate parallel courses," or, "beginning homological algebra, but requiring remedial work in long division."

Using the time-honored device of "thinking away" complexity by the use of simple diagrams, we picture A as shown in Fig. 1.

Denoting the real number system by R, we picture $A\times R$ as follows. R is treated as a "time axis," following common usage.

$A\times R$ will be called a (mathematics) *curriculum space* and a function F defined on the interval $[0, 4]$ in R and having values in A will be called a *pregradu-*

ate curriculum (in mathematics). The graph of one pregraduate curriculum is shown in Figure 2.

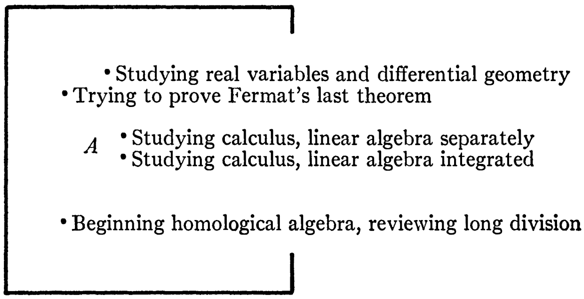

FIG. 1. Activities space.

$A \times R$ is the space of the direction field associated with our differential equation. The equation has a special name. It will be called "the CUPM Panel on Pregraduate Training" and denoted by $y' = P(t, y)$, or sometimes, by PPT.

The Panel on Pregraduate Training (PPT) acts as a differential equation in the following sense. It consists of approximately thirteen[3] mathematicians representing institutions ranging from liberal arts colleges to leading graduate departments. Each point in $A \times R$ represents the state of a hypothetical student engaged in a particular mathematical activity at a particular time during the period we have come to call, with nostalgia, "those bright college years." For each such point the Panel, after thorough and candid discussion, will decide what it feels that student ought to do next and how rapidly he ought to move on. In this sense the Panel establishes a "direction" in $A \times R$.

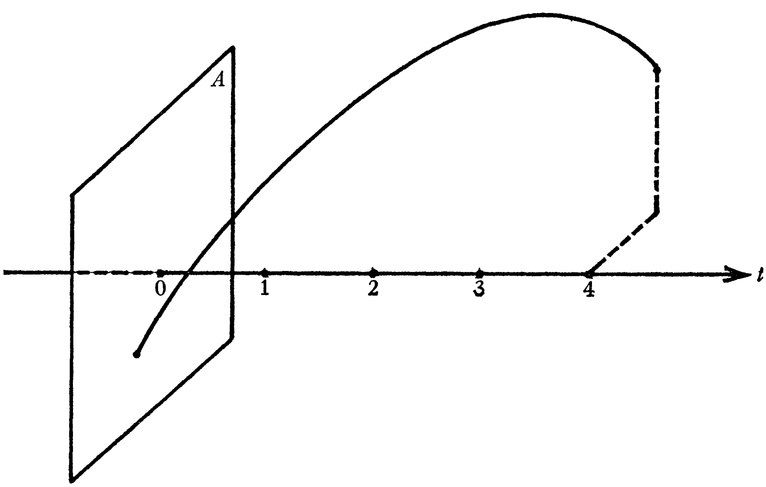

FIG. 2

The rest of the story need not be spelled out for the reader. A curve in $A \times R$ which is the graph of a "solution" of PPT will be called a *CUPM-recommended pregraduate curriculum*. Since the curve follows the direction field at each point, it represents a hypothetical "flow" of undergraduate mathematical experience which the Panel would call optimal under the circumstances.

This last phrase, "under the circumstances," brings up a final feature of our theory, not unexpected in view of the analogy already initiated. Solutions of PPT (or $y' = P(t, y)$) are not unique, but depend on certain boundary conditions. We identify two types of boundary conditions which we associate with parameters α and β. For each specified α, β[4] a unique[5] solution $F(\alpha, \beta)$ of PPT exists.

