

MATHEMATICAL ENGINEERING

A FIVE-YEAR PROGRAM

A Report of

The Panel on Mathematics for the Physical Sciences
and Engineering

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INTRODUCTION

During a conversation at the mathematics meetings in January, 1964, the late S. S. Wilks of Princeton University pointed out the absence of any special large-scale efforts to provide technical personnel for the nation's space program. As he saw it, there was a continuing need for persons really well-trained in the mathematical sciences and able to apply their fields to the complicated engineering problems of the space effort. His aim was to provide technical manpower specifically prepared for a strong combination of mathematics and engineering--in addition to the more customary converts from a great variety of backgrounds. He felt that CUPM, with its tradition of pervasive concern for all aspects of collegiate mathematical education, might undertake a study of this problem. The present report is, in part, a response to his ideas.

A pair of meetings with representatives of NASA and the space industry confirmed the eminent need for the projected product. At the same time it became clear that many other industries, such as electronics and communication, would have an equal interest in a mathematical engineer with the same basic background but possibly somewhat different specialization. All these industries face innumerable engineering problems with a common need for extensive and sophisticated mathematical analysis. For the solution of such problems it is no longer true that the prime requirement is a good physical intuition; rather, one also needs a well-developed mathematical intuition. Thus the mathematical engineering program must involve a heavy concentration in mathematics, but with a choice of topics that will give a useful basis for applications as well as solid grounding in theory. As the Panel came to grips with this multiplicity of purposes, it developed the notion of a common core, for all mathematical engineers, of material in the basic physical sciences and, more extensively, in the mathematical sciences. The core, in turn, is complemented by a number of options, which are more specialized developments in depth and which assure that the student will be fairly well acquainted with at least one branch of engineering. Orbit mechanics, operations research, and control theory are three options which are developed in detail in the present report.

It turns out that a minimum of five years, rather than four, is needed to carry out the desired sequences in depth. It is not immediately obvious what the student's area of concentration should be called. The dual emphasis on mathematics and engineering makes either field conceivable; in fact, the program comes close to being recognizable as a master's program in applied mathematics. However, the heavy emphasis on the physical sciences, the concern in each option with the building of mathematical models, and the rather heavily prescriptive nature of the program make a realization within the engineering school more suitable. Wherever the program may appear within an institution's offerings, it should involve close cooperation between the mathematicians and the engineers.

A natural question also arises as to the possible fields of further graduate study which a student could enter on completion of the present program. It is our opinion that relatively little, if any, "remedial" work will be necessary to qualify the student for a doctoral program in applied mathematics or, depending on the particular option, in engineering science, or in industrial or electrical engineering. Whether or not a master's degree should be given for the completion of this program is a matter for the offering institution to decide. As remarked above, the content of the program is of the right order of magnitude for this degree. Other considerations (e.g., requirement of a thesis) may be deciding factors.

A number of additional remarks about the program are in order. A most important aspect is the flexibility which would automatically be built into the mathematical engineer. With such a background and with a considerable facility in making connections between the real world and mathematical models thereof, such a man could easily retrain himself, say, from space science to oceanography, if a sudden shift of present national interest should make this desirable. Secondly, the similarity in spirit of this program to the recently introduced engineering physics and engineering science programs is worth noting. The idea of these programs is to give the student a solid background of the kind of physics that would be useful in a wide variety of engineering applications, along with enough engineering subjects to impart some feeling for the kinds of problems he would encounter. There is now no question of the value of such training. In just the same way, mathematical engineering combines a solid foundation in major areas of applicable mathematics with real strength in some particular area of engineering, and experience in connecting the two. Incidentally, it should be remarked that mathematical engineering has existed for some years, much in the spirit of the present report, at several universities in the Netherlands. It seems to be a successful program from the point of view of both employment opportunity and preparation for further graduate work.

DESCRIPTION OF PROGRAM

As remarked above, the program is constructed around a core consisting of a heavy concentration of mathematics and the physical sciences. Attached to the core there may be many options, each providing motivation, application, and extension of the core material to some phase of engineering. The core is fairly well defined and will probably not vary greatly from one institution to another. The options, on the other hand, will necessarily have much local flavor both in their general subject matter and in the particular courses that compose them. The three options that we present here are thus to be regarded as samples of what can be done.

The Core

Modern engineering is built upon a three-part foundation consisting of mathematics, the physical sciences, and automatic computing. The last of these is a newcomer whose precise role and manner of development are still matters of speculation, but there is no question as to its basic importance. These three topics, then, compose the core.

The mathematical portion, which is, for this program, the most extensive, is based on Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists, page 628. The courses recommended there as preparation for graduate work have been modified somewhat and about nine semester hours have been added, much of it as additional work in topics already begun. This gives us the following list (the initial number refers to the course outlines given in the Appendix [or elsewhere in this COMPENDIUM], and "hours" means "semester hours"):

- Calculus and Linear Algebra (12-15 hours)
- 1. Functions of Several Variables (3 hours)
- 2. Intermediate Ordinary Differential Equations (3 hours)
- 4. Numerical Analysis (3 hours)
- 5. Probability and Statistics (6 hours)
- 6. Complex Variables (3 hours)
- 7. Functional Analysis (3 hours)
- 10. Partial Differential Equations (6 hours)
- 11. Optimization (3 hours)

Discussion of the individual courses is deferred until the whole core has been described.

The resulting 42-45 hours of mathematics in the core is about the magnitude of a good undergraduate major in mathematics, but the emphasis is quite different. This program stresses analysis heavily. Indeed, the minimal treatment of algebra and geometry is perhaps the most vulnerable point of this curriculum. However, for the foreseeable future the topics included are certainly of first importance. Other mathematical topics needed in certain courses can be developed when needed to the extent required.

We recognize full well that the value of these courses depends on the spirit in which they are taught. One must keep in mind that their ultimate purpose is application, either directly or as preparation for more obviously applicable topics. Such courses as Partial Differential Equations and Optimization should lean heavily on applied problems. Computational methods should be stressed throughout. Further, as much interconnection as possible should be built into the whole program. It is planned that both the mathematics and the engineering courses should reinforce one another to an unprecedented degree.

