

# Environmental Mathematics

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## Introduction

Environmental mathematics is an emerging field, waiting to be delineated. Whatever its final definition, a synthesis of a subject whose central issue is the survival of life on this planet and a subject that is crucial for modern science and technology will command attention.

The environment is the most important problem facing the human race today, and the only effective response involves educating the general population. I can see no better place to begin the educational process than in the mathematics curriculum.

—*Dean Hoover, Alfred University*

This project is perhaps the most valuable one that mathematical educators can be involved in at this time.

—*Marty Walter, University of Colorado, Boulder*

Richard Schwartz of College of Staten Island, a pioneer whose efforts to raise environmental consciousness via introductory mathematics goes back fifteen years, thinks it is essential that we marshal our efforts to "...make mathematicians and others much more aware of the critical nature of environmental problems."

The term "environmental mathematics" made its first appearance in a national publication in the April 1990 issue of *Focus* ("Solving Environmental Problems: Where Are the Mathematicians?", Ben A. Fusaro and Marcia P. Sward). The topic was first part of a national AMS-MAA program as a panel discussion at the San Francisco meeting in January 1991. The success of this panel led the 1992 Program Committee to call for a proposal for increased environmental activities at the Baltimore meeting. Subsequently, environmental mathematics was called to the attention of the full AMS-MAA community by being featured in announcements for this meeting. One nice side effect of this publicity is that mathematicians who were not initially part of our Focus Group wrote in with suggestions. Thus, there will be names in this document that do not appear among the list of Focus Group participants at the end. (These volunteers turned out to be very important. Our e-mail system, faltering almost from the beginning of the project, got worse as time went on.)

## Setting the Stage

I tried to set the stage by providing an implicit definition of environmental mathematics (EM), by listing some characteristics, and by suggesting how EM should be introduced to the curriculum.

Environmental mathematics needs to be distinguished from courses in ecosystems, eco-modelling, mathematical ecology, and similar subjects. Ecology is usually defined as the

study of the relation between organisms and their environment. The “eco” courses tend to be in biology departments, or are taught as upper-division or graduate applied mathematics courses. Our intention is that environmental mathematics refer to undergraduate activity (including high school).

It could be said that environmental mathematics has the same relation to the environment as engineering mathematics has to engineering. This definition by analogy gives the topic ample scope, and received the implicit approval of the Focus Group participants, with one word of caution:

If the word “mathematics” is emphasized in the title “environmental mathematics,” then the environmental science students will not take it. If the environmental part is emphasized, then it doesn’t sound like a mathematics course.”

—Lothar Dohse, *University of North Carolina, Asheville*

A crucial difference between “eco-” courses and environmental mathematics is that the former can be—and often are—taught in a clinical “white coat” fashion. An article that was part of a nationally-distributed announcement illustrates this point. The author labored for several paragraphs to convince the reader how important it was for scientists to classify and study all endangered species before they disappear. Only at the end of the article was there a “by the way” that perhaps something ought to be done to keep these species from disappearing.

Tom Hallam of the University of Tennessee, Knoxville sounded a warning. He was concerned that *too* strong of a partisan approach could lead one to imposing one’s views on students. Bob McKelvey of the University of Montana had sounded a similar warning at the EM panel discussion in San Francisco. He suggested that one had to be careful not to let a strong environmental concern intrude on—and perhaps distort—a scientific message. McKelvey emphasizes that working in environmental mathematics often leads one to deal with policy, and therefore controversy.

Initially, the main effort in environmental mathematics should be directed at developing courses suitable for general education. Here is where the biggest impact could be made. The effort could then be shifted to introducing environmental topics into sophomore or junior modelling courses. The topics might deal with pollution, recycling, (so-called) timber management, water quality, energy, etc.

## Characteristics of Environmental Mathematics

The concern about partisan teaching suggests a need to clarify the “white-coat science” issue. No one wants “PC”-type pressures as have surfaced in several prominent universities. However, environmental mathematics needs to include an awareness of the environmental cost of implementing a model, and beyond that, a general concern for environmental degradation. The subject cannot avoid dealing with *values*.

Indeed, competing value systems are often built into the problem.

Resolving the relative harm to those with differing views is often the *point* of the problems we address: economics vs. wildlife, jobs vs. spotted owls, erosion and logging vs. salmon runs.