We will call α the *input boundary condition*. That is, α refers to a combination of the student's ability, motivation, high school training, and the ability of his teachers and facilities of his college. In other words, α is a measure of the limitations imposed on a four-year undergraduate mathematics program by the anticipated raw materials.

We will call β the *output boundary condition*. That is, β refers to the quantity and variety of mathematical knowledge the student is expected to have when he enters graduate school. In other words, β is a measure of the limitations imposed by a set of minimum standards which the graduating senior, or, alternately, the entering graduate student, is expected to meet. While α reflects the raw materials, β represents the desired end result (see Fig. 3).

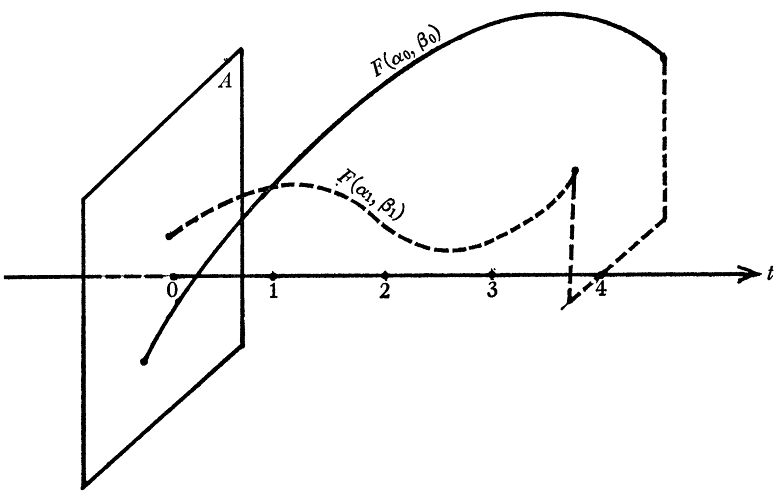

FIG. 3

2. One set of boundary conditions: existence of a solution. To introduce a discussion of the recommended curriculum which has resulted from a particular set of boundary conditions, we remark on the nature and purpose of PPT. The CUPM Panel on Pregraduate Training has been given the task of studying the college training of prospective graduate students in mathematics and making recommendations for the improvement of this training in the United States. Its tools are the knowledge, ingenuity, and experience of its members and the information and guidance it can obtain from conferences and conversations with other mathematicians and teachers.

Early in its deliberations on this subject, the Panel recognized the folly of suggesting small and immediate improvements in pregraduate mathematics programs in the absence of a clearly understood set of long-range goals. Accordingly, it selected as its first task the construction of an idealized program which, while keeping in touch with reality, would also help set the pace for curricular improvements for some years to come.

In the language of our mathematical theory of curriculum construction, PPT selected for its first consideration a pair of boundary conditions α_0, β_0 which we will call *ideal boundary conditions*.[6] The boundary condition α_0 represents a combination of a student of high ability and motivation, with the best high school training which can reasonably be expected, and competent, well-trained college teachers. It refers to a student who already knows, when he enters college, that he desires to be a research mathematician. β_0 represents what mathematicians representing first-rate graduate schools would like a student of this ability and motivation to know at the end of four years of college, years which can be thought of as the first four in a seven-year program leading to a Ph.D.

The members having been selected, in part, for their ability to identify these boundary conditions from their own experience, the Panel set about its task, and the resulting solution, $F(\alpha_0, \beta_0)$, was communicated to the mathematical community early in 1964. It is contained in a booklet, "Preliminary Recommendations for the Pregraduate Training of Research Mathematicians," which has been distributed by CUPM to mathematics departments throughout the country.

The Panel has this to say about $F(\alpha_0, \beta_0)$[7]: " . . . The present document does much more than merely recommend a program for the gifted students of today. It will also serve as a guide and a source of ideas to persons interested in evaluating and modifying existing mathematical curricula. The Panel spent more than two years in a critical examination of the basic structure of college mathematics, in relation to present mathematical research. It has attempted to discern important underlying patterns and to effect some unity, both of viewpoint and of technique, within a four-year curriculum. It has attempted to make use of the simplification of concept and technique resulting from recent discoveries, without sacrificing intelligibility. . . .

