Mathematical Sciences

In 1981 the Committee on the Undergraduate Program in Mathematics (CUPM) published a major report entitled RECOMMENDATIONS FOR A GENERAL MATHEMATICAL SCIENCES PROGRAM. *This report comprises six chapters that are reprinted here, with minor editing, as the first six chapters of the present volume. Alan Tucker, Chairman of the CUPM Panel that wrote the 1981 report, has written a new Preface to introduce this reprinting.*

1989 Preface

In the eight years since the CUPM Recommendations on a General Mathematical Science Program appeared, issues in mathematics curriculum, such as calculus reform and discrete mathematics, have become hot topics in the mathematics community and have even received extensive coverage in the popular press. The CUPM Panel on a General Mathematical Sciences Program had the luxury of working in comparative anonymity, although ten panel discussions at national and regional mathematics meetings gave the panel some professional visibility. The Panel's basic goal was to give long-term, general objectives for undergraduate training in mathematics.

The 1960's and 1970's had seen a variety of specialized appeals made to college students interested in mathematics. For example, the discipline of computer science emerged as an exciting career for mathematics students. The earliest CUPM recommendations for the mathematics major were aimed at preparing students for doctoral work in mathematics. By the late 1970's, there was a sense that the mathematics major had lost its way, with upper-division enrollments in traditional core courses like analysis and number theory down by 60% from their levels five years earlier and with industrial employers showing little interest in hiring mathematics majors.

To put these recent events in perspective, the Panel obtained a historical briefing from Bill Duren (the founding chairman of CUPM). He recounted over a century of swings of the pendulum between the theoretical and the practical in American collegiate mathematics education, and between training for careers of the future and training in classical, old-fashioned methods.

The Mathematical Sciences Panel sought to find a common ground for the mathematics major which taught abstraction and application, emerging new problem areas and time-tested old ones. The Panel sought to persuade mathematicians that the curriculum in the mathematics major should be shared among the various intellectual and societal constituencies of mathematics. The challenge was to be diverse without being superficial.

The most concrete consequence of the Panel's work was its name, Panel on a General Mathematical Sciences Program. It asked that the mathematics major be renamed the mathematical sciences major—a change explicitly adopted by hundreds of colleges and universities and implicitly adopted by the vast majority of institutions. The Panel recommended that first courses in most subjects should have a good dose of motivating applications, particularly linear algebra and statistics, and that one advanced course should have a mathematical modeling project. This recommendation also seems to have wide acceptance. There were several panel recommendations that reflected trends already occurring but being resisted by some mathematicians: requiring an introductory course in computer science; not requiring linear algebra as a prerequisite for multivariable calculus; encouraging weaker students to delay core abstract courses until the senior year; and not requiring every mathematics major to take courses in real analysis and abstract algebra (i.e., other mathematics courses at comparable levels of abstraction could be substituted).

Although it was unhappy with calculus, the Mathematical Sciences Panel consciously avoided recommending changes in calculus for fear that the inevitable controversy and the complexity of such an undertaking would undermine acceptance of its basic recommendations about the structure of a mathematics major. The Panel touched only lightly on the issue of discrete versus continuous mathematics, recommending exposure to "more combinatorially-oriented mathematics associated with computer and decision sciences" (Tony Ralston's provocative essays about discrete mathematics had not yet appeared).

It was gratifying to the Mathematical Sciences Panel that its report was well-accepted: all two-thousand copies printed have been sold (another two-thousand copies had been sent gratis to department heads). In reviewing the report for this reprinting, the only changes have been to add a few additional references. On the other hand, there was one panel suggestion that has been ignored thus far and which merits consideration.

It concerns the "modest" version of abstract algebra (in Section III) in which time would be spent sensitizing students to recognize how algebraic systems arise naturally in many situations in other areas of mathematics and outside mathematics (to keep algebra alive in their minds after they leave college).

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March, 1989

1981 Preface

This report of the CUPM Panel on a General Mathematical Sciences Program (MSP) presents recommendations for a mathematical sciences major. The panel has concentrated its efforts on general curricular themes and guiding pedagogical principles for a mathematical sciences major. It has tried to frame its recommendations in general terms that will permit a variety of implementations, tailored to the needs of individual institutions. A prime objective of the original 1960's CUPM curriculum recommendations for upper-level mathematics courses was easing the trauma of a student's first year of graduate study in mathematics. This report refocuses the upper-level courses on the traditional objectives of general training in mathematical reasoning and mastery of mathematical tools needed for a life-long series of different jobs and continuing education.

The MSP panel has tried to avoid highly innovative approaches to the mathematics curriculum. The emphasis, instead, has been on using historically rooted principles to organize and unify the mathematical sciences curriculum. The MSP panel believes that the primary goal of a mathematical sciences major should be to develop rigorous mathematical reasoning. The word 'rigorous' is used here in the sense of 'intellectually demanding' and 'in-depth.' Such reasoning is taught through a combination of problem solving and abstract theory. Most topics should initially be developed with a problem-solving approach. When theory is introduced, it usually should be theory for a purpose, theory to simplify, unify, and explain questions of interest to the students.

CUPM now believes that the undergraduate major offered by a mathematics department at most American colleges and universities should be called a Mathematical Sciences major. Enrollment data show that for several years less than half the courses, after calculus, in a typical mathematics major have been in pure mathematics. Furthermore, applied mathematics, probability and statistics, computer science, and operations research are important subjects which should be incorporated in undergraduate training in the general area of mathematics.

Computer science has become such a large, multifaceted field, with ties to engineering and decision sciences, that it no longer can be categorized as a mathematical science (at the National Science Foundation, computer science and mathematical sciences are different research categories). A mathematical sciences major must involve coursework in computer science because of the usefulness of computing and because of computer science's close ties to mathematics. Undergraduate majors in mathematical sciences and in computer science should complement each other.

The new course recommendations presented in this report do not, in most instances, replace past CUPM syllabi. They describe different approaches to courses; for example, a one-semester combined probability and statistics course, or a multivariate calculus course without a linear algebra prerequisite.

The work of the CUPM Panel on a General Mathematical Sciences Program was supported by a grant from the Sloan Foundation. The chairmen of CUPM during this project, Donald Bushaw and William Lucas, deserve special thanks for their assistance.

For information about other CUPM documents and related MAA mathematics education publications, write to: Director of Publications, The Mathematical Association of America, 1529 Eighteenth Street, N.W., Washington, D.C. 20036.

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Panel Background

The CUPM Panel on a General Mathematical Sciences Program (MSP) was constituted in June, 1977 at a CUPM conference in Berkeley. CUPM members decided that a major re-examination of the mathematics major was needed. The CUPM model for the mathematics major contained in the 1965 CUPM reports on Pregraduate Training in Mathematics and a General Curriculum in Mathematics in Colleges (revised in 1972) was felt to be out of date. Following a six-month study, MSP reported to CUPM that the CUPM mathematics major curriculum should be substantially revised and broadened to define a mathematical sciences major. MSP was charged then with developing mathematical sciences recommendations.

Five subpanels were created to develop course recommendations in:

- The calculus sequence,
- Computer science,
- Modeling and operations research,
- Statistics, and
- Upper-level core mathematics.

The MSP project has had the cooperation of curriculum groups in the American Statistical Association, the Association for Computing Machinery, the Operations Research Society of America, and the Society for Industrial and Applied Mathematics. Graduate programs in the subjects covered by those societies draw heavily on undergraduate mathematics students, and except for computer science, undergraduate courses in these subjects are usually taught by mathematicians. Hence these curriculum groups had a major interest in the design of a mathematical sciences major.

