Curriculum for Grades 11-13

The following report was approved in 1987 by the Board of Directors of the National Council of Teachers of Mathematics and by the Board of Governors of the Mathematical Association of America. The joint NCTM-MAA committee that prepared the report was chaired by Joan Leitzel, Associate Provost of The Ohio State University. It is printed here in its entirety for the first time.

The Joint Task Force on Curriculum for Grades 11-13 was formed in Spring 1986 by John Dossey, President of the National Council of Teachers of Mathematics, and Lynn Steen, President of the Mathematical Association of America. The Task Force was charged to focus on curriculum for the mainstream of students who take a college preparatory program in high school and who go on in college to a standard freshman mathematics course such as calculus, finite mathematics, statistics, or discrete mathematics. The Task Force was asked to advise on the need for new recommendations to high schools and colleges concerning curriculum for students who intend to pursue careers that depend on mathematics.

The Task Force has studied and synthesized recent national reports on the state of mathematics education, as well as the recommendations of many national and state boards, and the reports of several recent curriculum projects. It is clear that the present situation in mathematics education is dynamic and that significant changes in curriculum in grades 11-13 may occur in the next few years. The Task Force has been impressed and excited by the quality and scope of curricular projects currently underway. We note particularly the K-12 Curriculum Project undertaken by the Mathematical Sciences Education Board (MSEB), the frameworks of which will be released during 1989, and the collegiate mathematics project undertaken jointly by MSEB and the Board of Mathematical Sciences of the National Research Council (The Mathematical Sciences in the Year 2000: Assessment for Renewal in U.S. Colleges and Universities).

It is not the purpose of this report to resolve present conflicts or to determine new directions for mathematics education. Rather, it is the purpose of this report to summarize those areas related to grades 11-13 curriculum in which it appears the profession now has a level of consensus, and also to summarize those issues where there is lack of consensus and apparent need of additional study.

Although it is tempting to make no statements during this time of rapid evolution, we are persuaded that school and university students, parents, and teachers will welcome some clarification from the professional associations. The joint statement of the NCTM and MAA on college preparation has not been updated since it was released in the mid-70s. Thus, although we do not in any way want to suggest that the status quo is acceptable for education in mathematics, we do wish to summarize prevailing opinions in the hope that some guidance will be helpful to students and teachers at this time.

This Task Force was formed to consider curricular matters. Even though it is beyond the charge of the Task Force, we are compelled to mention that attention to curriculum will be meaningless without concurrent attention to other matters. One such concern is the increasing shortage of qualified teachers at all levels, especially in the middle grades and the secondary schools. In addition, colleges and universities frequently depend heavily on teaching assistants and part-time teachers.

Secondly, it is difficult to discuss curriculum in grades 11-13 without reference to the mathematics of the earlier grades. The Task Force has been reminded frequently of the critical role that middle school mathematics and the first year of algebra play in a student's success in college preparatory mathematics. Continued attention needs to be given to the curriculum of these grades. Indeed, some of the recommendations we make in this report for grades 11-13 apply as well to earlier grades.

Issues of Apparent Consensus

After reviewing numerous reports and talking with teachers, mathematicians, and mathematics educators, the Task Force believes that there is agreement within the profession on many issues related to curriculum in grades 11-13 for those students who are directed toward mathematics-dependent collegiate programs. From this consensus, it appears that the following recommendations can be made:

1. College-bound students should take mathematics in all years of secondary school; this mathematics should include the content of intermediate algebra and geometry.

2. Geometry in both two and three dimensions, coordinate geometry and the development of geometric perception are essential parts of college preparatory mathematics.
3. Although computer programming is an important tool for all college-bound students, courses in computer programming should not be regarded as substitutes for college preparatory mathematics.
4. Students should learn the content of a full four years of college preparatory mathematics before taking a calculus course. Students directed toward twelfth grade calculus need an enriched program as early as grades 7 and 8.
5. A full program of college preparatory mathematics should be provided in every secondary school.
6. Calculus, if taught in the schools, should be equivalent to college-level calculus. NCTM and MAA have made a joint recommendation to the schools that students should expect to write proficiency examinations (either Advanced Placement or university credit exams) to establish that they have learned beginning calculus, and to obtain college credit for high school calculus courses. Watered-down calculus courses in the secondary schools stressing manipulations but slighting subtle processes do not help students. Introducing polynomial calculus in precalculus courses uses time better spent on other topics.
7. If mathematics is required in a student's college program, the student should enroll in mathematics courses as a college freshman. There should not be a gap of a year or more during which a student takes no mathematics.
8. Students should expect to make use of calculators and computers in their mathematics courses in grades 11-13. Calculators and computers should be used as tools to enhance and expand the learning of mathematics in these grades.
9. The availability of technology should permit computational approaches to college preparatory mathematics that result in this mathematics being accessible to more students. Further, the availability of technology should permit students in these grades to investigate problem situations that are not approachable without computational tools, and to be introduced to different mathematics than was possible without calculators and computers.
10. Mathematics in grades 11-13 needs to have a clear connection to real world problems, and students should be expected to acquire a growing ability to use mathematics to model real situations. Students should become aware of new applications of mathematics as these applications develop. In addition, students should understand that mathematics itself is a developing discipline, and, where possible, students should come to appreciate new developments in mathematics.
11. The curriculum of grades 11-13 should contain units in statistics, probability, and fundamental topics in discrete mathematics for all college-bound students.
12. Mathematics in grades 11-13 should have goals beyond the acquisition of computational techniques. Mathematical understanding and analytical reasoning are basic goals for mathematics at this level. Problem solving strategies should be stressed, and manipulative and computational techniques, although important, should not predominate.
13. Teachers of mathematics in grades 11-13 should employ strategies that encourage student reading, writing, and reflection. Assignments and examinations should be designed to help students become more independent learners of mathematics and to increase their abilities to discuss both orally and in writing the mathematical ideas they are learning. Courses should not cover an excessive number of topics at the expense of reflection and independent learning on the part of students. Teachers should take advantage of mathematics competitions and science fairs to encourage independent learning in students.
14. While there is need for meaningful review within new mathematics, the amount of time spent on review at the beginning of a course in grades 11-13 should not be excessive. Typically, review should be integrated into the learning of new mathematics.
15. Expectations of students with regard to homework, examinations, and knowledge of previous courses need to be raised in many grade 11-13 programs. These expectations should include daily homework, cumulative examinations, and examination questions that require problem-solving skills.
16. To overcome the effects of socialization that discourages girls and American minorities from studying mathematics, these groups of students should be especially encouraged in the study of mathematics in grades 11-13, and efforts should be made to identify applications of mathematics that hold particular interest for under-represented groups. Furthermore, the perceived preponderance of negative attitudes toward mathematics in this country should be studied to determine what aspects of curriculum and what features of culture contribute most heavily to these attitudes.