"Mathematics and mathematicians being what they are, there can be no single 'best' pregraduate program, even with an assumed idealized setting. It is taken for granted that there is a clear need to bridge the gap between undergraduate instruction and contemporary mathematics, but the bridge can be erected in many ways. Hence, we constructed a sample program which serves the purpose of showing the type, amount, level, and distribution of mathematical topics which the Panel deems appropriate. To make explicit some ways in which the program can be executed, several course outlines are given . . . . These outlines serve as a guide to the spirit and content of the recommendations, and their multiplicity should dispel any notion that there is a prescribed, narrow, CUPM version of optimum undergraduate preparation. . . .

"Viewed broadly, the suggested program consists of calculus, algebra, analysis, and geometry (with the term 'geometry' being used in a sense sweeping enough to include differential geometry, differential topology, algebraic topology, etc.). Certain topics and areas are regarded as basic and should be generally available to pregraduate students. Additional courses are recommended which should be considered by departments having an adequate staff. . . .

"We have recognized the importance of linearity by recommending the early introduction of linear algebra and have made linearity a recurrent theme in the analysis courses. We have tried to show the unity of mathematics by emphasizing algebraic and topological ideas throughout. Breadth is important for research mathematicians but difficult to achieve at the early undergraduate level; nevertheless, students at all levels ought to be presented with as integrated a picture of mathematics as they can assimilate. We have attempted to give students a chance to see some of the structures and some of the applications of modern mathematics. In the analysis courses, we recognize the traditional role of the sciences as sources of mathematical ideas and methods. The spirit of classical geometry has been retained by emphasizing theorems which have a geometric formulation. . . .

"The program we have suggested is divided into two parts: Introductory Undergraduate Mathematics covers the first two years, and Higher Undergraduate Mathematics covers the last two years."

To complete a summary description of $F(\alpha_0, \beta_0)$ we offer several more detailed comments on its major features. Under the assumption of a student audience with strong mathematical training in high school and with excellent motivation, a unified two-year sequence of what might be called "vector space calculus" is recommended as the basis for the pregraduate program. This sequence, to occupy the first two years, is what the Panel calls Introductory Undergraduate Mathematics. The program suggests that calculus be presented so as to introduce and utilize significant notions of linear algebra and geometry in the construction of analytic tools for the study of transformations of one Euclidean space into another.

The Panel has constructed three detailed course outlines for the first two years[8] each of which covers, in approximately 15 semester hours, about nine semester hours of analytic geometry and calculus and six semester hours divided between linear algebra and differential equations. The three course outlines are constructed from different points of view concerning how these subject matters should be integrated. One combines linear algebra with the calculus but presents differential equations in a separate course. Another has a separate course in linear algebra but includes topics in differential equations in the main body. The third has no separate courses, but has a section on differential equations, and develops linear algebra topics as they are needed to solve various problems in analysis. The outlines differ also in such things as the order of introduction of integration and differentiation, the particular method of introducing the definite integral and in the degree to which abstract concepts from elementary point set topology are utilized. Further differences are revealed in the basic motivation of the separate outlines. In two of the outlines the prime motivation is concern for the internal structure of the calculus and of linear algebra, applications being made when appropriate. In these outlines the generalized Stokes theorem is considered a fitting climax for the first two years because of the merging of concepts in algebra, topology, and analysis which are needed in reaching it and because of its important applications in mathematics and physics. In the third outline the approach is to develop mathematical concepts directly as needed for the solution of important problems that arise in mathematics and physics. In particular, linear algebra is so treated. This outline is more oriented in the direction of classical analysis. Stokes theorem, for instance, is thought of as but one of a number of important theorems beyond the traditional calculus.

In presenting these three separate outlines for the first two years, the Panel emphasizes again its recognition of the fact that there are many paths leading to a recognized set of goals.

