QUALIFICATIONS FOR TEACHING UNIVERSITY-PARALLEL MATHEMATICS COURSES IN TWO-YEAR COLLEGES

A report of

The ad hoc Committee on Qualifications for a Two-Year College Faculty in Mathematics

August 1969

INTRODUCTION

CUPM has published reports on the qualifications needed by teachers of the GCMC curriculum and on the adaptation of that curriculum to the circumstances of university-parallel programs in two-year colleges (Qualifications for a College Faculty in Mathematics (1967) and A Transfer Curriculum in Mathematics for Two-Year Colleges (1969)). The present report is an effort to describe the qualifications desirable for faculty members teaching courses in the university-parallel or transfer programs in two-year colleges.

Our comments and recommendations are addressed to administrators of two-year colleges, to university mathematics departments, to mathematics teachers in two-year colleges, and to those contemplating careers as mathematics teachers in two-year colleges. The concluding section of our report offers specific advice to each of these four groups.

We discuss the qualifications of teachers of the following set of courses, whose subject matter can be thought of as a working definition of university-parallel mathematics.

Mathematics 0. Elementary Functions and Coordinate Geometry. A one-semester course in coordinate geometry and the properties of the elementary functions.

Mathematics A. Elementary Functions and Coordinate Geometry (with Algebra and Trigonometry). A more slowly paced version of Mathematics 0 in which are embedded some topics from high school algebra and trigonometry. This course is to be thought of as extending over more than one semester.

Mathematics B. Introductory Calculus. An intuitive one-semester course covering the basic concepts of single-variable calculus.

Mathematics C. Mathematical Analysis. A two-semester course completing the study of elementary calculus.

Mathematics L. Linear Algebra. A sophomore-level one-semester introduction.

Mathematics PS. Probability and Statistics. An elementary one-semester course (not having calculus as a prerequisite) suitable for students in business and social sciences.

Mathematics NS. The Structure of the Number System. A two-semester course recommended by the CUPM Panel on Teacher Training for beginning the preparation of elementary school teachers. [Since this report was written, the recommendations on teacher training have been revised. See the 1971 publication Recommendations on Course Content for the Training of Teachers of Mathematics, page 158.]

The detailed discussion of these courses will be found in the CUPM report A Transfer Curriculum in Mathematics for Two-Year Colleges, mentioned above. In addition, this report suggests that, under certain circumstances, it may be advisable for a two-year college to offer additional courses and suggests a selection from among the following: further courses for elementary school teachers; finite mathematics; a calculus-based course in probability; numerical analysis and intermediate differential equations (or differential equations with topics from advanced calculus). For suggested work in computing, see the course C1 in Recommendations for an Undergraduate Program in Computational Mathematics, page 563.

Our recommendations are intended to apply to all instructors who teach any such university-parallel courses. We are aware of the great importance in two-year colleges of courses in mathematics for students in occupational and technical curricula and of courses designed for students lacking even basic mathematical skills. We are also aware of the existence of difficult and challenging pedagogical and curricular questions related to such courses. We have chosen to wait until there is a better resolution of these questions before seeking to formulate recommendations about the proper qualifications for teaching courses that are not parallel to those commonly offered by four-year colleges and universities. [Some of these questions are discussed in A Course in Basic Mathematics for Colleges.]

The university-parallel role of the two-year college is of increasing importance in the educational system. For example, in California 86% of all freshmen in publicly supported institutions in 1966-67 were in two-year colleges. The percentage of college students enrolled in two-year colleges has been increasing rapidly nationwide. There are already a number of universities in which the junior class is larger than the freshman class. Moreover, a majority of students entering two-year colleges intend to continue their education at least to the bachelor's degree. Thus, university mathematics departments must recognize that the university-parallel courses taught in two-year colleges are becoming an integral part of the university program in mathematics.