A corollary of this requirement is that the courses should be taught by whoever is capable of doing the best job, regardless of the department he happens to be in. For the more standard and the more theoretical courses the mathematics department would be the natural place to look for teachers, but where applications are heavily stressed the best teacher may well be found in some other field.

Beyond a fairly standard 15- to 18-hour introduction to physics and chemistry, the program calls for 12 more hours in the basic sciences. Six of these are accounted for by a mechanics course, intended to be a coordinated combination of physics, mathematics, and computing. Considerable time is spent on variational methods and continuum mechanics, as well as on the standard mechanics of particles and rigid bodies.

The remaining six hours is divided between electromagnetics and thermodynamics (including statistical mechanics). Of the many possible continuations of the basic material it was felt that these two, because of their fundamental nature, their wide applicability, and their susceptibility to interesting mathematical analysis, are particularly appropriate to this program.

Modern computing facilities and the techniques for using them are still developing with bewildering rapidity, and no program fixed now will give adequate coverage for very long. We are painfully aware of these rapid developments and claim no special powers of prophecy. The proposal delineated here provides two realistic approaches to current problems of computing. A direct approach is the inclusion of two courses devoted to computation, the Numerical Analysis mentioned above and an Introduction to Computer Science [such as the course C1 in Recommendations for an Undergraduate Program in Computational Mathematics]. These give the principles of modern computation, including the use of a programming language, and their basic applications to mathematical problems.

Less direct, but perhaps of more ultimate importance, is the inclusion of computational methods in connection with each appropriate topic in other courses. Such topics occur in almost all the core courses, but particularly in Differential Equations (both Ordinary and Partial), Mechanics, and Optimization. It is expected that significant problems to program and run on a computer will be part of the work in these courses. Only in this way can a real understanding of the power and (especially) of the limitations of modern computing techniques be communicated.

The overall structure of the core can be seen in the table. Here each entry represents a 3-hour course.

<u>Year</u>	<u>Core Courses</u>		<u>Electives*</u>
I	Calculus	Physics Chemistry	6
	Calculus	Physics Chemistry	6

II	Linear Algebra	Physics Computer Science	6
	Calculus	Physics	9

III	1. Functions of Several Variables	3. Mechanics 4. Numerical Analysis 5. Probability	3
	2. Ordinary Differential Equations	3. Mechanics 5. Statistics	6

IV	6. Complex Variables	8. Electromagnetics	9
	7. Functional Analysis	9. Thermodynamics	9

V	10. Partial Differential Equations	11. Optimization	9
	10. Partial Differential Equations		12

* semester hours			

Of an assumed total of 150 semester hours for the five years, 45 hours are devoted to mathematics and computing and 30 to basic sciences. This leaves the remaining 75 hours for humanities electives and for additional courses in engineering, mathematics, and science. With a roughly even split this should satisfy normal requirements. The most demanding of our sample options, Control Theory, specifies only 24 hours, leaving, say, 15 hours for basic engineering and technical electives and 36 hours for the humanities.

The first two years of the program are fairly standard. For the mathematics portion we recommend the sequence of courses described in Commentary on A General Curriculum in Mathematics for Colleges, page 33. In addition to linear algebra and the usual elementary calculus of one or more variables this includes an introduction to differential equations.

Introductory physics and chemistry courses are at present under intensive review by professional groups; some radically new physics courses have recently appeared and other experimental programs are underway. We therefore refrain from specifying these courses in any detail but urge the reader to consult the publications of the Commission on College Physics and the Advisory Council on College Chemistry. Widespread adoption of new elementary courses could require great changes in the content, or even in the selection, of the later science courses in the core.

The introductory computer science course should include a discussion of the nature of an automatic computer and the manner in which it solves problems, an introduction to a specific computer language and its role in this process, and some practice in the actual solution of various types of problems. Either by means of this course or from supplementary instruction, students should be able to program and run simple problems early in their sophomore year.

The third year has the heaviest concentration of core courses, foundation for the more advanced material in the core and for the technical applications in the options. These courses are of fairly standard type except for Mechanics, which has been described, and Intermediate Ordinary Differential Equations. The occurrence of considerable material on differential equations in the calculus course justifies the initial adjective in the latter title and permits the course to concentrate on linear equations with variable coefficients, boundary value problems, and special functions. There is also a brief introduction to nonlinear equations.

The report Commentary on A General Curriculum in Mathematics for Colleges outlines three courses in Functions of Several Variables (Mathematics 5. Multivariable Calculus II.). The first of these has a classical vector analysis approach, while the second uses differential forms; the third is particularly suited for students in statistics. We recommend the second course, outlined on page 77, partly because the vector technique is covered in the physics courses but also because the more general approach is a valuable background for fourth-year Functional Analysis.

With the exception of the Probability and Statistics all the third-year work is closely interconnected, and considerable thought should be given to the sequence of topics so as to get the most coordination. In particular, linear algebra, numerical techniques, and the use of a computer to solve problems are ever-recurring themes in the year's work.

The fourth and fifth years of the core are fairly light, since here will come most of the work in the options. In the mathematics courses, in addition to the obvious requirements of Complex Variables and Partial Differential Equations--six hours of the latter is necessary for any sort of coverage, we have included courses in Functional Analysis and Optimization. The first of these provides an introduction to some hitherto abstract topics that are proving useful in a

variety of applications, such as numerical analysis, communication theory, quantum physics, and many branches of mathematics. General metric and linear spaces, operators and functionals, with an introduction to measure theory, are the central topics.

Optimization is another introductory course, but one tied very closely to applications. Based on the notions of compactness, convexity, and Lagrange multipliers, it treats briefly the various types of mathematical programming, some combinatorial problems, and the calculus of variations.

The two science courses in the fourth year, Electromagnetics and Thermodynamics, could vary considerably in content. In any case, however, they should take advantage of the students' exceptional background in analysis, probability, mechanics, and computing to give a considerably more sophisticated treatment than could commonly be contemplated.