Recommendation

The Task Force is persuaded that improvement in the curriculum of grades 11-13 and in student performance in this curriculum requires strong collaborative effort among mathematicians, university faculty in mathematics education, teachers, school leaders, counselors, students, and parents. The Task Force recommends that funding be sought so that the curricular issues cited above and other essential related information can be communicated to these groups. Particularly targeted brochures and flyers need to be developed and circulated to teachers, school leaders, counselors, students, and parents.

We have included at the end of the report a fuller description of the rationale and content of these proposed communiques and recommendations for the desired collaboration.

Issues Requiring Further Study

In addition to the issues listed above on which the profession appears to have a consensus, we have identified many issues where there is lack of consensus. In many cases, studies, reviews, and experimentation are underway in a variety of projects. Where we know of such projects, we have cited them here. In some cases it appears that more study and discussion will be needed to clarify these issues.

Gifted Students: Acceleration or Enrichment?

The types of courses that students are capable of taking in grades 11-13 will depend upon the preparation of these students in earlier years. Should the very best students be accelerated or should their programs instead be enriched with topics they may otherwise miss? This question is still a matter of debate among mathematics educators.

Advocates of early introduction of algebra usually argue that the current curriculum for grades 7 and 8 is mainly just a review of topics taught in previous grades with little new material introduced. The Report of the MSEB Task Force on Curriculum Frameworks for K-12 Mathematics (Draft, October 1986) states that:

> We applaud the current attempts to make algebra an eighth grade subject. There is ample evidence from other countries that eighth graders can handle algebra. More generally, we think that grades 7 and 8 should look forward to high school mathematics as much or more than they look backward to elementary school mathematics.

The University of Chicago School Mathematics Program is being designed for the general school population. Its eighth grade course is mainly algebra, but heavily manipulative techniques are postponed to later courses. In his paper, "Why Elementary Algebra Can, Should, and Must be an 8th Grade Course for Average Students," Zalman Usiskin argues that in other countries algebra is usually done with all students at grades 7 and 8 and that with proper curriculum in grades 1-6, algebra could be mastered in grade 8 by U.S. students as well.

In *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective,* a report on the Second International Mathematics Study, the authors recommend:

> The content of the mathematics curriculum needs to be re-examined and revitalized. The domination of the lower secondary school curriculum by the arithmetic of the elementary school has resulted in a program that, from an international point of view, is very lean. The curriculum should be broadened and enriched by including a substantial treatment of topics such as geometry, probability, statistics and algebra, as well as promoting higher-level process goals such as estimation and problem-solving.

In "Let's Not Teach Algebra to Eighth Graders!" (*Mathematics Teacher,* November 1985), Fernand Prevost provides evidence that offering algebra in eighth grade has unwanted consequences. His study of New Hampshire schools showed that "only about half of the students who take algebra as eighth graders continue their study of mathematics through a fifth year." Prevost recommends an enriched program rather than algebra, with only the top 3 to 5 percent being truly accelerated. Two very relevant questions regarding his study are:

1. Do the sixth and seventh grade programs adequately prepare students for algebra in eighth grade?
2. Would the retention rate be greater if there were alternative twelfth grade courses?

Several letters of rebuttal were submitted by readers in response to Prevost's article.

There has been specific concern about students who study calculus in grade 11. ETS reports that 4,000 of the 60,000 students now taking AP calculus exams are in eleventh grade and lower. Very special arrangements are needed to guarantee that these students have appropriate mathematics in grade 12.

The NCTM's position statement on Provisions for Mathematically Talented and Gifted Students (October 1986) contends that:

> The needs of mathematically talented and gifted students cannot be met by programs of study that only accelerate these students through the standard school curriculum, nor can they be met by programs that al-

> low students to terminate their study of mathematics before their graduation from high school.