Conversely, recent trends in four-year institutions are placing new demands on teaching of mathematics in two-year colleges. As students come to colleges with better preparation in mathematics, many courses are moving downward toward the freshman year. It should be recognized that those now being trained or hired as teachers in two-year colleges must be prepared to deal at some time in the near future with subjects that are now thought of as belonging to the junior or senior years.

The degree most commonly held by mathematics teachers in two-year colleges is the master's degree, but this degree is of such varying quality that it is scarcely useful as a measure of qualification for appointment, promotion, or tenure. We feel it necessary to make recommendations which are independent of degrees held or of total credit hours earned in mathematics courses but which deal,

rather, with the substance of the mathematical training of prospective faculty members.

It should be understood that no academic program or degree in itself qualifies an individual to teach effectively at any level unless this preparation is accompanied by a genuine interest in teaching and by professional activities reflecting continuing mathematical growth. These activities may assume many forms:

a. taking additional course work

b. reading and studying to keep aware of new developments and to explore new fields

c. engaging in research for new mathematical results (even if unpublished)

d. developing new courses, new ways of teaching, and new classroom material

e. publishing expository or research articles

f. participating in the activities of professional mathematical organizations

This list reflects our conviction that an effective teacher must maintain an active interest in the communication of ideas and have a dedication to studying, learning, and understanding mathematics at levels significantly beyond those at which he is teaching.

A two-year college mathematics department, whose staff members are engaged in activities such as those described above and have the academic qualifications to be described below, should have confidence in its ability to provide the quality of teaching required of it.

THE FORMAL EDUCATION OF MATHEMATICS TEACHERS IN TWO-YEAR COLLEGES

The university-parallel courses in mathematics that a teacher in a two-year college should be able to teach effectively have been described in the previous section. We shall now set forth our recommendations for the mathematical qualifications for the teachers of these courses.

This mathematical background falls into two distinct components: a basic component which consists of a strong mathematics major program and a graduate component which embodies the require-

ment that a teacher at a two-year college must have a knowledge of mathematics well beyond that which he will be asked to teach.

Basic Component

The basic component of mathematics courses for the two-year college teacher is most succinctly described as a solid grounding in analysis and algebra, with additional courses in geometry, computer science, and probability providing greater breadth of knowledge.

We assume that the prospective teacher has mastered the following lower-division undergraduate material, as described in the CUPM publication Commentary on A General Curriculum in Mathematics for Colleges.

a. Calculus courses in one and several variables, including an introduction to differential equations (Mathematics 1, 2, 4)

b. The fundamentals of computer science, including experience in programming as well as the use of a computer [See, for instance, the course C1, page 563]

c. A semester course in linear algebra employing both matrices and a basis-free, linear transformation approach (Mathematics 3)

In addition, the prospective teacher should attempt to obtain as many of the following upper-division courses as he can at the undergraduate level.

a. A semester course in advanced multivariable calculus, covering differential and integral vector calculus, including the theorems of Green and Stokes, and an introduction to Fourier series and boundary value problems (Mathematics 5, first version)

b. A year's work in abstract algebra, treating the important algebraic systems (groups, rings, modules, vector spaces, and fields) and thoroughly developing the basic concepts of homomorphism, kernel, and quotient construction, with applications and consequences of these ideas. (This course is described in the CUPM report Preparation for Graduate Study in Mathematics, page 453. See also the courses Mathematics 6M and 6L in Commentary on A General Curriculum in Mathematics for Colleges, page 65 .)

c. A thorough year's course dealing with the important theorems in real analysis, with emphasis on rigor and detailed proofs. The treatment should use metric space notions and should lead to a detailed examination of the Riemann-Stieltjes integral. (Mathematics 11-12)

d. A semester course in complex analysis, covering Cauchy's theorem, Taylor and Laurent expansions, the calculus of residues, and analytic continuation, with application of these ideas to transforms and boundary value problems (Mathematics 13)

e. A semester course in applied mathematics. The student should be introduced to applications of mathematics in order that his teaching might better reflect the relevance of mathematical ideas. [The courses described in the 1972 publication Applied Mathematics in the Undergraduate Curriculum are suitable.]