By the time a student has completed the introductory undergraduate mathematics curriculum and is ready to enter the third year of his program, he will be ready to dig deeper, he will have an appreciation for abstraction and an insistence upon rigor, and he will already have developed a desire to pursue to some extent his special mathematical interests. The advanced part of the program therefore reflects a flexibility to suit both the needs of the student and the interests of the professors. The courses in this part of the program have much in common with courses ordinarily labeled "undergraduate-graduate" at many universities. It is impossible to defend the idea of a sharp break at the end of the first four years, particularly in view of the historical accident by which undergraduate and graduate education are separated. Such separation should not reflect a point of intrinsic change in the nature of the courses.

The Panel has suggested that there are several courses which every college department undertaking this program should provide for its "upper division" students. These courses could be roughly described as follows: real analysis, complex analysis, abstract algebra, geometry-topology, probability or mathematical physics.

In addition, to achieve a richer and more comprehensive program, a department should offer, as far as its resources will permit, a balanced selection of courses in: algebra, analysis, applied mathematics, foundations and logic, geometry (algebraic, differential, projective), mathematical statistics, number theory, topology.

For each of these areas the Panel has provided one or more outlines of suggested courses. Space will not permit a description of these in detail. It must suffice to say again that the courses are at what is generally regarded as the beginning graduate or advanced undergraduate level.

The Panel offers the following principles to guide the student in constructing a major program in mathematics: a) For the upper class years, at least three of the following four categories should be represented in the course program: 1. algebra; 2. analysis; 3. applied mathematics; 4. geometry-topology. b) Included in the program there should be, in order to achieve depth, at least two full-year courses—that is, courses in which the first semester is an essential prerequisite to the second. c) A major in mathematics should have at least seven semester courses beyond the introductory undergraduate mathematics.

The reader who wishes to examine $F(\alpha_0, \beta_0)$ in detail, and who does not have access to the booklet "Preliminary Recommendations for Pregraduate Preparation of Research Mathematicians" may obtain a free copy by writing to the CUPM Central Office, P.O. Box 1024, Berkeley, California 94701.

3. Another set of boundary conditions: a progress report. The Panel did not have long to rest after completing the construction of $F(\alpha_0, \beta_0)$. Even as it accepted the program and gave its approval to the publishing of a booklet describing it, CUPM asked the Panel to place a high priority on the construction of a solution for another set of boundary conditions.

The Commission recognized that widely circulated recommendations for curricular improvement, especially such idealized and pace-setting recommendations as these, are in danger of being misinterpreted by many readers. The need for enlightened suggestions for the *immediate* improvement of undergraduate mathematics in the nation, to keep pace with rapid developments both at the secondary school level and on the frontiers of mathematics, is so widely recognized and so close to the lives of college mathematics teachers that *any* recommendations from CUPM tend to be projected upon tomorrow rather than into the future. Imagine the shock wave which would have been produced if the Report of the Cambridge Conference on School Mathematics had been released in a mathematical world which had not been prepared by several years of SMSG, UISCM, and similar projects. Having produced the "College Cambridge Report," the Panel was asked to proceed with deliberate speed to prepare the "College SMSG."

Anticipating this charge, the Panel had already begun discussions of a new set of boundary conditions, and by the fall of 1963 it had decided to work seriously on an undergraduate mathematics program sufficiently strong for entrance into good graduate programs but appropriate for the "ordinary" undergraduate mathematics major in a majority of American colleges and universities.

The first step in the construction of a new solution of PPT was the identification of the desired set of boundary conditions, which we will call *real* input and output conditions, α_1 and β_1. The Panel had general descriptions of α_1 and β_1: α_1 represents the capabilities of an "ordinary" undergraduate student, in a "good average" college or university, who intends to do graduate work in mathematics; β_1 represents what good graduate schools require, or strongly recommend, as prerequisites for admission into a graduate program in mathematics. Confident that these descriptions determine a meaningful set of boundary conditions, but unsure of the exact nature of these conditions, the Panel embarked on several projects.