The NCTM paper goes on to recommend that:

> All mathematically talented and gifted students should be enrolled in a program that provides a broad and enriched view of mathematics in a context of higher expectation. Acceleration within such a program is recommended only for those students whose interests, attitudes, and participation clearly reflect the ability to persevere and excel throughout the entire program.

Greater Integration of Topics

It can hardly be disputed that the curriculum for grades 11-13 must be closely related to the curriculum of the preceding years. An issue on which there is no apparent consensus and which therefore requires further study is the extent to which the secondary school mathematics curriculum should be integrated or unified.

In most countries mathematics is not compartmentalized into algebra, geometry, etc., as is conventional in the U.S. Since World War II there have been numerous attempts to break down these compartments in this country. One such attempt in the 1950's was Florida's Functional Mathematics Program which was short-lived mainly because of the advent of the School Mathematics Study Group (SMSG) and the "new mathematics." In the late 1960's and early 1970's, the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), directed by Howard Fehr at Columbia, developed a unified program intended for the top 15 to 20 percent of secondary school students.

In 1984, after several years of experimentation, the New York State Board of Regents adopted an integrated approach as the high school Regents program in mathematics with intentions of gradually phasing out the traditional program. At the present time, the University of Chicago School Mathematics Project is developing an applications-oriented curriculum that is integrated to some extent although algebra prevails in grade eight and geometry receives the emphasis in grade nine.

Despite these moves toward unification, it is still the case in the U.S. that most college preparatory mathematics programs begin with a year of algebra, followed by a year of geometry and another year of algebra. The Second International Mathematics Study Summary Report for the United States (1985) contends:

> It is plausible that the "fragmentation" and "low intensity" found in many of our mathematics programs could be allayed by a more integrated approach to the high school mathematics curriculum.

The Report of the MSEB Task Force on Curriculum Frameworks for K-12 Mathematics (Draft, October 1986) states:

> Not only do we believe that an integrated curriculum offers the possibility of a richer, more coherent program than the alternative but, further, we believe that the introduction of a variety of new subject matter into the secondary school mathematics curriculum will inevitably signal the demise of the segregated curriculum if only for logistics reasons.

How inevitable this is remains to be seen. One factor that must always be taken into consideration when making curriculum decisions in the U.S. is the mobility of our population. It must be admitted that the ability to transfer a credit in algebra or a credit in geometry has simplified matters for many students. Unified curricula may make the process of transferring more traumatic. Such curricula may also cause much rethinking on the part of colleges that are accustomed to accepting well defined units of credit, although placement by examination may suffice. An additional concern is that teaching unified courses requires a breadth of understanding beyond what many teachers have been prepared to provide.

It should be pointed out that there is yet another aspect of integration that needs further study. This is the possibility of integrating mathematics with other disciplines, particularly science.

Role of Statistics & Discrete Mathematics

There appears to be agreement that topics in discrete mathematics and in statistics and probability should be included in the mathematics curriculum for college-bound students. However, there is lack of agreement on the appropriate place for these subjects in the mathematics curriculum and on the number of hours of study required, especially for students who study calculus in grades 12 or 13. The traditional mathematics curriculum from elementary algebra, geometry, intermediate algebra, through precalculus is largely a calculus preparatory curriculum. Usually these courses do not include substantial study of discrete mathematics or of statistics and probability even though foundational topics in these areas are often in the back of textbooks used in the courses.

STATISTICS AND PROBABILITY

Nearly every major committee making recommendations on the high school curriculum has said that familiarity with the basic concepts of statistics and statistical reasoning should be a fundamental goal for high school mathematics (new state frameworks in California, Illinois, Wisconsin, and New York; The College Board's *Academic Preparation in Mathematics;* MSEB Task Force Draft Report on Curriculum Frameworks for K-12 Mathematics; *A Nation at Risk* from the National Commission on Excellence in Education). Typical is the statement in ***Educating Americans for the 21st Century*** (1983) from the National Science Board Commission on Precollege Education in Mathematics, Science and Technology: "Elementary statistics and probability should now be considered fundamental for all high school students." Similarly, after studying the performance of 12th grade college preparatory mathematics students on the Second International Mathematics Study, the U.S. National Committee recommends that, "The curriculum should be broadened and enriched by including a substantial treatment of topics such as geometry, probability, statistics and algebra, as well as promoting higher-level process goals such as estimation and problem-solving."

Committees have been making recommendations for the inclusion of statistics for thirty years. In 1959, the Commission on Mathematics of the CEEB in its report, *Program for College Preparatory Mathematics,* recommended a one-semester course in probability and statistics as an alternative for grade twelve. The Commission published an experimental text, *Introductory Probability and Statistical Inference,* the same year. In 1975, the Euclid conference sponsored by the NIE identified probability and organization and interpretation of numerical data as two of ten basic goals for mathematics education. Also in 1975, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education (NACOME) reported:

> While probability instruction seems to have made some progress, statistics instruction has yet to get off the ground ....