The MSP panel coordinated its work with the National Research Council's Panel on Training in Applied Mathematics (chaired by P. Hilton, a member of MSP). The Hilton panel had a much broader mandate than the MSP panel. Its report addresses the unification of the mathematical sciences, the attitudes of mathematicians, academic-industrial linkages, and society's image of the mathematical sciences, as well as curricula. The Hilton report presented a limited number of general curriculum principles with the expectation that the MSP panel would develop fuller curriculum recommendations. The MSP panel recommendations have incorporated these principles (although the Hilton panel's stress on differential equations has been diminished). The MSP panel strongly endorses the Hilton report's emphasis on the importance within mathematics departments of proper attitudes towards the uses and users of mathematics and of a unified view that respects the content and teaching of pure and applied mathematics equally.

While CUPM and the Hilton panel have been recommending changes in the collegiate mathematics program, the National Council of Teachers of Mathematics has been assessing priorities in school mathematics. The 1980 NCTM booklet, *An Agenda for Action,* recommends "that problem solving be the focus of school mathematics in the 1980s ... that basic skills in mathematics be defined to encompass more than computational facility." Recent nation-wide mathematics tests administered to students in several grades showed uniformly poor performance on questions of a problem solving or application nature. Inevitably these mathematical weaknesses will become more of a problem with college students.

The tentative MSP ideas for curriculum revision were discussed by panel members at sectional and national MAA meetings, at the PRIME 80 Conference, and individually with dozens of mathematics department chairpersons. The helpful criticisms received on these occasions played a vital role in shaping the panel's thinking. It should be noted that several people warned that a mathematical sciences major was unworkable because of the diversity of techniques and modes of reasoning in the mathematical sciences today. Others stated that student course preferences had already "redefined" the mathematics major along the lines being proposed by the MSP panel.

Curriculum Background

Many students today start mathematics in college at a lower level and yet have specific (but uninformed) career goals that require a broad scope of new topics of varying mathematical sophistication. Student changes are reflected in recent upper-level enrollment shifts and the explosion of new theory and applications in all the mathematical sciences. Uncertainties in curriculum produced by these developments have led the MSP panel to look for guidance from past CUPM curriculum development experiences and, farther back, from the traditional goals of the mathematics major before CUPM's creation. No matter how great the advances in the past generation, the traditional intellectual objectives of training in mathematics, defined over scores of years, should be the basis of any mathematical sciences program.

Until the 1950s, mathematics departments were primarily service departments, teaching necessary skills to science and engineering students and teaching mathematics to most students solely for its liberal-arts role as a valuable intellectual training of the mind. The average student majoring in mathematics at a better college in the 1930s took courses in trigonometry, analytic geometry, and college algebra (including calculus preparatory work on series and limits) in the freshman year followed by two years of calculus. While this program may today seem to have unnecessarily delayed calculus, and subsequent courses based on calculus, it did provide students with a background that permitted calculus to be taught in a more rigorous (i.e., more demanding) fashion than it is today.

The mathematics major was filled out with five or six electives in subjects such as differential equations (a second course), projective geometry, theory of equations, vector analysis, mathematics of finance, history of mathematics, probability and statistics, complex analysis, and advanced calculus. Most mathematics majors also took a substantial amount of physics. Training of secondary school mathematics teachers rarely included more than a year of calculus. In the early 1950s, twenty years later, the situation had changed only a little; top schools did now offer modern algebra and abstract analysis.

In 1953, amid reports of widespread dissatisfaction with the undergraduate program, the Mathematical Association of America formed the Committee on Undergraduate Program (CUP, later to be renamed CUPM). CUPM concentrated initially on a unified introductory mathematics sequence Universal Mathematics, consisting of a first semester analysis/college algebra course (finishing with some calculus) followed by a semester of "mathematics of sets" (discrete mathematics). CUPM hoped its Universal Mathematics would "halt the pessimistic retreat to remedial mathematics ...(and) ... modernize and upgrade the curriculum."

The first comprehensive curriculum report of CUPM, entitled Pregraduate Training for Research Mathematicians (1963), outlined a model program designed to prepare outstanding undergraduates for Ph.D. studies in mathematics. Emphasis on Ph.D. preparation represented a major departure from the traditional mathematics program and was the source of continuing controversy. A more standard mathematics major curriculum was published in 1965 (revised in 1972), but many colleges also found it to be too ambitious for their students.

For a fuller history of CUPM, see the article of W. Duren (founder of CUPM), "CUPM, The History of an Idea," *Amer. Math. Monthly* 74 (1967), pp. 22-35.

Current Issues

In 1970, 23,000 mathematics majors were graduated. The numbers of Bachelors, Masters, and Doctoral graduates in mathematics had been doubling about every six years since the late 1950s. The 1970 CBMS estimate for the number of Bachelors graduates in mathematics in 1975 was 50,000, but by the late 1970s only 12,000 were graduating annually. Enrollments in many upper-level pure mathematics courses declined even more dramatically in the 1970s as students turned to applied and computer-related courses.

Yet while the number of mathematics majors is decreasing, the demand for broadly-trained mathematics graduates is increasing in government and industry. Mathematical problems inherent in projects to optimize the use of scarce resources and, more generally, to make industry and government operations more efficient guarantee a strong future demand for mathematicians. These problems require people who, foremost, are trained in disciplined logical reasoning and, secondarily, are versed in basic techniques and models of the mathematical sciences. In Warren Weaver's words, these are problems of "organized complexity" as well as well-structured applied mathematics of the physical sciences. If mathematics departments do not train these quantitative problem-solvers, then departments in engineering and decision sciences will.

The unprecedented growth of computer science as a major new college subject parallels the theoretical growth of the discipline and its ever-expanding impact on business and day-to-day living. The number of computer science majors now substantially exceeds the number of mathematics majors at most schools offering programs in both subjects. However, computer science has not "taken" students from mathematics, any more than science and engineering take students from mathematics. Rather, computers have generated the need for more quantitative problem-solvers, as noted above.

The shortage of secondary school mathematics teachers nation wide has become worse than ever before. This shortage appears to be due in large measure to the greater attractiveness of computing careers to college mathematics students (indeed high-paying computer jobs are currently luring many teachers out of the classroom). Although the training of future teachers should include course work in computing and applications, such course work heightens the probability that these students will switch to careers in computing.

On another front, pre-calculus enrollments have soared as the mathematical skills of incoming freshmen have been declining (a problem that concerned CUP in its first year). The mathematics curriculum may soon need to allow for majors who do not begin calculus until their sophomore year, as was common a generation ago.

At universities, the decline in graduate enrollments has frequently over-shadowed the decline in undergraduate majors. Faced with heavy precalculus workloads, shrinking graduate programs, and competition from other mathematical sciences departments, university mathematics departments appear less able to broaden and restructure the mathematics major than most liberal-arts college mathematics departments. Many university mathematicians prefer to retain their current pure mathematics major for a small number of talented students.

There are also several encouraging developments. A natural evolution in the mathematics major is occurring at many schools. Students and faculty have developed an informal "contract" for a major that includes traditional core courses in algebra and analysis along with electives weighted in computing and applied mathematics (a formal "contract" major at one school is discussed below).

Another important development is the emphasis on systems design, as opposed to mathematical computation, in current computer science curricula. The Association for Computing Machinery Curriculum 78 Report delegates the responsibility for teaching numerical analysis, discrete structures, and computational modeling to mathematics departments. This ACM curriculum implicitly encourages students interested in computer-based mathematical problem solving to be mathematical sciences majors. The MSP panel has been careful to coordinate its work with computer science curriculum groups in order to minimize potential conflicts and maximize compatibility between computer science and mathematical sciences programs.