In an attempt to identify α_1 the Panel made plans to hold two conferences with representatives of mathematics departments in two types of institutions. One conference, with thirty-nine representatives of large universities in the South, was held on April 17–18, 1964 in Dallas, Texas; another, with representatives of small liberal arts colleges in and near Ohio was held on May 7–8 in Cleveland. At both of these conferences participants were asked to react, both in small groups and in general sessions, to the Panel's description of $F(\alpha_0, \beta_0)$ from the point of view of their present programs and their present students. They were asked how much of $F(\alpha_0, \beta_0)$ seemed reasonable at their institutions, how long it might be before a substantial part of $F(\alpha_0, \beta_0)$ might be implemented there, what specific portions of $F(\alpha_0, \beta_0)$ seemed most feasible for their students and with their staffs and what portions seemed least feasible and/or least important.

The reactions of these mathematics teachers to the Panel's recommendations and the comments which they made on the capabilities of their students and staffs were extremely enlightening to the Panel. Just as enlightening, however, was their reaction to remarks made by Panel members explaining how the Panel feels its recommendations might be read with existing programs for "ordinary" pregraduate students in mind.

True to the predictions of CUPM and the Panel most of these participants had received the Panel's recommendations, read them from the point of view of the feasibility of the entire program for their students and their staffs, and, understandably, reacted with some shock. At each conference a member of the Panel explained the long-range nature of the recommendations and suggested that they be used in a different way by a teacher who is looking for help in deciding what changes ought to be made in his course or his program next year. Instead of reading the report with the question "Can we adopt this program?" in mind, the Panel member suggested that the teacher ask "How much of this can we do in our college?" or "Are there ideas here which will give us some direction in changes we make next year?" or even "Can we design a course which preserves the spirit and direction of this course description even though it contains only a portion of the material?"

On being told that the Panel was this flexible in its notion of the best use for its description of $F(\alpha_0, \beta_0)$ a marked change was observed in the attitudes of the participants toward the recommendations. People began to discuss the portions of the program which were similar to existing programs and portions which might reasonably be adopted in the near future. They were also quick to point out portions of the program which appear to be so ambitious that they cannot expect to influence existing undergraduate programs for many years ahead.

Out of these discussions, a consensus began to emerge. The portion of $F(\alpha_0, \beta_0)$ which the Panel calls Basic Undergraduate Mathematics plus a good one-semester course in abstract algebra (modeled, perhaps, after portions of the outline on pages 55–59 of the Recommendations) represents a program which exists in many colleges today and is a reasonable goal for many others in the immediate future. Many colleges and universities might now begin to think of compressing this package into, say, three years, adding new courses along the lines recommended by the Panel in the section, Higher Undergraduate Mathematics. Each of these course descriptions represents a goal to aim for; each can be suitably adjusted to fit existing needs and potential.

Thus, conferences which were intended to help identify the real input boundary conditions also produced the first outlines of a program looking very much like a good approximation of a solution of PPT satisfying at least the boundary condition α_1. Moreover, this program bore a close resemblance to a "segment" of $F(\alpha_0, \beta_0)$ "spread out" over a four-year period.

At the same time that these conferences were being planned, the Panel began a study designed to help identify β_1. This meant learning what graduate schools require for entrance, and the obvious way to obtain this information was to ask graduate schools. Wishing to do this in a way which would produce *some* information quickly without burdening department chairmen with lengthy questionnaires or involving them in extended correspondence, the Panel adopted the following procedure. Forty-one graduate departments of mathematics were asked to send to the Central Office printed materials which they use to inform students of admission requirements, the normal first-year program and the nature of preliminary or qualifying examinations. When replies from twenty-nine had been received, the Executive Director and a graduate student, employed for this project, read through these materials and indicated on a chart pertinent information which they contained.

Results were rough in many cases. To reduce catalog statements on entrance requirements to yes-no answers to a small collection of questions is to smooth over many of the nuances of carefully worded statements designed to be flexible in interpretation and, in some cases, to give lip service to certain standards while actually imposing no conditions. Moreover, in many graduate schools there is no "normal first year program"; students simply begin working at the level of their training and ability. However, viewed broadly and with these problems in mind, a meaningful picture emerged.