The situation has not changed much since this NACOME report. For example, Bruce Williamson in *The Statistics Teacher Network Newsletter* (1983) reported that a study of approximately 350 high schools in Wisconsin found that the percentage of schools which allot more than three weeks in the total high school program to statistics declined from 26% in 1975 to 23% in 1983. In 1975, 43% allotted more than three weeks to probability; this declined to 34% in 1983. However, Wisconsin now requires some elements of statistics for all students so it is likely the decline has been reversed.

The dearth of good materials may be the main reason why more statistics and probability is not being taught. Usiskin (1985 NCTM Yearbook) states, "The content of any new curriculum must be specified in as much detail as current content. ...Materials must be available to implement recommendations." Materials currently available consist primarily of locally produced handouts, a recently published book for high school students (Travers, et al.), and four booklets published by the ASA-NCTM Joint Committee on the Curriculum in Statistics and Probability.

Arguments against including more statistics are raised by educators who feel that students in grades 11-13 need to spend their time on "basic" mathematics topics. For example, the integrated New York State mathematics program is criticized by the IEEE Long Island Section because the introduction of new subjects such as statistics and probability reduces the time spent on basic algebra, geometry, and trigonometry.

DISCRETE MATHEMATICS

Recently, topics in discrete mathematics have been recommended for inclusion in the high school curriculum in several reports including the Report of The MSEB Task Force on Curriculum Frameworks for K-12 Mathematics (Draft, October 1986). The CBMS ("The Mathematical Sciences Curriculum K-12: What Is Still Fundamental and What Is Not") recommends that discrete mathematics now be regarded as "fundamental."

While certain topics (induction, matrices, discrete probability, and combinatorics) are found at the back of many high school textbooks, they are not always taught. Additional topics such as graph theory, difference equations, recurrence relations, and game theory are also recommended by some. No high school curriculum has yet been standardized in discrete mathematics. Some relatively short units, such as the HiMap modules, are now available and finite mathematics texts are sometimes adapted for this instruction.

Many colleges and universities now offer a lower division course in discrete mathematics particularly suited to students in computer science. There has been discussion of how students should be prepared for such a course. To quote from the preliminary report (1984) of the MAA Panel on Discrete Mathematics in the First Two Years:

> What should be taught in the high schools or on the remedial level in the colleges to prepare students adequately for this course? Our suggestion is tentative: some of us feel that perhaps a revived emphasis on the use of both formal and informal proof in geometry courses as a means for teaching methods of proof and analytic thinking would be a step in the right direction. Others of us are not so sure. Increased use of algorithmic thinking in problem solving could be easily adapted to many high school courses .... Simple restoration of some of the classical topics (the binomial theorem, mathematical induction, natural logarithms) and increased emphasis on problem solving might make the proposed course much easier for the student.

The issue of what curriculum adequately prepares students for college-level discrete mathematics or whether any particular preparation is essential requires further study.

Twelfth Grade Mathematics Courses

For students in a college preparatory program who take algebra I in grade 9, the twelfth grade course is traditionally a year of precalculus mathematics that includes trigonometry as well as topics such as exponential and logarithmic functions and equations, conic sections, rational functions and their graphs, polar coordinates, parametric equations and their graphs.

Two groups of students are identified by some as students who need alternatives to traditional twelfth grade courses. The first are students who have studied precalculus in grade 11 but who may not benefit particularly from the study of calculus in grade 12 and who may benefit more from the study of other topics in mathematics, delaying calculus until grade 13. The second are students who do not expect to need calculus in their college programs, but do expect to take college mathematics. It is argued by some that standard precalculus in grade 12 (or calculus, in the case of students who are eligible for calculus) does not provide these students with the best college preparation.

The three most frequently mentioned semester-length courses proposed as 12th grade options are given below. We also include a new course under development at the North Carolina School of Science and Mathematics.

STATISTICS AND PROBABILITY

A one-semester course in statistics and probability has been proposed by the new *Mathematics Framework for California Public Schools.* The tenth grade course being designed by the University of Chicago School Mathematics Project is Statistics and Computers. Currently, statistics courses are not widely taught. For example, of the 42 high schools in New Hampshire, only five offer a course in statistics (Prevost in *The Statistics Teacher Network Newsletter,* 1983).

DISCRETE MATHEMATICS

Using their own lecture notes, Georgetown University lecturers have taught a summer course in discrete mathematics/mathematical modeling to selected high school students (Sandefur in 1985 NCTM Yearbook). The North Carolina School of Science and Mathematics has offered a one-semester course called "Topics in Discrete Mathematics" which follows multivariate calculus, again to very select students. We know of no experiments in traditional high schools. The curricula from the six colleges and universities that were funded by the Sloan Foundation to integrate discrete mathematics into the first two years of the college program may provide some guidance for secondary schools seeking a 12th grade course in discrete mathematics.

LINEAR ALGEBRA

Full courses in linear algebra are not common in secondary schools although various individuals argue the appropriateness of this mathematics for grade 12 (e.g., John Thorpe, "Algebra: What Should We Teach and How Should We Teach It," NCTM Research Agenda Project Conference on the Teaching and Learning of Algebra, Athens, Georgia, March 25-28, 1987). In the Chicago area, high school students involved in accelerated programs through the Johns Hopkins talent search do take linear algebra in several junior colleges.