Curricular Principles

The goal of this panel was to produce a flexible set of recommendations for a mathematical sciences major, a major with a broad, historically rooted foundation for dealing with current and future changes in the mathematical sciences. The panel sought a unifying philosophy for diverse course work in analysis, algebra, computer science, applied mathematics, statistics, and operations research.

Program Philosophy

I. The curriculum should have a primary goal of developing attitudes of mind and analytical skills required for efficient use and understanding of mathematics. The development of rigorous mathematical reasoning and abstraction from the particular to the general are two themes that should unify the curriculum.

II. The mathematical sciences curriculum should be designed around the abilities and academic needs of the average mathematical sciences student (with supplementary work to attract and challenge talented students).

III. A mathematical sciences program should use interactive classroom teaching to involve students actively in the development of new material. Whenever possible, the teacher should guide students to discover new mathematics for themselves rather than present students with concisely sculptured theories.

IV. Applications should be used to illustrate and motivate material in abstract and applied courses. The development of most topics should involve an interplay of applications, mathematical problem-solving, and theory. Theory should be seen as useful and enlightening for all mathematical sciences.

V. First courses in a subject should be designed to appeal to as broad an audience as is academically reasonable. Many mathematics majors do not enter college planning to be mathematics majors, but rather are attracted by beginning mathematics courses. Broad introductory courses are important for a mathematical sciences minor.

Course Work

VI. The first two years of the curriculum should be broadened to cover more than the traditional four semesters of calculus-linear algebra-differential equations. Calculus courses should *include more numerical methods and non*-physical-sciences applications. Also, other mathematical sciences courses, such as computer science and applied probability and statistics, should be an integral part of the first two years of study.

VII. All mathematical sciences students should take a sequence of two upper-division courses leading to the study of some subject(s) in depth. Rigorous, proof-like arguments are used throughout the mathematical sciences, and so all students should have some proof-oriented course work. Real analysis or algebra are natural choices but need not be the only possibilities. Proofs and abstraction can equally well be developed through other courses such as applied algebra, differential equations, probability, or combinatorics.

VIII. Every mathematical sciences student should have some course work in the less theoretically structured, more combinatorially oriented mathematics associated with computer and decision sciences.

IX. Students should have an opportunity to undertake "real-world" mathematical modeling projects, either as term projects in an operations research or modeling course, as independent study, or as an internship in industry.

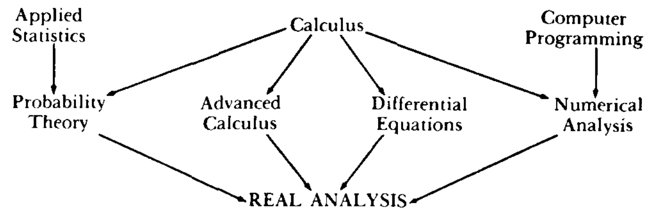
X. Students should have a minor in a discipline using mathematics, such as physics, computer science, or economics. In addition, there should be sensible breadth in physical and social sciences. For example, a student interested in statistics might minor in psychology but also take beginning courses in, say, economics or engineering (heavy users of statistics).

Building Mathematical Maturity

As noted in Principle I, a major in mathematical sciences should emphasize general mathematical reasoning as much as mastery of various subject matter. Implicit in this principle is that less material would be covered in many courses but that students would be expected to demonstrate a better understanding of what is taught, e.g., by solving problems that require careful mathematical analysis.

This mathematical sciences curriculum would model the historical evolution of mathematical subjects: some problems are introduced, formulas and techniques are developed for solving problems (usually with heuristic explanations), then common aspects of the problems are examined and abstracted with the purpose of better understanding "what is really going on." The difference in this scheme between beginning calculus and upper-division probability theory would be primarily a matter of the difficulty of the problems and techniques and the speed with which the material is covered and generalized, i.e., a matter of the mathematical maturity of the audience. In the course of two or three years of such course work, there would be a steady increase in sophistication of the material and more importantly, an increase in the student's ability to learn and organize the ideas of a new mathematical subject. Students should be able to read and learn mathematics on their own from texts. The MSP panel feels that such maturity is a function of how a subject is learned as much as what is learned.

All courses should have some proofs in class and, as the maturity of students increases, occasional proofs as homework exercises. In particular, students should acquire facility with induction arguments, a basic method of proof in the mathematical sciences. After reviewing performances of current students and programs of mathematics students 30 years ago, the MSP panel has concluded that many able students do not now have, nor were they previously expected to have, the mathematical maturity to take theoretical courses before their senior year. On the other hand, by the senior year, all students should be ready for some proof-oriented courses that show the power of mathematical abstraction in analyzing concepts that underlie a variety of concrete problems. For example, part of a flowchart of courses leading to a senior-year real analysis course might be:



Core Requirements

The panel has found the question of whether to require courses in algebra and analysis its most controversial problem. In light of the strongly differing opinions received on this subject, the MSP panel is making only a minimal recommendation (Principle VII) that it feels is reasonable for all students. Possible two course sequences besides a year of analysis or of algebra are: analysis and proof-oriented probability theory, analysis and differential equations, abstract algebra and (proof-oriented) combinatorics, applied algebra and theory of computation, or analysis and a topics-in-analysis seminar. While not a sequence, one course in analysis and one course in algebra also fulfill the spirit of this requirement. Some departments will want to make stronger requirements. The issue of theory requirements is discussed more fully below.

Students should not be required to study a subject with an approach whose rationale depends on material in later courses nor should they be required to memorize (blindly) proofs or formulas. Some upper-level elective courses should always be taught as mathematics-for-its-own-sake, but an instructor should be very careful not to skip the historical motivation and application of a subject in order to delve further into its modern theory.

The recommendation for interactive teaching (Principle III) seeks to encourage student participation in developing new mathematical ideas. It constrains an instructor to teach at a level that students can reasonably follow. Interactive teaching implicitly says that mathematics is learned by actively doing mathematics, not by passively studying lecture notes and mimicking methods in a book. Without needlessly slowing progress in class, an instructor should discuss how one can learn much from wrong approaches suggested by students. New mathematical theories are not divined with textbook-like compact proofs but rather involve a long train of trial-and-error creativity.

Henry Pollak expressed this need in the Conference Board of Mathematical Sciences book, *The Role of Ax-*

iomatics and Problem Solving in Mathematics (Ginn, 1966):

> A carefully organized course in mathematics is sometimes too much like a hiking trip in the mountains that never leaves the well-constructed trails. The tour manages to visit a steady sequence of the high spots in the natural scenery. It carefully avoids all false starts, dead ends and impossible barriers and arrives by five o'clock every afternoon at a well-stocked cabin. ...However, you miss the excitement of occasionally camping out or helping to find a trail and of making your way cross-country with only a good intuition and a compass as a guide. "Cross country" mathematics is a necessary ingredient of a good education.

Further details about the course work recommendations in Principles VI, VIII, and IX appear in later chapters of this report. Discussion of courses in discrete methods, applied algebra, and numerical analysis appears in the last section of this chapter.

Teaching Mathematical Reasoning

Because a mathematical sciences major must include a broader range of courses than a standard (pure) mathematics major, many mathematicians have expressed concern that it will be harder to teach the average mathematics student rigorous mathematical reasoning in a mathematical sciences major. They believe that the major will develop problem-solving skills but that without more abstract pure mathematics, students will never develop a true sense of rigorous mathematical reasoning. The MSP panel thinks that a mathematical sciences major with a strong emphasis on problem-solving is in keeping with time-tested ways of developing "mathematical reasoning." The question of whether to require "core" pure mathematics courses, such as abstract algebra and real analysis, in any mathematical sciences major is discussed in the next section.