—Rollie Lamberson, *Humboldt State University*

Our populace is becoming less able to evaluate or assess the trade-offs between alternatives. It would be exceptionally useful if the environmental courses would be able to open the eyes of the non-mathematics major to the evaluation of alternatives.

—Lee Seitelman, Pratt & Whitney, Connecticut

The health of the environment is so critical for us all, wildlife and *homo sapiens* alike, that we dare not hold the subject at arms length—we need to become engaged. A few years ago there was a mathematical conference on ecological modelling at which tuna was served for lunch. A sampling of the opinion of a few participants indicated that they were either unaware of the relation of yellow-fin tuna consumption to a protected species or, even worse, did not see what our lunch menu had to do with the topic of the conference. Are these the kind of role models we want to teach students the mathematics of the environment . . . ?

The challenge is to have mathematics serve as an instrument for modelling and for raising environmental consciousness in “a setting where scientific explorations can be pursued openly.”

## General Education and Entry-Level Courses

There was fairly strong agreement that first efforts should go to developing introductory courses or materials for general education. The next level of effort should be to develop materials for lower-level modelling courses, for calculus courses, and for other beginning courses for majors. Since part of the concept is to use mathematics to develop environmental awareness, the earlier the subject is introduced, the better.

Two participants, Fusaro and Schwartz, have developed materials for general education courses and have been teaching them for nine and sixteen years, respectively. Fusaro has been using an environmental text [2] for background and notes. Supporting packaged software is available [3]. Schwartz has published a text, *Mathematics and Global Survival* [5].

Marty Walter has recently proposed a course called “Mathematics for the Environment” with only high school algebra as a prerequisite. S.S. Dalal suggested a course built entirely around energy, noting that “energy is of great interest to students.”

Students are to explore different forms of energy and write papers which are presented in class for general discussion. Students are given about fifteen topics to choose from. Some examples are: solar energy, wind energy, renewable energy, energy from biofuels, small-scale hydropower systems, and municipal resource recovery.

—S.S. Dalal, Embry-Riddle Aeronautical University, Florida

Others are working in different ways to introduce environmental issues into first year courses. Christopher Schaefe and Nancy Zumoff of Kennesaw State College, Georgia presented an MAA paper “Applications of Algebra to the Environment” at the Orono, Maine summer meeting. They offered a replacement for college algebra called “Earth Algebra” [4].

Another interesting development was revealed in a letter from Geoffrey Beresford of Long Island University, New York. His letter started with “I am writing a calculus textbook” [Gasp!], but continues with “and am very interested in finding undergraduate-level applications to environmental science. I am making a special effort in my book to raise environmental consciousness.” [Ah . . . !]

## Modelling Courses

Most of the Focus Group participants described modelling courses in which environmental materials were introduced to varying degrees. Lothar Dohse has taught a Mathematics Modelling course in which most of the student projects were environmental or biological. Rollie Lamberson reports that Humboldt State began in 1980 to offer a sophomore-junior course with emphasis on biological and environmental problems.

Robert Wenger of the University of Wisconsin, Green Bay comes from a department that has been actively working on environmental problems for about fifteen years. He makes two important points. First, since environmental problems are interdisciplinary, mathematicians need to make the effort to interact with other disciplines. Second, sophisticated models or techniques are usually not required. He also notes that little is available in textbook form and suggests a series of UMAP-type modules focused on environmental problems. (There are a few modules in the COMAP 1992 catalog: Nos. 207, 607, 610, 628, 653, 670, 675, and 688.) Generally, modelling courses dealt with open-ended projects, required student projects and presentations, and encouraged team efforts.

## Conclusion

One of the striking aspects of most courses on environmental mathematics is the similarity of classroom management techniques and the alignment of instructional styles with contemporary recommendations about effective teaching. Respondents in our study made use of almost all of the following: experiential learning, classroom presentations, cooperation or team efforts, interdisciplinary approaches, open-ended problems, and term projects. It seems that in environmental mathematics courses, form follows content.

Events indicate that mathematics and environmental science are headed for a marriage. It is a marriage we should encourage as professionals and reflect in the curriculum as teachers. Getting environmental mathematics into the curriculum will be a parallel process, but the initial emphasis should be on introductory courses. An eventual goal might be the development of masters degree programs in environmental mathematics, perhaps along the lines of the Environmental Systems program at Humboldt State University. At all levels, these experiences need to be accompanied by an environmental consciousness in the context of open inquiry.