A SURVEY OF MODERN MATHEMATICS

A twelfth grade course is being developed with a Carnegie Foundation grant at the North Carolina High School of Science and Mathematics that is a survey of modern mathematics. The year-long course will consist of units (at least three weeks long) which introduce students to the kinds of mathematics that they could study in college. Students will presumably then be able to make a more informed choice of their first college courses. The topics proposed include calculus, discrete mathematics, computer programming, popular software such as SMPs, statistics, probability, mathematics of finance, linear programming, operations research, and linear algebra. The course focuses on the types of problems that are characteristic of each field and introduces students to the mathematical techniques that are used to solve them.

Calculus Review for College Freshmen

Each year a large number of students take a full year of high school calculus and either do not write the AP test or do not receive a 3, 4, or 5 on it, and do not test above the beginning calculus level on college placement tests. These students may not "fit" into a standard college calculus sequence. They typically view the introductory material as mathematics they have already learned and do not take beginning work seriously enough to succeed in the later work of their courses. It can be argued that these students are potentially capable students in mathematics. The CUPM Subcommittee on Calculus Articulation [Don Small, Chair] has recommended that colleges should develop a special course with the following characteristics:

1. The course should be different from high school calculus.

2. The course should contain a broad review component designed to provide depth missing in most high school courses.
3. The course should assume a high school calculus experience and build on it.
4. The course, when completed successfully, should provide one semester of beginning calculus credit in addition to the credit for the course.

Colby College has developed a two-semester calculus sequence integrating the treatment of one and several variables that has these characteristics. The course is offered for identified students instead of the regular three-semester calculus course.

On the other side, many argue that if colleges develop special courses for students who take calculus in high school but do not master it at a college level, students will be encouraged to be satisfied with less than full mastery of their high school calculus and high schools will be encouraged to offer watered-down courses in calculus in the 12th grade. (The MAA and the NCTM have prepared a joint statement for the schools indicating that students who take high school calculus should expect to establish college credit either through the AP exam or through a college proficiency exam.) These educators maintain that alternatives to calculus should be developed in grade 12 for students who are not ready to master calculus, and that it is not a good use of student time to spend one and a half to two years on the content of first-year calculus.

The Place of Deductive Reasoning

University and college faculty complain that too often students enter college with the view that mathematics is just a collection of rules and algorithms to be used to attack a variety of standard problems. They assert that students' abilities to reason either deductively or inductively have not been developed by their mathematics courses, and that their ability to attack problems of an unfamiliar nature has not been developed. If these statements are true, there is need to determine the cause. Some argue that the rush to streamline the high school program so that calculus can be taught in the 12th grade, jettisoning things deemed unimportant to this goal and omitting end-of-the-text topics, bears primary responsibility. Others claim that the emphasis on the formal structure of mathematics has stifled the ability to reason intuitively. Also it is not clear how much better entering college students a generation ago were at mathematical reasoning. There is general agreement that strengthening students' reasoning ability is a goal in grades 11-13, but little evidence that this is happening. The following comments by Phil Curtis, UCLA, suggest where attention may need to be focused:

> If this problem is to be corrected, more attention must be paid to the development of a student's reasoning ability at all levels, and the instruction should focus on inquiry and discovery rather than an over-riding emphasis on mastery of a body of rules and techniques. Is the following statement true or false? How can we decide? If true, why is it true? How can we see if the statement is a consequence of ideas that have been discussed before?
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> The emphasis on reasoning should focus on intuitive understanding rather than a development of formal logic. The structure of implication, however, should be stressed. What is the nature of mathematical implications? What is the hypothesis and what is the conclusion of an implication? What is the difference between a statement and its converse?
>
> Traditionally, the geometry course was the first course in which students met the notion of mathematical proof. Proofs were very formal but, since there was never an attempt to extend this form of reasoning to other mathematics courses, very little in the way of reasoning skills was retained when students came to college. There are many areas where mathematical implication can be stressed: solution of equations and inequalities in algebra as well as certain elementary ideas from number theory, use of coordinate geometry and vector techniques in geometry, mathematical induction, and counting arguments and elementary probability in advanced algebra or discrete mathematics.
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> To develop mathematical reasoning skills takes time and should be one of the primary goals of the high school program. Ideas should be developed leisurely; the focus on the program should not be just to make students ready for calculus in the 12th grade. To accomplish this development the program needs to be opened up at all levels. In particular, the necessity for inordinate amount of review present at all levels should be lessened. Rule bound students are on a mathematical dead end. The high school program should be able to do more.

Curricular Impact of Calculators

The introduction of single variable calculus as the "desired" 12th grade mathematics course in high school is thought by some to have resulted in a streamlining of the program prior to calculus. The resulting lack of seasoning or maturity on the part of many incoming college freshmen is perhaps the biggest complaint of college faculties. Students are said to know much more than students of a generation ago as far as calculus-related ideas are concerned, but geometric intuition, skill with coordinate geometry, the ability to organize applied problems and the ability to construct mathematical arguments are all too often reported to be lacking. Students may have an algorithmic facility with mathematics, but many lack a true understanding of the subject.

It is often expressed that when hand-held calculators are generally available at all grade levels the narrowing of the curriculum may get worse. This apprehension is typified by the remark: "If students are to use calculators, why should we teach all of this arithmetic (and possibly algebra) that can be done so much easier on a calculator?" Experience in other countries, e.g., Australia, shows that narrowing of the curriculum need not be a serious problem.

