Reading List

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Appendix A: COMAP Geometry Conference

In recent years, there has been a tremendous surge in research in geometry. This surge has been the consequence of the development of new methods, the refinement of old ones, and the stimulation of new ideas both from within mathematics and from other disciplines, including computer science. Yet during this period of growth, education in geometry has remained stagnant. Not only are few of the new ideas in geometry being taught, but also fewer students are studying geometry.

In March 1990, a group of college and university researchers and educators in geometry met to assess the directions of education and to make suggestions for invigorating it. These individuals represented a wide variety of branches of geometry as well as a wide spectrum of institutions. Discussions ensued on the causes of the decline in geometry education and on the steps that might be taken at all grade levels (K–graduate school) to energize the teaching of it. Special attention was given to the content of the survey course in geometry taught in many universities and colleges. This course has historically been taken by a large number of prospective high school teachers, and thus setting new directions for this course offers the hope of exposing future mathematics practitioners to new ideas in geometry, as well as for laying the basis for future changes in lower grades.

Despite the varied points of view expressed by the individuals who attended the conference, there was a broad core of common views, which, if implemented, can have a significant effect on geometry. This common core of views and recommendations is presented below.

These recommendations and the following article "Geometry: Yesterday, Today, and Tomorrow" by Joe Malkevitch are reproduced with permission from *Geometry's Future*, the proceedings of a March 1990 conference sponsored by COMAP, Inc. (57 Bedford Street, Suite 210, Lexington, MA 02173).

Conference Recommendations

Future directions for the teaching of geometry (especially for implementation in the college/university survey course):

- Geometric objects and concepts should be studied more from an experimental and inductive point of view rather than from an axiomatic point of view. (Results suggested by inductive approaches should be proved.)
- Combinatorial, topological, analytical, and computational aspects of geometry should be given equal footing with metric ideas.
- The broad applicability of geometry should be demonstrated: applications to business (linear programming and graph theory), to biology (knots and dynamical systems), to robotics (computational geometry and convexity), etc.
- A wide variety of computer environments should be explored (*Mathematica*, *Logo*, etc.) both as exploratory tools and for concept development.
- Recent developments in geometry should be included. (Geometry did not die with either Euclid or Bolyai and Lobachevsky.)
- The cross-fertilization of geometry with other parts of mathematics should be developed.
- The rich history of geometry and its practitioners should be shown. (Many of the greatest mathematicians of all time: Archimedes, Newton, Euler, Gauss, Poincaré, Hilbert, von Neumann, etc., have made significant contributions to geometry.)
- Both the depth and breadth of geometry should be treated. (Example: Knot theory, a part of geometry rarely discussed in either high school or survey geometry courses, connects with ideas in analysis, topology, algebra, etc., and is finding applications in biology and physics.)
- More use of diagrams and physical models as aids to conceptual development in geometry should be explored.
- Group learning methods, writing assignments, and projects should become an integral part of the format in which geometry is taught.
- More emphasis should be placed on central conceptual aspects of geometry, such as geometric transformations and their effects on point sets, distance concepts, surface concepts, etc.
- Mathematics departments should encourage prospective teachers to be exposed to both the depth and breadth of geometry.

Appendix B: Geometry: Yesterday, Today, and Tomorrow

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Despite the increased pace of exciting developments in both the theory and applications of geometry in the last 40 years, it appears that less geometry is being taught in college today than was taught in the recent or distant past. The purpose of this paper is to examine this "paradox" and to study how the teaching of geometry in colleges affects what geometry is and can be taught in high school, grade school, and graduate school mathematics.