Historically (before 1940), the main thrust of the mathematics major at most colleges was problem-solving. Most courses in the major could be classed as mathematics for the physical sciences: trigonometry, analytic geometry, calculus (first-year and advanced), differential equations, and vector analysis. Proofs in advanced calculus were symbolic computations. Proofs in number theory were, and still are, usually combinatorial problems. The one abstract "pure" course in the curriculum was logic. A "rigorous" course did not mean an abstract course, "mathematics done right." A rigorous course used to mean a demanding, more in-depth treatment that required more skill and ingenuity from the student. The past curriculum surely had some faults, but its problem-solving and close ties to physics came from traditions that go back to the roots of mathematics.

While problem solving may traditionally be the primary way of teaching mathematical reasoning to undergraduates, the complexity and breadth of modern mathematics and mathematical sciences require theory to help organize and simplify learning. Rigorous problem solving should lead students to appreciate theory and formal proofs. In a mathematical sciences major, theory should be primarily theory for a purpose, theory born from necessity (of course, this is also the historical motivation of most theory). Students may find theory difficult, but they should never find it irrelevant.

Most courses in a mathematical sciences major should be case studies in the pedagogical paradigm of real world questions leading to mathematical problem solving of increasing difficulty that forces some abstraction and theory. As mentioned earlier, lower-level courses would concentrate on problem solving to build technical skills with occasional statements of needed theorems, while typical upper-level courses would concentrate on problem solving to build technical skills with occasional statements of needed theorems, while typical upper-level courses would emphasize the transition from harder problem-solving to theory.

Instructors should resist pressures to survey fully fields such as numerical analysis, probability, statistics, combinatorics, or operations research in the one course a department may offer in the field. The instructor of such a course should give students a sense of the problems and modes of reasoning in the field, but after that, should be guided by the pedagogical model given above. All syllabi produced by MSP subpanels should be viewed in this light. Most instructors will cover most of a suggested syllabus, but general pedagogical goals should always take precedence over the demands of individual course syllabi.

The MSP panel believes that for generations mathematics instructors have used the paradigm mentioned above to develop rigorous mathematical reasoning. Implicit in this paradigm is a unity of purpose between students and instructor. Most students like to start with concrete real-world examples as a basis for mathematical problem solving. They expect the problems to get harder and require more skill and insight. And they certainly appreciate theory when it makes their work easier (although understanding formal proofs of useful theory requires maturity). Interactive teaching also becomes natural: students are interested in participating in a class that is developing a subject in a way that they can appreciate.

How Much Theory?

This section summarizes arguments for and against requiring upper-level analysis and algebra courses of all mathematical sciences majors, and why the MSP panel made its "compromise" decision.

Expecting controversy on several issues, the MSP panel organized sessions at national and regional MAA meetings to get input from the mathematics community. The main area of contention was how many courses to require in specific areas. The panel heard complaints that some areas were being neglected or that only one course in a certain area would be so superficial as to be worse than no course. However, most constituencies came to accept the need for compromise recommendations of limited exposure to several areas with students left to choose for themselves an area to study in greater depth. On the other hand, one important issue emerged on which a compromise position seemed to antagonize at least as many people as it pleased. This was the question of whether to require an analysis and/or an abstract algebra course and, more generally, how much proof-oriented course work should be required in a mathematical sciences major.

In the early 1970's, a majority of mathematics programs required at least these two upper-level "core mathematics" courses for all students. Recently, declining enrollments in these courses and student preference for more applied or computing courses have forced many departments either to relax this requirement or to introduce a new applied track which does not require these two courses. People favoring the requirement of analysis and algebra argue that:

- Not requiring them would speed an already dangerous deterioration in the intellectual basis of the mathematics major;
- A major without at least analysis and algebra would be a superficial potpourri of courses—a major of no real value to anyone, e.g., graduate study in statistics requires analysis and (linear) algebra;
- One cannot understand "what mathematics is about" without these two courses—a major without these two courses simply should not be offered by a mathematics department.

People in favor of not requiring analysis and algebra argue that:

- With a more applied emphasis the mathematical sciences major will attract more good students, whereas requiring these courses would mean no change (except for new applied electives) from the 1960s type of mathematics major that today attracts only a marginal number of students;
- Analysis and algebra are fine for some students but demand a mathematical maturity that many other undergraduates lack—these students memorize proofs blindly to pass examinations and never take the follow-on courses needed to appreciate the structure and elegance of these subjects; and
- Proofs and abstraction can equally well be developed through other courses such as applied algebra, probability, differential equations, or combinatorics.

Mathematicians must face the reality of a general change in the attitude of college students towards mathematics. The popularity of science and mathematics in the 1960s drew more of the brightest students to mathematics and also motivated all students to work harder at mathematics in high school. So the average mathematics student was capable of handling a more theoretical mathematics program.

Today, mathematics appears to be getting no more than its traditional (smaller) share of bright students, and high school study habits are less good. However, almost all of today's mathematics students still find a few subjects, pure or applied, particularly interesting and want to study this material in some depth. Also by the senior year, the MSP panel believes that mathematics majors do have the mathematical maturity to appreciate, say, a moderately abstract real analysis course. Examples of new approaches to teaching analysis and other core mathematics courses appear in subsequent chapters.

Since there was agreement on the importance of some theoretical depth, the MSP panel proposed the compromise of Principle VII, recommending "a sequence of two upper-division courses leading to the study of some subject in depth." Because of the lack of consensus on the analysis-algebra question, the MSP panel expects this issue to be debated and modified at individual institutions. The faculty should not require courses that most students strongly dislike, nor should faculty shy away from any theory requirements for fear of losing majors. The faculty rather must motivate students to appreciate the value of some theoretical course work.

Sample Majors

This section presents two 12 semester-course mathematical sciences majors. Many other sample majors could be given. The MSP panel believes that most majors should be a "convex combination" of the two majors given here. Major A contains much of a standard mathematics major, while Major B is a broader program designed for students interested in problem solving. Both majors should be accompanied by a minor in a related subject.

The common core of all majors would be three semesters of calculus, one course in linear algebra, one course in computer science plus either a second computer course or extensive use of computing in several other courses, one course in probability and statistics, the equivalent of a course in discrete methods, modeling experience, and two theoretical courses of continuing depth.

Mathematical Sciences Major A

- Three semesters of calculus
- Linear algebra
- Probability and statistics
- Discrete methods
- Differential equations (with computing)
- Abstract algebra (one-half linear algebra)
- Two semesters of advanced calculus/real analysis
- One course from the following set: abstract algebra (second course), applied algebra, geometry, topology, complex analysis, mathematical methods in physics
- Mathematical modeling
- Plus related course work: two semesters of computer science and two semesters of physics, to be taken in the first two years.

Mathematical Sciences Major B

- Three semesters of calculus
- Linear algebra
- Introduction to computer science
- Numerical analysis *or* second course in computer science
- Probability and statistics
- Advanced calculus *or* abstract algebra
- Discrete methods *or* differential equations
- Mathematical modeling or operations research
- Two electives continuing a subject with theoretical depth.

Subsequent sections in this report contain recommendations for discrete methods, applied algebra, and numerical analysis courses; for calculus, linear algebra, and differential equations courses; for upper-level core mathematics; for computer science; for modeling and operations research; and for probability and statistics.

Major A is meant to be close to the spirit of the major suggested by the NRC Panel on Training in Applied Mathematics. That panel viewed differential equations as a unifying theme in the major. The proper mixture of Majors A and B (with appropriate electives) would also allow students to make statistics or operations research a unifying theme.

The MSP panel feels that a set of courses similar to either of the above two majors, or a mixture thereof, would be reasonable for most mathematical sciences students. Some departments could offer several tracks for the mathematical sciences major. Special areas of faculty strength or student interest should obviously be reflected in the curriculum.