The selection and arrangement of the courses comprising the core represent the Panel's best judgment of the curriculum currently needed to develop the kind of highly trained but still flexible engineer described in the Introduction. Local conditions and opinions will undoubtedly suggest some changes, and future developments--for example, an upsurge of biological engineering--may call for a reappraisal of the whole program. But the basic framework of mathematics, science, and computing should still be appropriate for many years to come.

The Options

The role of the option is to provide the student with a solid acquaintance with some branch of engineering, at the same time giving background and applications of many of the subjects treated in the core. In general, serious work in the option will begin in the fourth year, following the heavy load of third-year core courses and some appropriate technical introduction. With this background the work in the option can begin, and proceed, at a higher level of sophistication than is usually possible.

As samples of what might be done we present three options, labeled, for want of better names, Operations Research in Systems Engineering, Orbit Mechanics, and Control Theory. There is nothing special about these; they simply happened to be topics of interest to some of the Panel members and consultants. Other topics of equal suitability might be, for example, Fluid Mechanics, Solid Mechanics, Electronics and Microwaves, Wave Propagation and Plasma Physics, Materials Engineering, and Nuclear Engineering.

For each of the sample options we give here a brief description of the program and an outline of its structure. Detailed syllabi of the courses are given in the Appendix.

OPERATIONS RESEARCH IN SYSTEMS ENGINEERING

Operations Research has been described as the application of mathematical methods to the solution of practical optimization problems in engineering, in business, and in government. The Operations Research Option builds onto the core those features of operations research that pertain especially to the design, development, and production of large-scale engineering systems. These require analysis of the complicated interrelationships among component and system performances, development and production costs, scheduling priorities, available manpower and facilities, and a host of other factors. Such considerations have made necessary the use of various optimization techniques, the application of probabilistic and statistical methods, the development of a highly mathematical reliability theory, Monte Carlo simulation methods, optimal control theory, linear, nonlinear and dynamic programming methods, and queueing theory. These mathematical topics provide the typical tools for operations research studies which find wide applicability in the evaluation (and comparison) of performance, programs, and policies in certain types of engineering and industrial situations.

The Operations Research Option, which has been fleshed out in some detail, represents an attempt to provide suitable training for engineers who have to cope with such problems. Building upon the 6-hour course in Probability and Statistics of the core, it provides a 6-hour course in mathematical methods of reliability engineering. The course includes both probabilistic models of reliability problems and statistical techniques of reliability estimation. The introductory optimization course of the core is supplemented by a further 6-hour course in linear programming techniques, dynamic programming, inventory and scheduling problems, queueing theory, and related topics.

The third course recommended in this option is a 3-hour course in System Simulation, which exploits the use of a computer in carrying out the analysis of such operations research activities.

Additional courses in economics, such as Economic Decision Theory, or in management science would constitute appropriate electives for certain students.

Note that the core course in Optimization has been moved into the fourth year to provide the necessary background for the fifth-year course in Operations Research.

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>	<u>Electives</u>
IV	6. Complex Variables	OR1. Reliability	9
	7. Functional Analysis	OR1. Reliability	
	8. Electromagnetics		
	9. Thermodynamics and Statistical Mechanics		
	11. Optimization		

V.	10. Partial Differential Equations	OR2. Operations Research	15
	10. Partial Differential Equations	OR2. Operations Research	
		OR3. Systems Simulation	

ORBIT MECHANICS

As with each of the options, the aim is to build upon the foundation supplied by the core to provide greater specialization in an aspect of mathematics of central importance in modern engineering, here space science.

The design of space vehicles; prediction, correction, and control of their space flight; transmission and evaluation of information collected in space--all such tasks place unusual new requirements on engineering skills and training. Additional problems arise from the necessity for real-time computations and corrections during space flight. Underlying all these difficulties is the problem of developing a correct physical intuition for the nature of space travel, vehicle control, and environmental conditions in space.

The Orbit Mechanics Option supplements the core courses in mechanics with substantial one-semester courses in celestial mechanics and in orbit theory. The addition of an advanced programming course and an introduction to control theory provides solid grounding for many problems of space vehicle engineering. The course in data smoothing and prediction provides training essential to the successful collection, retrieval, and interpretation of telemetered information.

Additional courses in astronomy or in space physics constitute natural electives for students in such a program.

Since some of Partial Differential Equations is needed for Advanced Numerical Analysis and Celestial Mechanics, the core course 10 must be put in the fourth year. This gives a rather heavy concentration of mathematics in the fourth year, but this could be relieved, if desired, by a further shifting of some of the other courses.

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>	<u>Electives</u>
IV	6. Complex Variables	OM1. Advanced Numerical Analysis	9
	7. Functional Analysis		
	8. Electromagnetics		
	9. Thermodynamics and Statistical Mechanics		
	10. Partial Differential Equations		
	10. Partial Differential Equations		

V	11. Optimization	OM2. Advanced Programming	12
		OM3. Celestial Mechanics	
		OM4. Orbit Theory	
		CT2. Control	
		CT5. Data Smoothing and Prediction	

CONTROL THEORY

The advances in computers and in instrumentation have brought an enormous increase in the sophistication of control systems. The instruments allow us to measure rapidly and precisely many variables which were previously hard to measure, and the computer allows us to

make use of all the data while it is still current. The space program has given a great impetus to control theory by bringing up a number of new problems with very strict requirements. Another aspect of many control problems is that they involve control loops which extend over great distances, thereby creating an interface problem between the control and the communications specialist.

The Control Theory Option starts in the third year with a one-semester course in circuit theory which exposes the student to the modeling problem, to some specific physical devices which he will encounter later, and to basic system concepts in simple physical situations. In the fourth year the control course will furnish the student the basic facts about control systems and the linear systems course will provide the common base for further courses in control, communications, and circuits. The fifth year includes a course on the techniques of optimization, one on advanced control, one on advanced communications, and one on information theory.