Indeed, many argue that an introduction of calculators can encourage a broadening of the curriculum rather than a further narrowing of it. At the elementary level, students can handle numerical data in much greater amounts and in wider practical situations than would be possible without calculators. A sense of 'reasonableness of solution' to more complicated calculational problems can and should be a goal of instruction. There is an increased opportunity to develop a student's mental arithmetic and estimation skills.

Rather than decrease students' algebraic skills, these proponents argue, the ability to confront practical problems of much greater computational complexity should be an impetus for development of the algebra skills necessary to organize the problems so they are amenable to numerical calculation. All of the calculations involving compound interest are possible, with the associated opportunity to do manipulations with geometric series. Transcendental equations in trigonometry are easily solved. This could be a spur to confront more meaningful applied problems in trigonometry involving manipulation and solution of trigonometric equations; computations that were impossible in the past. Certain topics of course can be dropped; for example, dependence on tables for teaching trigonometry as well as the use of logarithms to perform multiplication and division calculations.

Curricular materials are under development now on several levels and at several locations that make central use of calculators and computers. The answers to many questions concerning the effect of technology on the curriculum will emerge as these materials become widely available. For grades 11-13 the content of intermediate algebra, precalculus, discrete mathematics, and calculus are all likely to be affected. Ronald Douglas has noted in the Introduction to *Toward a Lean and Lively Calculus* (MAA, 1987):

> Anyone who has seen hand-held calculators which output the graph of an equation usually realizes that we can and, indeed, we will have to change what we ask students to learn and what we test them on. And this is just a start of developments that include the growing availability of programs for symbolic and algebraic methods as well as for numerical methods.

The Sloan Foundation has funded projects at seven colleges and universities to consider the potential impact of computer algebra systems on the teaching of calculus. The National Science Foundation has announced a major initiative in the area of calculus curriculum that can be expected to include some projects where calculators and computers play a central role.

Alternatives to Remedial Courses

An ever increasing fraction of resources is now devoted to remedial courses at both the college and high school levels. Algebra and precalculus courses are common at colleges for students who previously have been exposed to this material. Failure rates of 40-60% are reported in beginning algebra courses at the 10th grade level. At both levels, part of the problem seems to be that the necessary prerequisite material was not learned; and the remedy often is to teach material over again in the same way it was taught the first time. Some argue that there are alternatives.

Initially, students who have no prospect of success should not be placed in a beginning algebra course. There are two problems here: the need for an effective predictor test and the need for middle grade curricula to better prepare students for algebra. The California Mathematics Diagnostic Testing Project is developing an algebra readiness test. Several curriculum projects for the middle grades are seeking to strengthen curriculum at that level. A common assumption in these projects is that imaginative use of hand-held calculators, with the tremendous increase in calculational power they give students, should play a central role in stimulating student interest and providing different perspectives on the abstract ideas, for example that of a variable, that students will encounter.

There are proposals that more flexible scheduling could also ease the remediation problem. These proposals argue that students who fail the first semester of beginning algebra should have the opportunity of repeating it in the second semester and not be forced to wait until the following year to begin again. However, this flexibility is seldom present in school schedules. In some districts, students who fail an entire year are given the opportunity of making the course up in a summer session. When a student attempts to learn a year of algebra I, geometry, or algebra II in a shortened summer session, there is little chance the student will become proficient with the fundamental ideas of the course.

Some geometry texts attempt to help avoid remediation, or excessive review, at the intermediate algebra level by providing diagnosis in the geometry course of what algebra skills have been retained and what have not. A renewed emphasis on these topics parallel to the development of geometry can be used to achieve readiness for the second course in algebra. Coordinate geometry is a possible framework for renewal work in algebra.

In some colleges and universities, algebra and precalculus courses avoid being just a review of high school material by making actual use of computers and calculators and by blending precalculus topics with an introduction to the notion of limit and other fundamental ideas from calculus.

Use of Standardized Test Scores

Standardized tests are widely used in high school and beginning college mathematics programs throughout the United States. These tests range from achievement examinations, such as the mathematics achievement tests of the College Board and the New York State Regents Examinations, to mathematics aptitude tests such as the SAT, and various assessment instruments designed at the state level and often used at the 12th grade and other levels. In addition there are diagnostic instruments designed to assess readiness for the next level of the program and placement examinations, such as those available from the MAA, often used at the college freshman level and in high school.

These tests are often criticized on various grounds. First, since they are usually multiple choice exams, they are criticized for supposedly not testing higher order thinking or problem solving skills. Secondly, since the general assessment exams are used to compare schools and programs, there can be considerable political pressure "to get those scores up," with the result that there can be a narrow concentration by the teacher on just those basic skills covered in the examination.

Also, tests designed for one purpose are often used for another. Diagnostic examinations are used as assessment instruments to compare the performance of classes and schools. When this is done the pressure to narrow the curriculum to just those basic skills necessary for success in the next course can become great. On the other hand if this comparative pressure is removed, diagnostic tests can be quite effective in indicating areas of the curriculum which are not being retained by the student but which are absolutely necessary for success at the next level of the program. Strategies for dealing with these deficiencies can then be constructed which should result in a broadening of the curriculum rather than a narrowing of it to a concentration on just basic skills.