Computing assignments should be used in most courses. When a liberal arts college mathematics department teaches computer science, such computing course work must frequently be counted within the college limit of 12 or 13 courses permitted in one department. This regulation is assumed in Major B. However, the MSP panel believes that counting computer courses this way unfairly restricts a mathematical sciences major. One alternative is to list computer courses through the Computing Center.

The one fundamental new course in these sample majors is discrete methods. As mentioned in Principle VIII, the MSP panel feels that the central role of combinatorial reasoning in computer and decision sciences requires that some combinatorial problem solving should be taught in light of the three semesters devoted to analysis-related problem solving in the calculus sequence. To this end, the modeling course should be heavily combinatorial if students have not taken a formal discrete methods course.

Major A would be good preparation for graduate study in mathematics, applied mathematics, statistics, or operations research as well as many industrial positions as a mathematical analyst or programmer. Major B would be good preparation for most industrial positions and for graduate study in applied mathematics, statistics, or operations research (for such graduate study, both advanced calculus and upper-level linear algebra are usually needed). Representatives from many good mathematics graduate programs have stated that they would accept strong students with Major B-type training.

Many computer science graduate programs would accept Major B if the two electives were in computer science (although some other undergraduate computer science course deficiencies may still have to be made up in the first year of graduate study). In a computer science concentration within a mathematical sciences major, modern algebra might be replaced by applied algebra (see below for more details). Major B with an elective in the theory of interest and a second probability-statistics course would be excellent preparation for actuarial careers. Students interested in physical sciences-related applied mathematics could modify either sample major to get a good program. Both majors provide preparation for secondary school mathematics teaching, when supplemented with teaching methodology and practicum courses (theory courses must include algebra and geometry).

Many smaller schools are being forced to offer a program in the spirit of Major B because almost all of B's courses have the needed enrollment base of students drawn from outside mathematics.

The courses involving numerical analysis, probability and statistics, discrete methods, and modeling all can be designed as lower-level or upper-level courses. A large amount of flexibility is possible in "repackaging" the mathematical sciences material. For example, a Computational Models course (see the 1971 CUPM *Report on Computational Mathematics*) could cover some numerical analysis along with a little applied probability and statistics to be used in simulation modeling. A quarter system institution would have even greater flexibility in implementing this major.

Mathematical Sciences Minor

Just as a mathematical sciences major should be accompanied by a minor in a related subject, so also do many other disciplines encourage their students to have a minor, or double major, in mathematics. At some colleges, as many as half the mathematics majors have another major. Unfortunately, while mathematical methods are playing an increasingly critical role in social and biological sciences and in business administration, students are generally ignorant or misinformed in high school and early college years about the importance of mathematics in these areas.

The result is that many students either do not realize the value of further course work in the mathematical sciences until their junior or senior year, or their poor high school preparation forces them to take a year of remedial mathematics before they can begin to learn any of the college mathematics they need. For such students, a traditional six to eight course minor in mathematics, starting with (at least) three semesters of calculus, is not feasible. When students in the social and biological sciences come to realize the value of mathematics in the junior year, they have frequently had only one semester of calculus, or perhaps a year of calculus with probability.

The MSP panel believes that these students would be well served by a six to eight course mathematical sciences minor consisting of two semesters of calculus, one semester of (calculus-based) probability and statistics, one semester of introductory computer science, plus two to four electives chosen from courses such as numerical analysis, discrete methods, linear algebra, differential equations, linear programming, mathematical modeling, and additional courses in calculus, probability or statistics, and computer science. Such a minor could easily be completed in three semesters. It has little prerequisite structure so that students can immediately pick courses based on personal interests rather than initially "mark time" waiting to complete the calculus sequence.

Such a minor has several important points in its favor. First of all, this minor is a collection of useful mathematical sciences courses which present concepts and techniques that arise frequently in the social and biological sciences. While this minor lacks the mathematical depth of the traditional type of mathematics minor, it nonetheless introduces students to important modes of mathematical reasoning. Second, such a minor will be attractive to students because it enhances employment opportunities and prospects for admission to graduate or professional schools. Third, after the exposure to interesting mathematical sciences topics, some students will want to study these subjects further in graduate school, either in a mathematical sciences graduate program or as electives in other graduate programs. Fourth, this minor will bring more students into mathematical sciences courses, making it possible to offer these courses more frequently. Conversely, offering more mathematical sciences courses each semester will make a mathematical sciences minor, as well as the regular mathematical sciences major, more attractive to students. In addition, when more students are taking mathematical sciences courses and finding out how useful mathematics is, the campus-wide student awareness of the value of mathematics will increase.

Examples of Successful Programs

Proper curriculum is the heart of a mathematical sciences program, but there are many non-academic aspects that also must be considered. A wide variety of course offerings is not as important as the spirit with which the general program is offered. This section discusses salient features of some successful mathematics programs. "Successful" means attracting a large number of students into a program that develops rigorous mathematical thinking and also offers a spectrum of (well taught) courses in pure and applied mathematics. Successful programs typically produce 5% to 8% of their college's graduates, although nation wide, mathematics majors constitute only about 1% of college graduates. Faculty and student morale is uniformly high in these programs. As one would expect, teaching and related student-oriented activities consume most of the faculty's time in such successful programs, and there is little faculty research. The professors' pride in good teaching and in the successes of their students leaves them with few regrets about not publishing. The set of programs mentioned here is only a sampling of successful programs that have come to the attention of this CUPM panel. More detailed information about these mathematics programs is available from individual colleges.

Saint Olaf College, a 2800-student liberal arts college in Northfield, Minnesota, has a contract mathematics major. Each mathematics student presents a proposed contract to the Mathematics Department. The contract consists of at least nine courses (college regulations limit the maximum number of courses that can be taken in one department to 14). The department normally will not accept a contract without at least one upper-level applied and one upper-level pure mathematics course, a computing course or evidence of computing skills, and some sort of independent study (research program, problem-solving proseminar, colloquium participation, or work-study).

Frequently a student and an advisor will negotiate a proposed contract. For example, a faculty member will try to persuade a student interested in scientific computing and statistics that some real analysis and upper-level linear algebra should be included in the contract by showing that this material is needed for graduate study in applied areas, and in any case a liberal arts education entails a more broadly based mathematics major. Conversely, a student proposing a pure mathematics contract would be confronted with arguments about not being able to appreciate theory without knowledge of its uses. In the end, the student and the faculty member understand and respect each other's point of view.

This understanding of each other's interests naturally carries into the classroom. Also, the contract negotiations "break the ice" and make students more at ease in talking to faculty (and encourage constructive criticism). The Mathematics Department offers minors in computing and statistics, but the attractiveness of a contract major in mathematics leads most students interested in these areas eventually to become mathematics majors.

Lebanon Valley College, a small (1000-student) liberal arts college in Pennsylvania, has only five mathematics faculty but its Department of Mathematical Sciences offers majors in Mathematics, Actuarial Science, Computer Science, and Operations Research. The course work in the mathematics graduate preparation track involves a problem seminar, Putnam team sessions, and formal and informal topics courses (because of the limited demand in this area). All mathematical sciences majors must take a rigorous 25 semester-hour core of calculus, differential equations, linear algebra, foundations, and computer science. Most courses are peppered with applications and computing assignments.

The mathematics faculty are heavily involved in recruiting students by attending College Fairs and College Nights and by visiting regional high schools to explain to students and counselors the many diverse and attractive careers in the mathematical sciences, and the importance of mathematics in other professions. As a result of this effort, 10% of the incoming Lebanon Valley freshmen plan majors in the mathematical sciences (the national average is 1%), and 7% of Lebanon Valley graduates are mathematical sciences majors. Many students are initially attracted by the major in actuarial science (an historically established profession) and then move into other areas of applied and pure mathematics, but this pattern may change with the newly established computer science major.