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>	<u>Electives</u>
IV	6. Complex Variables	CT2. Control	6
	7. Functional Analysis	CT3. Laboratory	
	8. Electromagnetics	CT4. Linear Systems	
	9. Thermodynamics and Statistical Mechanics	CT5. Data Smoothing and Prediction	

V	10. Partial Differential Equations	CT6. Advanced Control	12
	10. Partial Differential Equations	CT7. Information Theory	
	11. Optimization	CT8. Advanced Communications	

APPENDIX

Sample Outlines of the Courses

The course outlines given in this Appendix or elsewhere in the COMPENDIUM are intended in part as extended expositions of the ideas

that we have in mind, in part as feasibility studies, and in part as proposals for the design of courses and textbooks. They have a wide variety of origins. Some are standard courses now given in universities and some are experiments that have never yet been tried. Most of them, however, are modifications or combinations, more or less radical, of familiar material. They have been prepared by many different persons, with a broad spectrum of interests in mathematics and related fields, and representing industrial as well as academic interests. However, all outlines were carefully scrutinized by the whole Panel and were not accepted until their value to the whole program was clear. For better or worse, this is a committee product.

It will be observed that there is considerable overlapping in some of the content of the courses, for example in Intermediate Ordinary Differential Equations and in Numerical Analysis. This is inevitable in the courses in any modern university, where most courses are taken by a variety of students in different programs and with different backgrounds. If a neater dovetailing of these courses is possible in particular cases, the contents should of course be modified accordingly.

The Core

I, II	Physics	(12)	Calculus and Linear Algebra	(12-15)
	Chemistry	(6)	Computer Science	(3)

III	1.	Functions of Several Variables		(3)
	2.	Intermediate Ordinary Differential Equations		(3)
	3.	Mechanics		(6)
	4.	Numerical Analysis		(3)
	5.	Probability and Statistics		(6)

IV	6.	Complex Variables		(3)
	7.	Functional Analysis		(3)
	8.	Electromagnetics		(3)
	9.	Thermodynamics and Statistical Mechanics		(3)

V	10.	Partial Differential Equations		(6)
	11.	Optimization		(3)

1. Functions of Several Variables. (3 semester hours)

For an outline of this course, see Mathematics 5 (Multivariable Calculus II--alternate version) in Commentary on A General Curriculum in Mathematics for Colleges, page 77.

2. Intermediate Ordinary Differential Equations. (3 semester hours)

For an outline of this course, see Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists, page 643.

3. Mechanics. (6 semester hours)

The course outlined below differs from that given in certain textbooks in that the discussion of mechanics is interrupted at various stages in order to deal with topics in numerical analysis; e.g., after the equations of motion are formulated, various methods for numerically integrating initial value problems are discussed and analyzed. It is assumed that the student has had the linear algebra course as well as the computer science course. Homework assignments that involve use of a computer should be made.

a. Kinematics. (8 lessons) Cartesian coordinates in Euclidean 3-space, cartesian tensors, the numerical tensors δ_{ij} , ϵ_{ijk} . Parametric equations of curves. Velocity and acceleration in cartesian coordinates, in general coordinates. Moving general coordinates and the velocity and acceleration in such coordinates. Equations of straight lines in moving general coordinates. Characterization of inertial coordinate frames.

b. Particle mechanics. (10 lessons) Equations of motion. Initial value problems for a system of ordinary differential equations, existence, uniqueness, continuous dependence on parameters and initial values. Numerical methods for integrating initial value problems, their stability.

c. Perturbation theory. (8 lessons) Physical stability. Numerical stability. Linearization of nonlinear problems.

d. Central forces. (10 lessons) Planetary orbits. Energy integrals, angular momentum integrals. Constants of motion and symmetry properties.

e. Variational principles and rigid body motion. (13 lessons)
Hamilton's principle, generalized coordinates of Lagrange, canonical equations, contact transformations, partial differential equations of Hamilton and Jacobi. Rigid body motion.

f. Multidimensional variational principles. (8 lessons)
Variation of multiple integrals and applications to problems in statics and dynamics of deformable bodies. Vibrating strings and membranes. Rayleigh-Ritz method. Use of polynomials to derive difference equation approximation to the boundary value differential equations that are the Euler equations of a variational principle. Numerical integration of boundary value problems on the line and in the plane.

g. Continuum mechanics. (21 lessons) Stress and strain tensors. Conservation of mass, momentum and energy. Partial differential equations describing the motion of a perfect fluid. One-dimensional isentropic motions (simple and compound waves). Numerical integration of 1-dimensional motions. Existence of shocks. Numerical integration in the presence of shocks.

4. Numerical Analysis. (3 semester hours)

For an outline of this course, see Mathematics 8 (Introduction to Numerical Analysis) in Commentary on A General Curriculum in Mathematics for Colleges, page 83.

5. Probability and Statistics. (6 semester hours)

For an outline of this course, see Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists, page 642.

6. Functions of a Complex Variable. (3 semester hours)

For an outline of this course, see Mathematics 13 (Complex Analysis) in Commentary on A General Curriculum in Mathematics for Colleges, page 97.

7. Introduction to Functional Analysis. (3 semester hours)

For an outline of this course, see Mathematics Q (Functional Analysis) in A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates, page 125.

8. Electromagnetism. (3 semester hours)

This course combines the essentials of classical electromagnetic theory with the foundations of applications to plasma media. It should take advantage of the preparation in mechanics, especially continuum mechanics, as well as of the fundamentals of electricity and magnetism in the introductory physics course.

a. Electrostatics. (4 lessons) Vacuum field and potential theorems. Dielectrics. Boundary conditions. Energy relations and forces.

b. Magnetic fields. (5 lessons) Current and moving charges. The vector potential. Magnetic media. Energy relations and forces.

c. Maxwell's equations. (6 lessons) The law of induction. Maxwell's equations, extended to moving media. Energy, force, and momentum relations in the electromagnetic field.

d. Wave propagation. (8 lessons) The scalar wave equation. Plane, cylindrical, spherical waves. Homogeneous isotropic media. Dispersion. Nonhomogeneous isotropic media. Rays; the geometrical optics approximation. Wave packets, including nonhomogeneous media and absorption.

e. Electromagnetic waves. (7 lessons) Free space and homogeneous isotropic media. Homogeneous plasmas. Inhomogeneous media. Anisotropic media, including plasma with magnetic field.

f. Radiation. (9 lessons) Simple radiating systems. Radiation by moving charges. Radiation in ionized gases. Synchrotron radiation.