f. A semester course in which the student studies some geometric subject such as topology, convexity, affine and projective geometries, differential geometry, or a comparative investigation of Euclidean and non-Euclidean geometries (Mathematics 9)

g. A year's course in probability and statistics that reflects the growing importance of this subject to engineering and the biological, social, and management sciences [Mathematics 7 or the course outlined in Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists, page 642]

If the student has not completed all of the upper-division courses of this strong mathematics major as an undergraduate, then he should cover material comparable to that of the omitted courses during his graduate training.

Graduate Component

Graduate (one-semester) courses which are especially appropriate for the graduate component are:

P Measure and Integration

Q Functional Analysis

R Complex Analysis

S General Topology

T Homology and Multivariable Integration

U Topology and Geometry of Manifolds

V Galois and Field Theory

W Ring Theory and Multilinear Algebra

X Advanced Ordinary Differential Equations with Applications

Y Problem-oriented Numerical Analysis

Z Seminar in Applications

Of these, P, S, and X should be in the program of every prospective two-year college teacher.

Detailed descriptions of these courses are given in the CUPM report A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates, page 113. The program presented there is designed to prepare teachers to function in the first two years of a four-year college with occasional teaching of upper-division courses. It does not differ greatly from our program: In the four-year college report, instead of P, S, and X, the courses P, Q, S, and T are regarded as essential, and the material on probability and statistics, applied mathematics, and differential equations serves as a pool of courses on the applications of mathematics from which a year sequence is to be elected by the student. These differences are due to the fact that a two-year college teacher has less access to the services of experts in specific fields and, consequently, needs a somewhat broader training.

It should be emphasized that course X in differential equations is not a second undergraduate course in the subject, but is to be a genuine graduate course at least on the same level as the course in measure and integration. The graduate course in applied mathematics which the Committee most strongly favors is one (not yet commonly offered) which stresses the formulation and analysis of mathematical models in diverse fields, using the calculus, probability, and linear algebra of the first two undergraduate years. Course Z is of this type.

Students who plan to continue into advanced graduate work and to specialize in some area of pure mathematics are advised to take as many as possible of the other courses in the list. Other students may substitute electives to obtain a deeper knowledge of some other area of mathematics or computer science.

Undergraduate mathematics, especially in the lower division, is heavily slanted toward real analysis; courses in general topology and measure theory provide essential background for teaching courses in calculus and probability. If further work in analysis is elected, we recommend a study of functional analysis (course Q) in preference to complex analysis (course R) as a sequel to measure theory, as the former will further develop the ideas of linear algebra and the concept of uniform convergence. In spite of its importance for more advanced work in pure mathematics, a second year of abstract algebra to follow the strong undergraduate algebra course described in the basic component is not recommended as essential for teachers of mathematics in a two-year college.

The graduate component of courses should be augmented by two particular features to prepare the prospective teacher for the two-year college mathematics faculty.

First, a year's work focused on the problems of lower-division undergraduate teaching, such as an apprenticeship in teaching as

described below, preferably carried out at a nearby two-year college.

Second, a comprehensive examination designed specifically to test the breadth and depth of a candidate's understanding of mathematics relevant to the undergraduate curriculum.

A student who has a strong undergraduate major in mathematics will be able to complete the program in one year, even if he has not completed all of the courses listed in the basic component. For a student whose prior training is not substantially that of the basic component, the completion of the graduate component may require two years of study beyond his bachelor's degree. For example, if his undergraduate major does not include strong preparation in algebra and analysis, his program might be as follows:

First Year (both semesters)
- Abstract Algebra
- Real Analysis (Mathematics 11-12)
- Probability and Statistics
- Apprenticeship in Teaching

Second Year

First Semester
- Measure and Integration (P)
- Complex Analysis (Mathematics 13)
- Topology (S)
- Apprenticeship in Teaching

Second Semester
- Applied Mathematics (Z or one of the courses outlined in Applied Mathematics in the Undergraduate Curriculum)
- Advanced Differential Equations (X)
- Apprenticeship in Teaching
- Comprehensive Examination

The mathematical background in the graduate component, if satisfactorily completed, will permit the student to teach with confidence the university-parallel courses of the two-year college. Moreover, new courses, as they arise, should be well within his competence to prepare.