Once the faculty have the "students' attention," they work the students hard. The students respond positively to the demands of the faculty for three reasons. First, known rewards await those who do well in mathematics (besides the obvious long-term rewards, the department awards outstanding students with membership in various professional societies in the mathematical sciences). Second, a personal sense of intellectual achievement is carefully nurtured starting in the freshman year with honors calculus for mathematics majors. Finally, as at St. Olaf, a continuing dialogue between students and faculty allows students to help shape the mathematics program. In fact, students interview candidates for faculty positions and their recommendations carry great weight. The department keeps in close touch with alumni by sending each one a personal letter every other year with news about the department and fellow alumni.

Nearby Gettysburg College has a special vitality in its mathematics program that comes from an interdisciplinary emphasis. The department has held joint departmental faculty meetings with each natural and social science department at Gettysburg to discuss common curriculum and research interests. Several interdisciplinary team-taught courses have been developed, such as a course on symmetry taught jointly by a mathematician and a chemist. An interdepartmental group organized two recent summer workshops in statistics which drew faculty from eight departments. Mathematics faculty have audited a variety of basic and advanced courses in related sciences to learn to talk the language of mathematics users. Mathematics faculty bring this interdisciplinary point of view into every course they teach, giving interesting applications and showing, say, how a physicist would approach a certain problem. Needless to say, a large number of mathematics majors at Gettysburg are double majors.

Frequently a separate computer science department with its own major spells disaster for the mathematics major at a college. But Potsdam State College (in the economically depressed northeast corner of New York) has possibly the greatest percentage of mathematics graduates of any public institution in the country—close to 10%—despite competition from a popular computer science major. The most striking feature to a visitor to the Potsdam State Mathematics Department is the great enthusiasm among the students and the sense of pride students have in their ability to think mathematically. (While it is hard to measure objectively these students' mathematical development, leading technological companies, such as Bell Labs, IBM, and General Dynamics, annually hire several dozen Potsdam mathematics graduates.)

Classes have a limited amount of formal lectures. Most time is spent discussing work of the students. The emphasis on giving students a sense of achievement is due in large part to experiences of the Potsdam chairman when he taught in a Black southern institution. By instilling self confidence, he had helped able but ill-prepared students excel in calculus and even saw some go on to good mathematics graduate programs. The department has various awards for top students, a very active Pi Mu Epsilon chapter, publications about careers in mathematics and successes of former students, and a large student-alumni newsletter. Upper-class mathematics students are used to tutor (and encourage) beginning students. They also communicate their enthusiasm about mathematics to friends and teachers back home. As a result, half the incoming Potsdam freshmen sign up for calculus (although few departments require it).

The computer science major at Potsdam State is viewed by the mathematics faculty as a great asset to the Mathematics Department. The computer science major helps attract good students to Potsdam who often decide to switch to, or double major with, mathematics. Also the computer science program offers career skills and needed mathematical breadth. Numerical analysis, operations research, and modeling are taught in computer science (the Mathematics Department has had to limit severely their upper-level electives in order to keep class size down and preserve small group seminars).

As noted at the start of this section, the preceding mathematics programs represent only a small sampling of the excellent programs in this country. Several women's colleges offer fine programs worth noting. For example, the Goucher College Mathematics Department has integrated computing in almost all courses and has a broad curriculum in pure and applied mathematics; and the Mills College Mathematics Department has successfully promoted the critical role of mathematics for careers in science and engineering. The cornerstone of Ohio Wesleyan's excellent mathematics program is an innovative calculus sequence (with computing, probability, and diverse mathematical modeling). Georgia State University, an urban public institution with a highly vocational orientation, has a Mathematics Department that has broken out of the typical low-level service function mode to offer a fine, well-populated mathematical sciences major. While research and graduate programs often dominate concerns about the undergraduate mathematics major at universities, mathematics faculty at many universities work closely with undergraduate majors in excellent unified mathematical sciences programs. Three such institutions are Clemson University, Lamar University (Texas), and Rensselaer Polytechnical Institute.

Most universities today have separate departments in computing and mathematical sciences. To counter this division, the University of Iowa and Oregon State University have developed unified inter-departmental mathematical sciences majors. The MSP panel strongly endorses such inter-departmental majors. At some universities, most of the mathematical sciences, outside of pure mathematics, have been housed in one department. Although the MSP panel prefers a unified mathematical sciences major (ideally in one department), several of these non-pure mathematical sciences departments have good undergraduate programs that may be of interest to other institutions: the Mathematical Sciences Department at Johns Hopkins University, the Mathematical Sciences Department at Rice University, and the Department of Applied Mathematics and Statistics at the State University of New York at Stony Brook.

Departmental Self-Study and Publicity

The MSP panel urges all mathematics departments to engage in serious self-study to identify one or more major themes to emphasize in their mathematical sciences programs: an interdisciplinary focus in cooperation with other departments; an innovative calculus sequence (integrating computing, applications, etc.); a work-study program or other individualized learning experience; special strength in one area of the mathematical sciences (pure or applied); or a track directed towards employment in a regional industry (such as aerospace, automative, insurance). Some colleges have successfully developed a multi-option major, but usually such programs are the outgrowth of successful one-theme programs that slowly added new options (for example, the multiple-major mathematical sciences program at Lebanon Valley College, mentioned in the preceding section, started with just an Actuarial Science option). The MSP panel's advice is first to do one thing well.

A departmental emphasis should be consistent with the general educational purposes of the whole institution and the academic interests of the high school graduates who have historically gone to that institution. It is very risky to design a mathematical sciences program about a theme that the mathematics faculty find attractive and then to try to recruit a new group of high school students to come to the institution for this program. Note that a thematic emphasis does not mean that basic parts of the mathematical sciences program discussed earlier in this chapter can be neglected.

Following a departmental self-study and implementation of its recommendations for new courses or development of industrial work-study contacts, etc., it is next necessary to publicize the mathematics department's program with brochures and visits to regional high schools and College Fairs. Virtually all mathematics departments with large programs (where mathematical sciences majors constitute over 4% of the school's graduates) have extensive publicity programs. Such publicity should emphasize the general usefulness of mathematics in the modern world, whether a student is a prospective mathematical sciences major or minor or an undecided liberal arts student.

High school guidance counselors often do not realize that there are other attractive mathematics-related careers outside straight computing. Counselors tend to be afraid of mathematics because of their own personal difficulties with the subject. Some counselors have been known to discourage students from taking more than the minimum required amount of high school mathematics with the warning that students risk getting poor grades in (hard) mathematics courses and thus hurting their chances of college admission.

College faculty trying to publicize the value of mathematics and its study at their institution should seek the cooperation of local associations of the National Council of Teachers of Mathematics, which have long been working to promote interest in mathematics in the high schools.

New Course Descriptions

Finite structures are used throughout the mathematical sciences today. Two new basic courses about finite structures belong in the mathematical sciences curriculum, one addressing combinatorial aspects and one addressing algebraic aspects. Another topic, numerical analysis, has become more important with the growth of computer science. This section describes a numerical analysis course that is more applied and at a lower level than the previous CUPM numerical analysis recommendations (Course 8 in the CUPM report *A General Curriculum for Mathematics in Colleges.*)

Discrete Methods Course

This course introduces the basic techniques and modes of reasoning of combinatorial problem solving in the same spirit that calculus introduces continuous problem solving. The growing importance of computer science and mathematical sciences such as operations research that depend heavily on combinatorial methods justifies at least one semester of combinatorial problem solving to balance calculus' three semesters of analysis problem solving.