9. Thermodynamics and Statistical Mechanics. (3 semester hours)

There is a current trend to combine the macroscopic and the microscopic aspects of thermal physics from the beginning, instead of giving a careful treatment of classical thermodynamics, with applications, as in Thermal Physics by Philip M. Morse (second

edition; Menlo Park, California, W. A. Benjamin, Inc., 1969). The suggested outline follows the latter plan, as probably more appropriate as a background for varied applications.

Thermodynamics.

- a. State variables and equations of state. (3 lessons) Temperature, pressure, heat, and energy. Extensive and intensive variables. Pairs of mechanical variables. The perfect gas and other equations of state.
- b. The first law of thermodynamics. (4 lessons) Work, internal energy, heat. Heat capacities. Isothermal and adiabatic processes.
- c. The second law of thermodynamics. (6 lessons) Heat cycles. Reversible and irreversible processes. Entropy. Applications to simple thermodynamic systems.
- d. The thermodynamic potentials. (3 lessons) Internal energy, enthalpy, Gibbs and Helmholtz potentials. Examples and procedures for calculation.
- e. Phase equilibria. (3 lessons) Melting, evaporation, triple point, and critical point.
- f. Chemical applications. (2 lessons) Reaction heats, electrochemical processes.

Statistical Mechanics (Equilibrium).

- a. Statistical methods. (3 lessons) Random walk; probability distributions; mean values; binomial, Poisson, and Gaussian distributions.
- b. Statistical description of systems of particles. (3 lessons) Ensembles, ergodic hypothesis, postulates, limiting behavior for large N , fluctuations.
- c. Quantum statistics. (5 lessons) Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distributions with applications (solids, gases, electron gas, blackbody radiation, etc.).

Microscopic Description of Nonequilibrium.

- a. Elementary kinetic theory. (2 lessons)
- b. Transport theory. (2 lessons) Based on Boltzmann's equation, in simplified form.

c. Brownian motion. (3 lessons) Possibly including the Fokker-Planck equation. More on random variables. Markov processes, fluctuations, irreversible processes.

10. Partial Differential Equations. (6 semester hours)

This course ordinarily occurs in the fifth year of the sequence, although with certain options (e.g., Orbit Mechanics) it should be taken in the fourth year. The material should strike a reasonable balance between the classical analytical theory of partial differential equations and modern computational aspects of the subject. For that reason, existence theorems and the like should be of the constructive type whenever possible. Further, application to problems in classical and modern physics should constantly be borne in mind. Physical models should be used both to predict results concerning the behavior of solutions to partial differential equations and to interpret phenomena revealed analytically or computationally.

a. Introduction. (6 lessons) Derivation of some equations; discussion of mathematical models, continuous dependence theorems, and relation to physics.

b. Classification and characteristics. (9 lessons) Cauchy problem for first-order equations, formulation and statement of Cauchy-Kowalewski theorem.

c. Hyperbolic equations. (12 lessons) Existence and continuous dependence for second-order equations. Riemann method. Three-dimensional wave equation. Retarded potentials. Numerical methods--finite difference schemes and stability.

d. Elliptic equations. (21 lessons) Potential theory in three dimensions with smooth boundaries. Eigenvalue problems--estimates. Numerical methods.

e. Parabolic equations. (12 lessons) Thermal potential theory. Convergence to steady state and relation to potential problems. Numerical methods and connection, in steady state, to numerical methods for elliptic problems.

f. Integral representation of solutions. (12 lessons) Green's functions. Integral equations.

g. Equations of hydrodynamics. (6 lessons) Shock phenomena, weak solutions. Numerical methods.

11. Optimization. (3 semester hours)

For an outline of this course, see Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists, page 647. A natural successor to this course is the 6-semester-hour course Operations Research in the Operations Research Option, where linear programming techniques are developed in depth and additional topics in dynamic programming, inventory and scheduling problems, Monte Carlo simulation techniques, and queueing theory are introduced.

OPERATIONS RESEARCH IN SYSTEMS ENGINEERING OPTION

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>
III	1. Functions of Several Variables (3)	
	2. Intermediate Ordinary Differential Equations (3)	
	3. Mechanics (6)	
	4. Numerical Analysis (3)	
	5. Probability and Statistics (6)	

IV	6. Complex Variables (3)	OR1. Reliability (6)
	7. Functional Analysis (3)	
	8. Electromagnetics (3)	
	9. Thermodynamics and Statistical Mechanics (3)	
	11. Optimization (3)	

V.	10. Partial Differential Equations (6)	OR2. Operations Research (6)
		OR3. Systems Simulation (3)

OR1. Quantitative Methods in Reliability Engineering. (6 semester hours)

The growing complexity of systems over the last two decades has inspired the development of a body of quantitative methods for developing, improving, and measuring system reliability. Since reliability is such a critical factor in the successful completion of every highly technical program, it is of value to the engineer to learn quantitative reliability theory, as presented in the following course. The mathematical and statistical methods of the course are not simply routine applications of well-known theory, but in many cases represent new developments motivated by reliability problems.

The 6-semester-hour course in Probability and Statistics of the core is an essential prerequisite for this course.

First Semester: Probabilistic Models in Reliability.

a. Failure distributions in reliability theory. (8 lessons) Typical failure laws. The exponential as the failure law of complex equipment. Monotone failure rates. Bounds for distributions with monotone failure rate. General failure rates.

b. Prediction of system reliability from a knowledge of component reliabilities. (3 lessons) Analytical methods for computing reliability exactly. Bounds on system reliability based on paths or cuts. Monte Carlo methods. Qualitative relationships for multi-component structures.

c. Redundancy optimization. (5 lessons) Optimal allocation of redundancy subject to constraints. Optimal redundancy assuming two kinds of failure.

d. Operating characteristics of maintenance policies. (6 lessons) Renewal theory. Replacement based on age. Comparison of age and block replacement policies. Random replacement. Repair of a single unit.

e. Optimum maintenance policies. (4 lessons) Replacement policies. Inspection policies.

f. Stochastic models for complex systems. (8 lessons) Semi-Markov processes. Repairman problems. Marginal checking. Optimal maintenance policies under Markovian deterioration.