Apprenticeship in Teaching

An important component in the training of teachers of mathematics for two-year colleges is an understanding of the teaching and learning processes as they apply to these institutions. One of the best ways for the potential instructor to gain this kind of knowledge and experience is through a supervised teaching activity. This activity preferably should take place in a two-year college, but it can, if necessary, be carried out in a four-year institution in appropriate courses.

Most value will be obtained if apprentice teachers receive experience in a variety of courses involving a heterogeneous group of students with differing career aspirations, comparable to the situation that they will encounter in most two-year colleges.

The success of an apprenticeship program will depend significantly upon the attitude of the graduate faculty. If effective teaching is regarded as an important function of the department, and if senior mathematicians encourage excellence in teaching by precept and by example, the apprentice teachers will respond accordingly.

The work assignment of the apprentice should be carefully graduated and should always involve close contact with and supervision by a senior colleague. The apprentice should have frequent opportunities to go over purposes, methods, and content with his supervisor. Arrangements should be made for frequent post-teaching conferences in which the teaching and learning problems encountered are reviewed and solutions suggested. This can be done individually or in a group for all apprentices in the program. Valuable contributions can be made to such seminar sessions by mathematics instructors from two-year colleges and by experts in curricular construction and evaluation.

In total, the apprenticeship in teaching should constitute approximately one quarter of the work load of the student during his graduate experience.

Adequate budgetary provisions should be made for the extra burden of the apprenticeship program on the senior mathematicians, as well as financial support for the apprentices.

An apprenticeship system has a great potential for preparing two-year college mathematics teachers having a real attachment to the discipline and an understanding of the values and the rewards of the teaching profession. Done poorly, it will discourage candidates from the field. Done well, it will attract and retain competent and interested persons.

COMPOSITION OF A TWO-YEAR COLLEGE MATHEMATICS FACULTY

Although mathematics teachers at two-year colleges are called upon to teach specialized courses for a variety of students (remedial, general education, technical-occupational), our attention in the present report continues to be focused upon qualifications of persons who teach courses in the university-parallel curriculum.

It is our recommendation that all teachers of university-parallel courses at a two-year college have the mathematical preparation equivalent to our graduate component. Although the university-parallel courses that a two-year college teacher may be called upon to offer today are principally like those described in the introduction under 0, A, B, and C, it seems reasonable to expect that courses in finite mathematics, linear algebra, probability and statistics, and mathematics for prospective elementary school teachers will be standard offerings in two-year colleges in the near future. Our recommendation reflects a belief that the teacher of university-parallel courses should have mathematical training well beyond the course he is teaching. Moreover, the mathematical background we recommend will permit all faculty members to participate in knowledgeable discussions of curricular changes, both internally and with faculty members of four-year colleges and universities. The mathematical preparation we recommend will permit a faculty member to prepare new courses with confidence. Moreover, it will provide the individual faculty member with a basis for effective participation in mathematical organizations, which in turn will help him to maintain the intellectual curiosity and interest in mathematics that is essential to a successful mathematics teacher.

The Committee believes that a universally well-qualified faculty for the university-parallel courses is most important, with each member able to make a contribution in all of the ways already indicated. We do not, however, envision that all two-year college staff members will have exactly the same mathematical background. The choice available for individual preferences in the graduate component allows a staff which includes people with varying interests, and hence people especially well prepared to teach linear algebra or probability and statistics or computer science.