Of the twenty-nine graduate schools responding, nine did not specify in their materials any minimum entrance requirements for admission into graduate work in mathematics. Of the twenty whose catalogs do specify prerequisites, seventeen require (or strongly recommend) a course in advanced calculus and ten a course in abstract algebra and/or linear algebra. (An advanced calculus course is expected to follow the equivalent of two full-year courses in the calculus. It is evident from the descriptions that most graduate schools have in mind a substantial course at the level of, say, Taylor or Widder.)

Of the seventeen schools which mention advanced calculus as a prerequisite, eight require two more "post-calculus" courses and three require three additional courses at this level.

Of the ten schools that require both advanced calculus and abstract algebra, eight require an additional "post-calculus" course, four strongly recommend an introductory course in functions of a real variable (Buck or Apostol variety) and several mention courses in complex variables and point-set topology.

A typical first-year graduate program includes courses in algebra, real variables, complex variables and topology, at least two of which are above the introductory level.

The picture which emerges is of an undergraduate program containing a sequence in analysis proceeding through the level of advanced calculus and probably including at least a brief introduction to what is usually called real variable theory. This certainly involves a careful treatment of the differential and integral calculus of functions from E_n to E_m, an introduction to some of the standard theorems concerning continuous functions, a bit of the topology of the plane and 3-space, and may include an introduction to the Lebesgue integral. The program also contains some abstract algebra; certainly linear algebra, and very likely an introduction to the basic algebraic structures. Topology and complex variables are serious contenders for space in the program.

It is a program, in short, very much like the segment of $F(\alpha_0, \beta_0)$ which emerged from discussions at the Dallas and Cleveland Conferences. Moreover, it is a program which, according to official catalog descriptions, is sufficiently rich for entrance into graduate study in mathematics in most of the ranking universities in the country.

It thus appears that a solution $F(\alpha_1, \beta_1)$ satisfying both real boundary conditions α_1, β_1 can be constructed by "spreading out" a segment of $F(\alpha_0, \beta_0)$. That is

$$F(\alpha_1, \beta_1; t) = F(\alpha_0, \beta_0; kt) \qquad t \in [0, 4],$$

where k is a constant probably satisfying, at the present time, $1/2 \leqq k \leqq 3/4$.

4. Implications for the future. The discovery of $F(\alpha_1, \beta_1)$ hidden away in $F(\alpha_0, \beta_0)$ has come as a surprise to many who have been associated with the work of the Panel. All indications were that the very teachers who need help most would be unable to read the recommendations in any meaningful way. It was generally expected that $F(\alpha_1, \beta_1)$ would have to be presented to the public in a new document, addressed to a wider audience than the first; a document which would suggest a more nourishing diet than we now have without causing indigestion, which would describe ways of making small steps toward $F(\alpha_0, \beta_0)$, $F(\alpha_1, \beta_1)$ being the first.

The discovery, through discussion with college and university teachers, that all this can be found in the present report by one who is willing to read it in the proper spirit, surprised and pleased the Panel. The result is a new emphasis in the Panel's plans for the future which we sketch in conclusion.