Second Semester: Statistical Reliability Theory.

a. Estimating reliability parameters assuming form of distribution known. (8 lessons) Maximum likelihood estimation in the

case of normal, exponential, gamma, Weibull, and binomial distributions. Confidence and tolerance limits in these cases. Minimum variance unbiased estimation in these cases.

b. Estimating reliability parameters under physically plausible assumptions. (9 lessons) Errors resulting from incorrect assumption as to form of failure distribution. Maximum likelihood estimation assuming a monotone failure rate. Maximum likelihood estimation assuming a decreasing and then increasing failure rate. Conservative confidence and tolerance limits.

c. Estimating reliability growth. (7 lessons) Form of growth assumed known. Only monotonicity of reliability assumed. Conservative confidence limits.

d. Confidence limits on system reliability using observations on individual components. (6 lessons) Success or failure observations. Life length observations. Asymptotic methods.

e. Hypothesis testing. (9 lessons) Acceptance sampling, fixed sample size, truncated and censored sampling, sequential sampling. Accelerated life testing. Testing for monotone failure rate.

OR2. Operations Research. (6 semester hours)

The accent is on the mathematical aspects of the subject, rather than the management or industrial engineering aspects. It is assumed that time and facilities are available for a computation laboratory in connection with both semester courses. A prerequisite of an introductory computer programming course is desirable.

The first semester develops linear programming in depth, building on the preparation given in a previous one-semester course in Optimization.

First Semester: Advanced Linear Programming.

a. Review of the simplex algorithm. (4 lessons) Variations of the simplex algorithm. Degeneracy, perturbation. Revised simplex method.

b. Games and linear programs. (4 lessons) Matrix games. Equivalence of matrix games and linear programs.

c. The transportation problems. (4 lessons) Elementary

transportation theory. The transshipment problem.

- d. Networks and the transshipment problem. (4 lessons)

Graphs and trees. Interpreting the simplex method on the network.
The shortest route problem.

- e. Variables with upper bounds. (3 lessons) The general case. The rounded variable transportation problem.

- f. Programs with variable coefficients. (3 lessons) Wolfe's generalized program. Special cases.

- g. Decomposition principle for linear programs. (9 lessons)
The general principle. Decomposing multistage programs.

- h. Convex programming. (4 lessons) General theory. Separable convex objectives. Quadratic programming.

- i. Discrete variable problems. (4 lessons) Survey of methods. Gomory's method of integer forms.

Second Semester: Dynamic Programming and Stochastic Models.

- a. Dynamic programming. (10 lessons) Principle of optimality. Multistage allocation problems. Arrow-Harris-Marschak inventory model.

- b. Dynamic programming and Markov processes. (5 lessons)
Discrete dynamic programming. Optimal policies with discounted returns.

- c. Monte Carlo techniques. (10 lessons) Production of random variables by computer. Simulating stochastic systems on the computer.

- d. Mathematical theory of queues. (14 lessons) Single server; Poisson input; exponential service. Many servers; Poisson input; exponential service. The busy period. Stochastic inventory models.

OR3. Systems Simulation. (3 semester hours)

This course examines those symbol manipulation applications of the computer that involve the numerical and logical representation of some existing or proposed system, for the purpose of experimenting with the model and of comparing methods of operating the system. The primary purpose of the computer is thus not a calculating adjunct to experimentation but is the experimental medium itself. A course in probability and statistics is a prerequisite.

a. Programming languages. (11 lessons) Special languages designed for use in simulation, such as SIMSCRIPT and GPSS. Additional study of the languages will arise in their use throughout the rest of the course.

b. Technical problems of simulation. (14 lessons) Synchronization of events, file maintenance, random number generation, random deviate sampling.

c. Statistical problems peculiar to simulation. (7 lessons) Sample size estimation, variance reducing techniques, problems of drawing inference from a continuous stochastic process.

d. Applications. (7 lessons) Queueing models; storage, traffic, and feedback systems; design of facilities and operating disciplines.

ORBIT MECHANICS OPTION

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>
III	1. Functions of Several Variables (3)	
	2. Intermediate Ordinary Differential Equations (3)	
	3. Mechanics (6)	
	4. Numerical Analysis (3)	
	5. Probability (3)	

IV	6. Complex Variables (3)	OM1. Advanced Numerical Analysis (3)
	7. Functional Analysis (3)	
	8. Electromagnetics (3)	
	9. Thermodynamics and Statistical Mechanics (3)	
	10. Partial Differential Equations (6)	

V	11. Optimization (3)	OM2. Advanced Programming (3)
		OM3. Celestial Mechanics (3)
		OM4. Orbit Theory (3)
		CT2. Control (3)
		CT5. Data Smoothing and Prediction (3)

OM1. Advanced Numerical Analysis. (3 semester hours)

This course, with its emphasis on topics in partial differential equations and elementary functional analysis, demands a reasonable amount of mathematical maturity. It should be taken after the first semester of Partial Differential Equations.

a. Matrix inversion and matrix eigenvalues. (10 lessons)

Review and extension of iterative methods. Jacobi, Householder, and other methods of finding eigenvalues. Ill-conditioning and error analysis.

b. Ordinary differential equations, boundary value problems, eigenvalue problems. (11 lessons) Finite difference methods, extremal principles.

c. Partial differential equations of second order. (18 lessons) Topics selected from the following: Classification, analytical solutions of well-posed problems for single equations; maximum principles for elliptic and parabolic equations, L_2 --or energy--estimates as well as pointwise estimates of solutions; hyperbolic equations, domain of dependence; Fourier analysis and stability for constant coefficient equations, eigenvalues for elliptic equations, iterative methods for difference equations arising from partial differential equations.

OM2. Advanced Programming. (3 semester hours)

This course deals with various types of computer programming and serves to introduce students to the concepts involved in current work in this area. An introductory course in computer science is a prerequisite, and it is assumed that the students have considerable facility in programming with FORTRAN or ALGOL.

a. Survey. (9 lessons) Assembly systems, methods of storage allocation when using these, pseudo-orders, macros, modify and load techniques, monitor and executive systems.

b. Structure of languages. (15 lessons) Study of a particular language such as ALGOL, its ambiguities, its method of dealing with recursions and procedures. List-processing languages, compiler-writing languages.

c. Theory of compilers. (15 lessons) Nature of syntax-

directed compilers, compilers for dealing with problem-oriented languages, compilers for dealing with compiler syntax languages. Discussion of the evolution of a translator from a simple language whose translator is given in machine language.