Placement exams can be misused if they are used as a barrier which prevents students from taking a given mathematics course, rather than providing a route covering necessary preparatory material. The latter then places the student in the course more properly prepared.

Other misuses of tests are commonly cited: standardized tests designed for comparative assessment purposes that "drive the curriculum," failure to allow for margin of error in interpreting scores, excessive use of multiple choice tests for classroom assessment.

The Mathematical Sciences Education Board has designed a comprehensive testing study that will survey testing practices, assess how test results are used, and the effects of testing on curriculum and teaching behavior. This study should provide a foundation for future decisions related to test construction and use.

The Form Geometry Should Take

There is general agreement that many entering college students lack geometric intuition and the ability to visualize geometric situations. This would seem to indicate that a strengthening of the geometric content of the high school program is sorely needed. But there is not a consensus as to how this should be done. Nor is there agreement on the place of such topics as transformations and vectors or the amount of emphasis that should be placed on formal proof or the desirability of introducing students to formal logic within the geometry course.

What ought to be taught in high school geometry has been debated for at least a half century. In an address to the 1958 annual meeting of the NCTM in Cleveland (published in the *Mathematics Teacher* as "The Nature and Content of Geometry in the High Schools"), Julius Hlavaty referred to the "continuing crisis in the teaching of geometry" that had endured for "fully 25 years." Titles of other articles in the *Mathematics Teacher* in the last two decades substantiate the existence of an ongoing debate (Adler, "What Shall We Teach in High School Geometry?" March 1968; Allendoerfer, "The Dilemma in Geometry," March 1969; Fehr, Eccles, and Meserve, "The Forum: What Should Become of the High School Geometry Course?" February 1972). The introduction to the 1973 NCTM Yearbook *Geometry in the Mathematics Curriculum* is entitled "Disparities in Viewing Geometry." The book contains chapters discussing several different approaches to high school geometry: conventional, coordinate, transformation, affine, vector, and an eclectic approach. In the latest yearbook of the NCTM, *Learning and Teaching Geometry K-12* (1987), Usiskin writes on "Resolving the Continuing Dilemmas in School Geometry," and Niven on "Can Geometry Survive in the Secondary Curriculum?" Usiskin offers suggestions for resolving the dilemmas, and Niven proposes recommendations that he thinks will make geometry a more attractive subject. Other chapters in the Yearbook discuss various geometric topics and applications.

The proliferation of computers, the growing popularity of Logo, and the development of software such as "Geometric Supposer" are adding new elements to the debate on geometry. Can they perhaps help to resolve issues that have been around for quite some time?

Collaborative Efforts

In recent years there have been many studies and reports released calling for significant changes and reforms in American education. While there has been considerable consensus in the identification of the ills within this system, there has not been universal agreement as to what the solutions should be in order to remediate the identified problems and shortcomings. Such has been the situation with issues involving education in mathematics, particularly those related to curriculum.

Issues related to curriculum have occupied an important position in many of the released studies and reports dealing with education in mathematics in the United States. In particular, discussions and recommendations pertaining to the mathematics curriculum of grades 11-13 have been prominent in many of the published reports and studies. Some of the consensus and lack of consensus items pertaining to the mathematics curriculum have been identified and dealt with in other portions of this report. While a listing of these items is the major thrust in this report, it is necessary to identify briefly several other considerations which pertain to the total mathematics experience for 11-13 grade students.

Many of the recent reports have cited the success of mathematics students in other countries, particularly those from Japan. The reports emphasize that the success of the students is due in part to the collaborative efforts of the school and family, and due to the emphasis, status, and importance accorded to learning and education. Enhanced success in mathematics education is dependent on extensive collaborative efforts involving several subgroups. We have recommended that targeted brochures and flyers be developed and circulated to these groups. We wish here to discuss further the various roles these groups must play in the desired collaboration and also the essential role of professional organizations in creating a climate for change.

Extensive partnership and collaborative efforts should involve school leadership, primarily the principal; public guidance personnel, primarily guidance counselors; the home, primarily the parents; the mathematics professional community, primarily the classroom teachers; and of course, the students. Collaboration within the schools needs to be linked also to colleges and universities. In these institutions, in addition to mathematics faculty members, admissions personnel play key roles. Some prototype regional programs for collaboration include the Bay Area Mathematics Project, and the Mathematics and Science Education Network of North Carolina. The more recently formed American Mathematics Project is chaired by R.O. Wells, Jr., of Rice University. It aims at encouraging and extending local cooperative efforts involving elementary, junior high school, and high school teachers, college and university faculty, and professionals in industries.

In the paragraphs which follow, several issues and questions relative to each of the essential groups will be addressed to provide evidence for a need of such partnerships. Through the raising and addressing of such issues it is apparent that educators and professional organizations will need to take advantage of many opportunities in collaborative efforts in order to affect curricular changes. It is also evident that curricular issues in and of themselves cannot be considered apart from factors which will have a significant effect upon any curricular proposals or modifications.