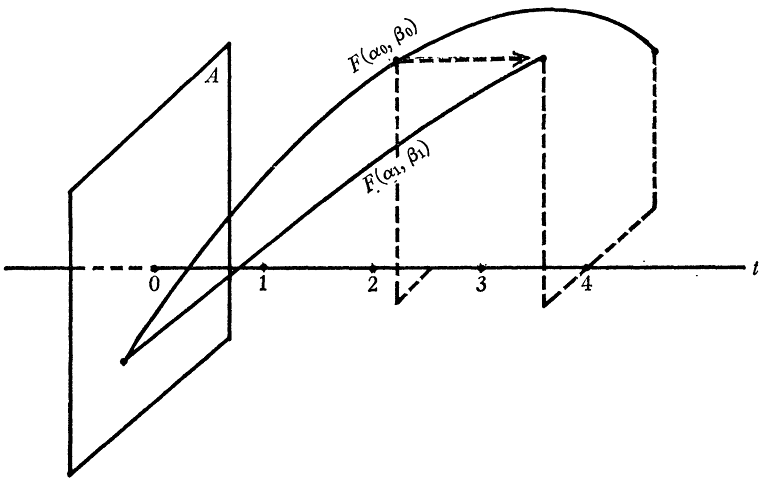

FIG. 4

i. The Panel presents its "Preliminary Recommendations for the Pregraduate Training of Research Mathematicians" to all those concerned with the continuing improvement of college mathematics curricula. While it continues to hope that the recommendations will serve to guide future efforts to improve these curricula, the Panel invites college mathematics teachers in all situations to consider them as a source of helpful ideas for improvements at all levels *now*. The program as a whole is a long-range goal, but no one should be so frightened by the goal that he cannot find a step which he is able to reach. The course descriptions are rich sources of mathematical ideas. The bibliographies contain books which can help the teacher translate the cryptic words and phrases of a sketchy outline into ideas which can be injected into classroom discussions at many levels. Finally, the recommendations should stir spirited discussions wherever mathematics teachers congregate; controversy *can* spawn enlightenment.

It can be hoped, in fact, that the Recommendations[9] will suggest a sequence (finite and, hopefully, not long) of solutions $F(\alpha_k, \beta_k)$ which will lead, over a period of time, from today's curricula to something like $F(\alpha_0, \beta_0)$.

ii. A description of a program, even a detailed sample outline, does not make a course. A course exists only when a group of students have had a certain educational experience during a period of time. The existence proof is communicated to the public as a textbook or as course notes. Recognizing that its recommendations do not constitute existence proofs for either $F(\alpha_0, \beta_0)$ or $F(\alpha_1, \beta_1)$, the Panel seeks to encourage experimentation with courses reflecting these recommendations and the production of course notes and textbooks from these experiments.

In this way we hope to maximize the number and variety of existence proofs which appear in the years immediately ahead. The Panel will publish, from time to time, information concerning the availability of such notes.

As a start in this direction, the Panel solicits information from the reader concerning any such existing experimentation and course notes. Information should be sent to the CUPM Central Office, P.O. Box 1024, Berkeley, California 94701.

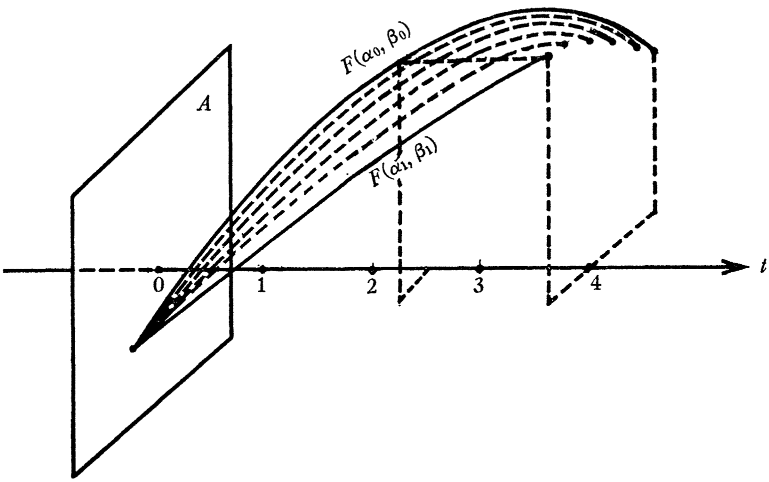

FIG. 5

iii. The Panel suggests another use for its recommendations. Contained in the booklet is a very good summary of the kind of mathematics with which a graduate student should be familiar before attempting his qualifying or preliminary examination for Ph.D. candidacy. For the program as a whole represents a good background for such graduate examinations by present day standards in most universities. That this should be the case is not surprising; a description of one of the better high school mathematics programs of today would have been an excellent syllabus for comprehensive examinations for a bachelor's degree in mathematics not so many years ago.