OM3. Celestial Mechanics. (3 semester hours)

This course in celestial mechanics concerns itself with the mathematical structures underlying the physical theory, and with deriving from them methods which are commonly used to attack the fundamental problems of interest to space research and technology.

a. Introduction. (6 lessons) Description of dynamical systems by means of Lagrangian functions. Lagrangian equations of motion. Ignorable coordinates, energy equation, and other elementary instances of first integrals. Liouville systems in general. Classical examples: harmonic oscillator, simple pendulum, spherical pendulum, central forces, a charged particle in an electromagnetic field, solid body. The principle of dynamical analogy.

b. Phase space. (6 lessons) Legendre duality (with a suggestion about how it is applied to derive state functions in classical thermodynamics). Transition from Lagrangian functions and Lagrangian equations to Hamiltonian functions and canonical equations. Canonical mapping: its definition, its multiplier, and its residual functions. Completely canonical mappings; canonical extensions of coordinate transformations. Generation of canonical mappings by numerical functions. Invariance with respect to the group of canonical mappings: canonical equations, Poisson brackets, Lagrange parentheses.

c. Canonical constants of a dynamical system. (6 lessons) Definition of a set of canonical constants of integration. Variation of canonical constants. The Hamilton-Jacobi equation as an algorithm for constructing sets of canonical constants. The action and angle variables. Separable Hamiltonians; Staeckel systems and Liouville systems. Applications: the problem of two bodies, the problem of two fixed centers. Normal modes of vibrations and vibrations of molecules.

d. Integrals of a dynamical system. (9 lessons) Poisson's theorem about the bracket of two integrals and its dual application to Lagrange parentheses. Integrals in involution; Liouville's theorem. Jacobi-last multiplier. Application to the motion of a solid body. Whittaker's adelphic integral. Application to the investigation of a dynamical system around the equilibrium. Isoenergetic reduction. Application to the regularization and the binary collisions in the problem of two bodies and in the restricted problem of three bodies.

e. Perturbation theory. (12 lessons) Poincaré's method of the small parameter. Birkhoff's method of iterative canonical mappings. Application to the motion of a satellite of an oblate planet.

OM4. Orbit Theory. (3 semester hours)

The purpose of this course is to offer illustrations of mathematical principles and to compel the student to master them securely by careful numerical examples. The material should be arranged to provide a nearly continuous flow of computational work for the laboratory sessions no matter at what level the course is set.

The instructor and his students should be directed to develop the topics all the way down to an efficient and reliable program in the FORTRAN or ALGOL language on an electronic computer. As most of the textbooks on the subject matter cater to computers who use logarithms or hand-operated desk-model calculating machines, special care should be taken to rearrange classical algorithms and formulas for use on an electronic computer.

Topical problems should be selected from at least these four main research areas:

a. Orbit determination. (10 lessons) The obvious reference here is P. Herget's The Computation of Orbits, published privately by the author at Cincinnati Observatory, 1948. This booklet has been updated by the author in his lectures on "Practical Astronomy" and on "Orbit Determination" given at the summer course in Space Mathematics, Cornell University, 1963.

b. Orbit analysis. (12 lessons) The question here is to gain physical information from the comparison between the orbit as

it has been observed and the orbit as it has been computed from a particular mathematical model.

c. Orbit design. (7 lessons) How to produce orbits that satisfy a priori conditions (e.g., given initial conditions, mission requirements, optimum characteristics, etc.).

The course should limit itself to well-tried problems and should aim at producing examples where good-quality results can be reached without too much effort. This can be achieved in the restricted problem of three bodies.

After an introduction to that problem, the instructor should review two or three methods for integrating numerically the equations of motion, either in cartesian coordinates or in regularized coordinates. Then should come a development on variational equations. Thereafter the theory of characteristic exponents should be applied to the analysis of a family of periodic orbits.

d. Analytical theories. (10 lessons) How to expand on literal theory by enabling an electronic computer to handle symbol manipulations in a given algebra.

This new field promises to provide mathematicians with powerful tools to develop literal theories in an extensive set of physical problems.

CONTROL THEORY OPTION

<u>Year</u>	<u>Core Courses</u>	<u>Option Courses</u>
III	1. Functions of Several Variables (3)	CT1. Electric Circuits (3)
	2. Intermediate Ordinary Differential Equations (3)	
	3. Mechanics (6)	
	4. Numerical Analysis (3)	
	5. Probability and Statistics (6)	

IV	6. Complex Variables (3)	CT2. Control (3)
	7. Functional Analysis (3)	CT3. Laboratory (3)
	8. Electromagnetics (3)	CT4. Linear Systems (3)
	9. Thermodynamics and Statistical Mechanics (3)	CT5. Data Smoothing and Prediction (3)

V	10. Partial Differential Equations (6)	CT6. Advanced Control (3)
		CT7. Information Theory (3)
	11. Optimization (3)	CT8. Advanced Communications (3)

CT1. Electric Circuits. (3 semester hours)

This is a basic course in circuit theory. The purpose is to teach in a precise language the fundamental facts of circuit theory while developing skills in writing and solving the circuit equations and keeping close contact with physical circuits (filters, amplifiers, digital circuits).

a. Lumped circuits. (3 lessons) Lumped circuit approximation. Kirchhoff laws, relation to Maxwell's equations. Circuit elements; including nonlinear and time-varying elements.

b. Simple circuits. (13 lessons) First- and second-order circuits: zero-input response and zero-state response; response to step, impulse, and sinusoid. Linearity and time-invariance: convolution. Impedance, phasors, frequency response, and resonance.

c. Coupling elements. (2 lessons) Coupled inductors, transformers, and dependent sources. Dependent sources as parts of models for electronic devices.

d. Power and energy. (2 lessons) Energy stored and power dissipated in elements; relation with real and imaginary part of impedance.

e. General methods of analysis. (6 lessons) Graph theory: trees, links, cut-sets, loops. Loop and cut-set analysis, mixed method. Duality. Computer programs for analysis of circuits.

f. Linear time-invariant circuits. (6 lessons) Reduction of systems of equations. Network functions: poles, zeros, gain and phase.

g. Network theorems. (3 lessons) Superposition, Thévenin, Norton, reciprocity. Careful discussion of range of applicability: comments on nonlinear and time-varying circuits.

h. Two-port description. (4 lessons) Two-port description of electronic devices: relation to their graphical characteristics. Linear time-invariant networks as two-ports. Interconnection of two-ports.