A two-year college may not be able at this time to recruit all such staff members from candidates with preparation equivalent to our recommended graduate component. In this case, they might seek on a temporary basis new candidates who have the mathematical preparation equivalent to our basic component; these candidates could be assigned to teach courses 0, A, and B. New faculty members whose qualifications are not equivalent to our graduate component should be required to augment their mathematical background so that in time they will be better prepared to have responsibility for any of the university-parallel courses.

a. To Administrators of Two-Year Colleges

These recommendations are addressed to those who appoint and promote two-year college faculty members and to those who, through accreditation and certification, influence the setting of qualifications for such teachers.

Although it has been traditional for college policies on the appointment, promotion, and tenure of faculty to include certain degree requirements, it is a fact that the course requirements for a particular degree in mathematics vary considerably from one institution to another; even minimum standards are not well defined. This is especially true of the master's degree. The Committee strongly encourages those concerned to note that this report recommends a set of courses which prospective members of a mathematics faculty should have taken. Successful completion of these courses should insure that the faculty member is adequately prepared, in terms of subject matter, to teach university-parallel courses.

The Committee urges all administrators to recognize proficiency in the content of the courses recommended in this report rather than academic degrees as a basis for faculty appointments and advancement. For example, graduate mathematics training of secondary school teachers, customarily and properly, differs from the training we have described. The Committee suggests that faculty members in mathematics be relied upon to determine the degree of proficiency possessed by those under consideration. Furthermore, it is recommended that orientation programs be developed for new faculty members who have had no previous experience teaching in two-year colleges.

b. To University Mathematics Departments

University mathematics departments should realize from the preceding sections that the major role in the training of mathematics instructors for two-year colleges is theirs. They must accept responsibility for establishing formal programs for the training of new instructors for two-year colleges and for retraining instructors who are now teaching in these institutions.

We believe that this can be done within existing frameworks of mathematics departments whose course offerings approximate in depth the detailed outlines to which we have referred. For such departments, this will not require extensive changes in curricula, except possibly for the introduction of a program for apprenticeship in teaching. However, it is necessary that the mathematics faculty be fully aware of the particular complexion, problems, and status of two-year colleges throughout the country. The mobility of instructors suggests the need for a national point of view. Moreover, in order to fulfill their responsibility, the university faculty must recognize

and respect the basic role of two-year colleges and be mindful of the problems that will be faced by mathematics instructors in two-year colleges.

c. To Those Currently Teaching Mathematics in Two-Year Colleges

All college teachers of mathematics, at one time or another, find it necessary to supplement their own mathematical training. Rapid changes are taking place in college mathematics. Hence, increasing numbers of college teachers are continuing their mathematical development by individual study and additional formal course work in mathematics.

Teachers of mathematics in two-year colleges should find that the course outlines referred to in this report provide useful guidelines for individual study, faculty seminars, and additional course work.

To serve the mathematical needs of two-year college students, a faculty member must maintain an awareness of contemporary curricula in both secondary schools and four-year colleges. He will find that the recommended courses provide a basis for effective communication with staff members of mathematics departments of four-year colleges. Personal knowledge of mathematics courses at four-year colleges is needed in order to be aware of the demands that will be made upon students after they transfer. This knowledge and the recommended strong preparation in mathematics make possible the necessary continuous evaluation and development of mathematics courses in two-year colleges.

d. To Prospective Teachers of Mathematics in Two-Year Colleges

The two-year college teacher of university-parallel mathematics courses has the responsibility for training students with a wide variety of goals. Some could be mathematicians, some scientists or engineers; there are others who will use mathematics in economics, psychology, or other social sciences. One who intends to teach mathematics in a two-year college could well use our description of a program of mathematics courses and apprenticeship in teaching as a guide in planning his own graduate work. He should also be aware that the program we have outlined is substantially different both in nature and extent from what we would regard as an optimal graduate program for teachers in secondary schools.