The Panel encourages the use of the booklet by advanced undergraduate and beginning graduate students for this purpose. A copy recently placed in a library reading room for graduate students in a leading university showed in a short time, unmistakable evidence of having been thoroughly and frequently used. The recommendations may be ordered by departmental representatives or interested students for this purpose. It is to be expected, of course, that the recommendations will lose their effectiveness for this purpose as improvement in undergraduate programs allows the raising of the level of graduate courses.

iv. For the future, the Panel plans to continue its efforts to obtain feed-back from the mathematical community concerning its Recommendations as well as to encourage the production of written course materials reflecting its ideas.

Additional conferences are planned to investigate further the relationship between $F(\alpha_0, \beta_0)$ and existing practice, immediate potential, and foreseeable demand. Written comments, mailed either to a Panel member or to the CUPM Central Office, are encouraged from readers of this article and of the Panel's booklet. With information and increased wisdom accumulated from such consultations with colleagues throughout the nation, the Panel plans to prepare a second set of guidelines in the near future which will reflect indicated modifications in $F(\alpha_0, \beta_0)$, outline clearly an appropriate $F(\alpha_1, \beta_1)$ for the immediate improvement of undergraduate mathematics programs, and offer suggestions for an orderly transition from $F(\alpha_1, \beta_1)$ to $F(\alpha_0, \beta_0)$.

Footnotes

[1] The author of an article which is whimsical refrains, if he is wise, from announcing the fact at the outset. The merits of the discovery method have long been recognized by this pedagogical school. We deviate reluctantly from the spirit of this proven rule to announce that the applications of the mathematical theory of curriculum construction found in sections 2–4 are *not* whimsical. They constitute, in effect, an informal report on the work of the CUPM Panel on Pregraduate Training by a person who has had an opportunity to observe the Panel at work for over a year. The reader who is intrigued by the whimsy of section 1 will find himself led into the possession of information about recent significant efforts to stimulate improvements in undergraduate preparation for graduate work in mathematics. The reader who is impatient with whimsy will find that sections 2–4 are, except for minor notational conventions, self-contained and may be read independently.

[2] We won't allow ourselves to be distracted from our goal by too much concern over the exact structure of this space. Assume, for conceptual simplicity, that A can be embedded in a Euclidean space.

[3] The exact number depends on the time and the fluctuations caused by an irregular system of rotating membership. At the time of this writing the Panel membership is as follows: Richard D. Anderson, Ralph P. Boas (Chairman), Dan E. Christie, Leon W. Cohen, Leslie A. Dwight, Samuel Eilenberg, George E. Hay, Leon A. Henkin, Peter D. Lax, John C. Moore, George F. Simmons, Gerard Washnitzer.

[4] Again, too much concern over the nature of the domains of α and β will divert us from our central purpose.

[5] In the weaker sense referred to in footnote 8.

[6] The reader is requested to try to live with the notion of a differential equation "selecting" boundary conditions for a solution of itself. This is not the only way in which our equation is extraordinary.

[7] The quotations are selections from pages 4 to 7 of the Panel's booklet.

[8] If a "curriculum" is thought of as a particular set of courses arranged in a sequence, then this multiplicity of suggested outlines appears to contradict uniqueness of a solution of PPT under a fixed set of boundary conditions. This brings up a subtlety of our theory which we have preferred not to bring into the open until this point. A "solution" of PPT is, in fact, an *equivalence class* of curricula. Recognizing that even sixteen mathematicians could not agree on a single "best" set of courses, the Panel offers alternate samples, any one of which it would recommend to the reader for careful consideration. These are elements of the equivlence class. The entire recommended program, in fact, is a sample element of the equivalence class of curricula which the Panel would call ideal. The purpose of the published recommendations is to suggest the equivalence class by displaying one or more sample elements. They are offered in the hope that each reader will construct his own element of the equivalence class. The only uniformity which the Panel envisions in the members of the class is a certain depth, flavor, and pace.

[9] Suitably revised on the basis of experience, of course. The Panel recognizes that criticism and revision are the only protection against rigor mortis.