CT2. Control. (3 semester hours)

In this course the student learns some basic facts about control systems, their analytical description, and techniques of design. This course is mostly concerned with single-variable control.

a. Description of feedback systems and components. (5 lessons) Advantages and disadvantages of feedback, importance of measuring device, noise problems. Basic components, electrical, hydraulic, pneumatic. Requirements and specifications of control systems. Examples.

b. Linear time-invariant control systems. (20 lessons) Analysis, illustrated by several extensive examples, based on differential equations and integral equations (convolution). Stability, root locus, Nyquist criterion. Design of compensating networks to obtain stability and meet the specifications.

c. Sampled systems. (6 lessons) Examples of systems where the feedback data are naturally sampled periodically. Analysis of sampled systems. Stability, root locus, Jury's criterion. Design of compensating networks.

d. Nonlinear systems. (8 lessons) Local stability near equilibrium. Example of limit cycles. Stability: approximate methods describing functions. Lyapunov's second method. Application to design.

CT3. Laboratory. (3 semester hours)

The purpose of the laboratory is to insure that students connect the classroom concepts and results to physical reality and appreciate the power and limitations of experimental work. Typically, one part of the laboratory could be devoted to circuits work: behavior of linear circuits (including resonance), effects of nonlinear elements on waveform and power spectrum, some pulse circuits. The second part of the laboratory would cover control: study of a typical control system, experiments with various compensations; stability; experiments with and simulation of a nonlinear control system.

CT4. Linear Systems. (3 semester hours)

Purpose: to provide a solid foundation of concepts, facts, and techniques to be used in later courses in control, communication, and circuits.

a. Systems. (6 lessons) State as a parametrization of input-output pairs and as part of the system description. Operator point of view. State equivalence. Linear systems. Linear systems obtained by linearization of ordinary differential equation about a nominal trajectory. Examples throughout.

b. Linear systems of the form
$$\begin{aligned}\dot{x} &= Ax + By \\ \dot{y} &= Cx + Dy\end{aligned}$$
 and their discrete time analogs. (6 lessons)

Time-invariant case: explicit solution by function of a matrix and Laplace and z-transforms. For simple linear operators, diagonalization, mode interpretation (including numerical techniques). Jordan form, analog computer interpretation. Time-varying case: properties of the state transition matrix. Periodic systems: Floquet theory, kinematic equivalence.

c. Impulse response and transfer functions. (12 lessons) Free use of Fourier and Laplace transforms. Superposition integral. Asymptotics of impulse response and transfer function. Minimum phase. Uncertainty principle. Group delay. Signal flow graphs.

d. Stability. (9 lessons) Characterization of stability for linear time-invariant, periodic and time-varying systems (zero-input stability: Liénard Chipart, Nyquist; bounded input implies bounded output; implications of an impulse response which is in L^1), Lyapunov method.

e. Input-output description and state equations. (6 lessons) Controllability, observability, and normality. Characterization in time-invariant and time-varying cases. Output controllability. Controllability and observability of an interconnection of systems.

CT5. Data Smoothing and Prediction. (3 semester hours)

a. Representation of functions by Fourier series and integrals. The Fourier transform in L^1 and L^2 . (8 lessons)

b. Random processes: definition, examples, representations; autocorrelation, power spectrum; estimation of spectral densities. (14 lessons)

c. Linear mean-square estimation, filtering and prediction. The Wiener-Hopf equation; solution by the Wiener filter and Kalman-Bucy filter. (11 lessons)

d. Detection and parameter estimation. Application to digital communications system and radar. (6 lessons)

CT6. Advanced Control. (3 semester hours)

This course treats advanced topics in control so that students can readily read the current literature. Multiple-input multiple-output systems are included.

a. Nonlinear control. (10 lessons) Describing function. Subharmonics. Stability: Sandberg circle criterion, Popov-type criteria. Lyapunov method used as a design tool. Lyapunov method for systems with inputs. Bounds on output.

b. Adaptive control. (8 lessons) Examples of adaptive control: identification techniques and parameter adjustment. Stochastic approximation.

c. Optimum control. (21 lessons) Formulation of the problem. Examples: systems described by ordinary differential equations and difference equations. Maximum principle for differential systems. Numerical methods. Relation of maximum principle with steepest descent.

CT7. Information Theory. (3 semester hours)

a. The concept of the source and of an information measure. Desirable properties of information measure, examples of simple sources. (3 lessons)

b. Codes, their efficiency and redundancy. The efficient

encoding of discrete independent sources. (4 lessons)

c. General discrete sources, Shannon's encoding theorem, the nature of written and spoken English. (4 lessons)

d. The concept of a channel, channel capacity, symmetry of a channel. (3 lessons)

e. The fundamental theorem of information theory, error-detecting and error-correcting codes, the geometric interpretation of coding problems. (17 lessons)

f. Generalization to continuous channels, channel capacity of continuous channels. (8 lessons)

CT8. Advanced Communications. (3 semester hours)

a. Review. (4 lessons) Signal and noise representations, the purpose of modulation.

b. Amplitude modulation. (7 lessons) The generation and detection of AM waves, power spectrum, single side-band and vestigial side-band transmission, effects of distortion and noise.

c. Frequency and angle modulation. (12 lessons) Generation, detection, power spectrum, effects of distortion and noise.

d. Pulse modulation. (10 lessons) Pulse amplitude modulation, pulse position modulation, pulse duration modulation. A brief introduction to pulse code modulation.

e. Design. (6 lessons) The design of optimum receivers in the presence of additive noise and fading.