There are numerous issues related to the leadership within the schools today, and there is research related to the role of leadership, particularly the role of principals, within successful learning environments. Administrators, as agents of Boards of Education, need to become partners with mathematics educators in dealing with curriculum-impacting issues such as the following:

1. The recruitment and retention of qualified staff members.
2. The development and maintenance of educational settings and environments which promote quality instruction, effective learning, and the maintenance of academic standards.
3. The providing of support and encouragement for professional growth, and the upgrading of content competencies and professional teaching skills.
4. The providing of support which enables the opportunities within schools for curricular change.
5. The providing of settings which enable participation in leadership by those having expertise related to education in mathematics.
6. The involvement of administrators as advocates for appropriate changes in the mathematics curriculum.

A second area where collaborative efforts need to be fostered involves the school guidance personnel. Efforts are needed to provide aid and support to counselors who have significant impact upon students. (In March, 1987 the NCTM made several recommendations concerning the advising of students in mathematics in the statement, "Counseling Students in Planning Their Mathematics Programs.") Often, guidance counselors do not have, nor can they be expected to have, a comprehensive understanding of the content or importance of the 11-13 grade mathematics curriculum. Mathematics teachers and guidance counselors need to address the following issues together:

1. How should students be placed in appropriate courses?
2. Which mathematics courses do students need in preparation for other mathematics courses, careers, and future academic work?
3. What communication with students and parents is necessary in order to direct students into appropriate mathematics courses?
4. What support should counselors be able to provide to encourage students in their studies of mathematics?
5. What support should counselors be able to provide to students who are struggling in their mathematics studies?
6. What efforts are needed to encourage all groups of students, regardless of gender or race, to take more mathematics courses?

Many of the recently released reports attribute the success of students in mathematics to the attitudes and the values developed in the home. Parental influence has been shown to have a significant impact upon the successes of the child in school. There are numerous issues and questions which need be addressed in developing stronger partnerships involving parents. (In Spring 1987 the National PTA sent a special mailing to the 25,000 local PTA presidents describing comparative data from international studies in mathematics and calling on parents to be more involved in finding solutions to problems in mathematics education in the schools.) Items for consideration include the following:

1. Parents need to provide an appropriate environment which encourages home study and the completion of homework.
2. Educators and parents need to work together to establish attitudes and priorities where learning becomes more valued.
3. There needs to be a support system within which class attendance is a high priority.
4. Parents need to become better informed about the need for appropriate mathematics backgrounds for their children. It is not enough that students take mathematics courses. They should take mathematics courses appropriate for their career goals.
5. Parents themselves need to be educated about the learning of mathematics. To many adults the study of mathematics is thought to be the performance of arithmetic computations. It needs to be stressed that the learning of mathematics is a continuous, sequential process to be pursued over a period of time. Parents need to understand the need for regular practice in the learning of mathematics.

Another component in the collaborative partnership is the teacher. Until the teacher becomes convinced that there is a need to improve the 11-13 grade curriculum, advances in the quality of 11-13 grade mathematics programs will be limited. Professional organizations will have significant opportunities to provide leadership, encouragement, and training to teachers. Significant thought and attention will be required to bring teachers into the process of affecting change in mathematics.

Students need to be involved fully in the educational process. They need to be made aware that an appropriate mathematics background is a necessity and that mathematics is a "hands-on activity" and a "do-it-yourself" activity. The teacher can be an aid or facilitator of learning, but cannot do the learning for the students. In the era of instant gratification it is important that students realize that solving a mathematical problem will not always be quick nor will it be easy; in fact, not every problem will have a solution. Students need to develop persistence and to realize that progress will often seem slow, especially in the reading of mathematics. Students also need to understand that it is important to keep up with their assignments and that class attendance is essential for success in the study of mathematics.

In the preceding paragraphs several subgroups have been identified as being necessary components for effective partnerships in addressing the issues and questions associated with the 11-13 grade mathematics curriculum and instructional program. The primary responsibility of this task force study has been to focus upon the areas of consensus or lack of consensus as identified in prior reports and studies. Nevertheless, it seems appropriate to address some of those issues which ultimately will determine much of the success or failure related to those curricular items for which there is consensus. There needs to be collaboration of school leaders, counselors, parents, teachers, and students—and linking with college and university personnel—in order to improve mathematics programs for the 11-13 grade student.

This Task Force has recommended that the NCTM and MAA prepare brochures and flyers targeted specifically at the various groups described in this section.

There are additional roles that the professional organizations will need to play if the collaborative efforts of these groups are to succeed. We urge the organizations to continue to enable greater communication among the various groups in these ways:

- To increase and expand the design of workshops and programs for policy makers, high school principals, and post high school level administrators in order to address effectively the issues relating to the recruitment and retention of qualified instructors and teachers of mathematics.
- To expand efforts to work at national, state, and local levels with and through the affiliated groups on issues related to the recruitment and retention of qualified mathematics teachers and instructors.
- To expand their roles in working with affiliated and non-affiliated groups in providing forums to address such issues as curriculum change, uniform standards of quality instruction, equity of opportunity, and expectations in mathematics education.
- To focus further on ways of being more effective in communicating the solutions to identified problems beyond their membership in order to provide the appropriate and necessary impact upon policy makers, administrators, counselors, teachers, parents, and students.
- To undertake actions and assume a significant role in developing programs where articulation with guidance counselors becomes a major priority.
- To work with school administrators, counselors, and teachers in establishing programs which will enhance and increase parental involvement and support in the educational process.
- To develop strategies that will involve the students more deeply in the educational process.

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