Unlike calculus, combinatorics is not largely reducible to a limited set of formulas and operations. Combinatorial problems are solved primarily through a careful logical analysis of possibilities. Simple ad hoc models, often unique to each different problem, are needed to count or analyze the possible outcomes. This need to constantly invent original solutions, different from class examples, is what makes the discrete methods course so valuable for students.

Like calculus, combinatorics is a subject which has a wide variety of applications. Many of them are related to computers and to operations research, but others relate to such diverse fields as genetics, organic chemistry, electrical engineering, political science, transportation, and health science. The basic discrete methods course should contain a variety of applications and use them both to motivate topics and to illustrate techniques.

The course has an enumeration part and a graph theory part. These parts can be covered in either order. While texts traditionally do enumeration first, the graph material is more intuitive and hence it seems natural to do graph theory first (as suggested below).

With the right point-of-view, many combinatorial problems have quite simple solutions. However, the object of this course is not to show students simple answers. It is to teach students how to discover such simple answers (as well as not so simple answers). The means for achieving solutions are of more concern than the ends. Learning how to solve problems requires an interactive teaching style. It requires extensive discussion of the logical faults in wrong analyses as much as presenting correct analyses.

Since the course should emphasize general combinatorial reasoning rather than techniques, a large degree of flexibility is possible in the choice of topics. The course outline given below contains many optional topics. Some of the core topics, such as the inclusion-exclusion formula, might also be skipped to allow the course to be tailored to the interests of students.

COURSE OUTLINE

I. Graph Theory
   - A. *Graphs as models.* Stress many applications.
   - B. *Basic properties of graphs and digraphs.* Chains, paths, and connectedness; isomorphism; planarity.
   - C. *Trees.* Basic properties; applications in searching; breadth-first and depth-first search; spanning trees and simple algorithms using spanning trees. Optional: branch and bound methods; tree-based analysis of sorting procedures.
   - D. *Graph coloring.* Chromatic number; coloring applications; map coloring. Optional: related graphical parameters such as independent numbers.
   - E. *Eulerian and Hamiltonian circuits.* Euler circuit theorem and extensions; existence and non-existence of Hamiltonian circuits; applications to scheduling, coding, and genetics.
   - F. *Optional topics:*
     - a. Tournaments
     - b. Network flows and matching
     - c. Intersection graphs
     - d. Connectivity
     - e. Coverings
     - f. Graph-based games

II. Combinatorics
   - A. *Motivating problems and applications.*
   - B. *Elementary counting principles.* Tree diagrams; sum and product role; solving problems that must be decomposed into several subcases. Optional: applications to complexity of computation, coding, genetic codes.
   - C. *Permutations and combinations.* Definitions and simple counting; sets and subsets; binomial coefficients; Pascal's triangle; multinomial coefficients; elementary probability notions and applications of counting. Optional: algorithms for enumerating arrangements and combinations; binomial identities; combinations with repetition and distributions; constrained repetition; equivalence of distribution problems, graph applications.
   - D. *Inclusion/exclusion principle.* Modeling with inclusion/exclusion; derangements; graph coloring. Optional: rook polynomials.
   - E. *Recurrence relations.* Recurrence relation models; solution of homogeneous linear recurrence relations; Fibonacci numbers and their applications.
   - F. *Optional topics:*
     - a. Generating functions
     - b. Polya's enumeration formula
     - c. Experimental design
     - d. Coding

The preceding course outline is for either a one-semester or a two-quarter course. A two-quarter course has a natural structure, covering enumerative material in one quarter and graph theory plus designs in another quarter. There are several books available for part or all of the discrete methods course. It is anticipated that as this discrete methods course becomes more widely taught, many more books will become available and the exact nature of the syllabus will evolve.

There are several obvious places where a computer can be used in this course: ways of representing graphs in a computer and performing simple tests (e.g., connectivity); asymptotic calculations in enumeration problems; network flow algorithm; and algorithms for enumerating permutations and combinations. The pedagogical problem is that computer programming takes time away from problem-solving exercises, possibly too much time if a school's computer operation runs in a batch processing mode.

A more advanced second course in combinatorics may also be considered. This course can treat core topics in the discrete methods course in greater depth, and some of the optional topics. Other important topics are Ramsey theory, matroids, and graph algorithms. The course could concentrate on combinatorics or on graph theory, or could be a topics course which varies from year to year. Some of the texts listed below would be suitable for this second combinatorics course.

COMBINATORICS & GRAPH THEORY TEXTS

1. Bogart, Kenneth, *Introductory Combinatorics*, Pitman, Boston, 1983.
2. Brualdi, Richard, *Introductory Combinatorics*, Elsevier-North Holland, New York, 1977.
3. Cohen, Daniel, *Basic Techniques of Combinatorial Theory*, J. Wiley & Sons, New York, 1978.
4. Liu, C.L., *Introduction to Combinatorial Mathematics*, McGraw Hill, New York, 1968.
5. Roberts, Fred, *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, New Jers., 1984.
6. Tucker, Alan, *Applied Combinatorics*, J. Wiley & Sons, New York, 1980.

GRAPH THEORY TEXTS

1. Bondy, J. and Murty, V.S.R., *Graph Theory with Applications*, American Elsevier, New York, 1976.
2. Chartrand, Gary, *Graphs as Mathematical Models*, Prindle, Weber, and Schmidt, Boston, 1977.
3. Ore, Oystein, *Graphs and Their Uses*, Math. Assoc. of America, Washington, D.C., 1963.
4. Roberts, Fred, *Discrete Mathematical Models*, Prentice-Hall, Englewood Cliffs, New Jersey, 1976.
5. Trudeau, Robert, *Dots and Lines*, Kent State Press, Kent, Ohio, 1976.

COMBINATORICS TEXTS

1. Berman, Gerald and Fryer, Kenneth, *Introduction to Combinatorics*, Academic Press, New York, 1969.
2. Eisen, Martin, *Elementary Combinatorial Analysis*, Gordon-Breach, New York, 1969.
3. Vilenkin, N., *Combinatorics*, Academic Press, New York, 1971.
4. Street, A. and Wallis, W., *Combinatorial Theory: An Introduction*, Charles Babbage, 1975.

Applied Algebra Course

(*Editorial Note in 1989 reprinting*: This course is now called **Discrete Structures** and is usually now taught at the freshman level. The course discussed here is more advanced and intended for the sophomore-junior level.)

A traditional time for an applied algebra course is in the junior year—when students would be ready for a modern algebra course. However, as noted above, many students will not be ready for algebraic abstraction until senior year. The course builds on experiences in beginning computer science courses that have implicitly imparted to students a sense of the underlying algebra of computer science structures, and formally presents topics like Boolean algebra, partial orders, finite-state machines, and formal languages that will be used in later computer science courses. At the same time, this course can also be very rewarding to regular mathematics majors who should appreciate the new algebraic structures such as formal languages and finite state machines that are so different from the structures in the regular abstract algebra course. Substantial class time should be spent on proofs with special emphasis on induction arguments. This course is just as mathematically sophisticated and capable of developing abstract reasoning as abstract algebra, but the topics stress set-relation systems rather than binary-operation systems. Indeed the abstract complexity of the basic structures is much greater in applied algebra, but this complexity precludes the construction of logical pyramids built of simple algebraic inferences common to many areas of abstract algebra.

This course is an advanced version of the lower-division B3 Discrete Structures course in ACM Curriculum 68. The B3 course was the source of much dissatisfaction because it contained a huge amount of material, and it required too great mathematical maturity for a lower-division course. The recent ACM Curriculum 78 recommends that the B3 course be treated as a more advanced course and that it should be taught in mathematics departments rather than computer science departments. The B3 course was the subject of several papers at meetings of the ACM Special Interest Group in Computer Science Education (SIGCSE); see the February issues (Proceedings of SIGCSE annual meeting) of the *SIGCSE Bulletin* in 1973, 1974, 1975, 1976.

The B3 course contained both applied algebra and discrete methods. The MSP panel recommends that a separate full course be devoted to discrete methods (see the discrete methods course description earlier in this Section). Because some computer science courses may devote a substantial amount of time introducing some of the topics in the above applied algebra syllabus, the exact content of this course will vary substantially from college to college. For this reason the syllabus outline was kept brief. At some colleges, applied algebra will still have to be combined with discrete methods in one course (the computer science major may not have the time for two separate courses). The applied algebra part of such a combined course would, in most cases, concentrate on topics 1, 2, 3, 4, 6 in the syllabus. Many of the discrete structures texts listed below cover both applied algebra and discrete methods.

COURSE TOPICS

A. Sets, binary relations, set functions, induction, basic graph terminology.

B. Partially ordered sets, order-preserving maps, weak orders.
C. Boolean algebra, relation to switching circuits.
D. Finite state machines, state diagrams, machine homomorphism.
E. Formal languages, context-free languages, recognition by machine.
F. Groups, semigroups, monoids, permutations and sorting, representations by machines, group codes.
G. Modular arithmetic, Euclidean algorithm.
H. Optional topics: linear machines, Turing machines and related automata; Polya's enumeration theorem; finite fields, Latin squares and block design; computational complexity.

APPLIED ALGEBRA TEXTS

1. Dornhoff, Lawrence and Hohn, Frantz, *Applied Modern Algebra,* Macmillan, New York, 1978.
2. Fisher, James, *Application-Oriented Algebra,* T. Crowell Publishers, New York, 1977.
3. Johnsonbaugh, Richard, *Discrete Mathematics,* Macmillan, New York, 1984.
4. Korfhage, Robert, *Discrete Computational Structures,* Academic Press, New York, 1974.
5. Liu, C.L., *Elements of Discrete Mathematics,* McGraw Hill, New York, 1977.
6. Preparata, Franco and Yeh, Robert, *Introduction to Discrete Structures,* Addison-Wesley, Reading, Mass., 1973.
7. Prather, Robert, *Discrete Mathematical Structures for Computer Sciences,* Houghton Mifflin, Boston, 1976.
8. Stone, Harold, *Discrete Mathematical Structures and Their Applications,* Science Research Associates, Chicago, 1973.
9. Tremblay, J. and Manohar, R., *Discrete Mathematical Structures with Applications in Computer Sciences,* McGraw Hill, New York, 1975.

Numerical Analysis Course

In any elementary numerical analysis course a balance must be maintained between the theoretical and the application portion of the subject. Normally, such a course is designed for sophomore and junior students in engineering, mathematics, science, and computer science. Students should be introduced to a wide selection of numerical procedures. The emphasis should be more on demonstrations than on rigorous proofs (however, this is not meant to slight necessary theoretical aspects of error analysis). At least one or two applied problems from each of the major topics should be included so that students have a good understanding of how the art of numerical analysis comes into play.

The course outline below presents a good selection of topics for a one-semester course. Error analysis should be continuously discussed throughout the duration of the course so as to stress the effectiveness and efficiency of the methods. Alternative methods should be contrasted and compared from the standpoint of the computational effort required to attain desired accuracy.

An optional approach to this course would emphasize a full discussion (with computer usage) of one procedure for each course topic (after the computer arithmetic introduction). A sample of five such procedures is:

1. The Dekker-Brent algorithm (see UMAP module No. 264).
2. A good linear equation solver involving LU-decomposition.
3. Cubic spline interpolation.
4. An adaptive quadrature code.
5. The Runge-Kutta-Fehlberg code RKF4 with adaptive step determination.

Weekly assignments should include some computer usage; in total, four or five computer exercises for each major topic. Students should do computer work for larger applied programs in small groups. However, the concept of utilizing "canned" programs with minor modifications should be stressed. Such an approach nicely brings out the strong interdependence between computers and numerical analysis yet does not overemphasize the efforts necessary to program a problem. An interactive computer system using video terminals is ideal for this course. Microcomputers and even hand-held calculators can also be used effectively. One or two applied homework problems from each of the main topics keep students aware of the balance that is necessary between the art and the science of numerical analysis. Prerequisites for this course should be a year of calculus including some basic elementary differential equations and a computer science course.

For schools on a quarter system, two quarters should be a minimal requirement and the above material would be more than ample. One should spend the first quarter on numerical solutions of algebraic equations and systems of algebraic equations and the last quarter on the other topics.

COURSE OUTLINE

A. *Computer arithmetic.* Discretization and round-off error; nested multiplication.
B. *Solution of a single algebraic equation.* Initial discussion of convergence problems with emphasis on meaning of convergence and order of convergence;

Newton's method, Bairstow's method; interpolation.

C. *Solution systems of equations.* Elementary matrix algebra; Gaussian methods, LU decomposition, iterative methods, matrix inversion; stability of algorithms (examples of unstable algorithms), errors in conditioned numbers.

D. *Interpolating polynomials.* Lagrange interpolation to demonstrate existence and uniqueness of interpolating polynomials and for calculation of truncation error terms; splines, least squares, inverse interpolation; truncation, inherent errors and their propagation.

E. *Numerical integration.* Gaussian quadrature, method of undetermined coefficients, Romberg and Richardson extrapolation (for both integration and differentiation), Newton-Cotes formulas, interpolating polynomials, local and global error analysis.

F. *Numerical solution of ordinary differential equations.* Both initial value and boundary value problems; Euler's method, Taylor series method, Runge-Kutta, predictor-corrector methods, multistep methods; convergence and accuracy criteria; systems of equations and higher order equations.

If this course has an enrollment of under 25 students, non-standard testing can be considered, such as a take-home midterm. At the end of the term, instead of the traditional three hour examination, each student can write an expository paper exploring in greater depth one of the topics introduced in class or investigating a subject not included in the work of the course, either approach to include computational examples with analysis of errors. (Since most of the students will not have had previous experience in writing a paper, topics may be suggested by the instructor or must be approved if student devised; scheduled conferences and preliminary critical reading of papers guard against disastrous attempts or procrastination.) Some examples of final projects are: spline approximations; relaxation methods; method of undetermined coefficients in differentiation and integration; least squares approximations; parabolic (or elliptic or hyperbolic) partial differential equations; numerical methods for multi-dimensional integrals; multi-step predictor-corrector methods.

NUMERICAL ANALYSIS TEXTS

1. Cheney, Ward and Kincaid, David, *Numerical Mathematics and Computing,* Brooks/Cole, Monterey, Calif., 1980.
2. Conte, S. and DeBoor, C., *Elementary Numerical Analysis,* McGraw Hill, New York, 1978.
3. Gerald, Curtis F., *Applied Numerical Analysis, 2nd Edition,* Addison-Wesley, Reading, Mass., 1978.
4. Forsythe, G.E. and Moler, C.B., *Computer Solutions of Linear Algebraic Systems,* Prentice-Hall, Englewood Cliffs, New Jersey, 1967.
5. Hamming, R.W., *Numerical Methods for Scientists and Engineers, 2nd Edition,* McGraw Hill, New York, 1973.
6. James, M.L.; Smith, G.M.; Wolford, J.C., *Applied Numerical Methods for Digital Computation,* Harper & Row, New York, 1985.
7. Ralston, Anthony and Rabinowitz, Philip, *First Course in Numerical Analysis,* McGraw Hill, New York, 1978.

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