## Annotated Bibliography

1. Harte, John. *Take a Spherical Cow: A Course in Environmental Problem Solving*. Mill Valley, CA: University Science Books, 1988.  
In this paperback the author covers a wide variety of environmental problems, from sulfur in coal to pollution of lakes. There is a great emphasis on the modelling process. The author relies heavily on knowledge of the natural sciences; many of the problems make use of calculus or differential equations. Neither calculators nor computers play much of a role in the text.
2. Odum, H.T. and Odum, E.C. *Energy Basis for Man and Nature*. New York: McGraw-Hill, 1978.  
This is a fascinating paperback introduction to Odum's energy analysis and language. Some knowledge of biology is required to appreciate the applications. Seven basic models are studied. BASIC code is provided in an Appendix.

3. Odum, H.T. and Odum, E.C. *Computer Minimodels and Simulation Exercises*. Gainesville, FL: Center for Wetlands, 1989.  
This paperback is intended to teach students to diagram, simulate, and experiment. There are 45 diagrammatic models, as well as BASIC code for the Apple II, IBM PC, and Macintosh. It supplies some background for each model, but does require the user to be familiar with the subject matter. The first nine models are general, the next seventeen are biologically or ecologically oriented, and the last nineteen are applications to economics.
4. Schaufele, C. and Zumoff, N. *Earth Algebra*. Glenview, IL: Harper Collins, 1991.  
The authors were led to this work by trying to answer the questions "What makes College Algebra boring to practically every student on this planet?", and "What can be done to make College Algebra more interesting?" They deal with these questions by extensive use of environmental contexts and by designing a text that exploits the power of a graphing calculator. These authors are not "white coat" scientists. They view environmental issues as vital, and their concern is reflected throughout the text.
5. Schwartz, R.H. *Mathematics and Global Survival, Second Edition*. Needham Heights, MA: Ginn Press, 1990.  
This is a freshmen-level paperback with a wide variety of environmental applications. It uses such issues as hunger, pollution, and resource allocation to motivate basic calculations and descriptive statistics. It is the basis for a course, Mathematics and the Environment, that the author has taught since 1975.

## Focus Group Participants

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## Appendix A: Three Course Outlines

### Mathematics and the Environment (Marty Walter)

Each student selects a problem for a semester-long project, with monthly progress reports required. Exams take the form of oral presentations to the class.

#### Warm-up Exercises:

- How many cobblers are there in the U.S.?
- How far will a drop of water spread on water?

- How large was the Ancient Asteroid that (perhaps) killed the dinosaurs?
- Population growth.
- What fraction of the total annual plant growth was eaten by humans last year?
- How much sulphur was put in the air by burning coal last year?

#### Modelling Tools:

- Steady-state box models and resident times.
- Thermodynamics and energy transfer.
- Chemical reactions and equilibria.
- Non-steady-state box models.

#### Open-Ended Problems:

- Acid rain.
- Mobilization of trace materials.
- Carbon cycle tracing.
- Global warming and the greenhouse effect.
- Optimal harvesting.
- Steady-state population in China.
- Road-killed rabbits in Nevada.

### Earth Algebra (Christopher Schaefe & Nancy Zumoff)

Both of us are “environmentalists” and began development of the course “Earth Algebra” in hopes of achieving two goals: to educate students at an early college level on environmental issues, and to demonstrate the effectiveness of mathematics as a decision making tool. Early evaluations indicate success in each category.

“Earth Algebra” is aimed at beginning college students. It utilizes all concepts from a traditional college algebra course to simplistically model environmental data, to make more decisions regarding predicted events, to evaluate alternative energy sources, and to formulate recommendations for changes which will improve environmental conditions. The course is very focused; everything in the text is relevant to the issue of global warming. This puts college algebra in a context which most students are already aware of, and to a certain degree, interested in; it gives relevance to mathematics, and hence generates interest in and purpose to its study.

This project is supported by grants from NSF and FIPSE; Harper Collins will publish the text in late 1992.

#### BRIEF TABLE OF CONTENTS

- I. Carbon dioxide concentration and global warming.
  1. Introduction: The greenhouse effect and global warming.
  2. Atmospheric carbon dioxide concentration.
  3. On the beach, ... or, what beach?
- II. Factors contributing to carbon dioxide build-up.
  4. Carbon dioxide emission from automobiles.
  5. Carbon dioxide emission from energy consumption.

6. Carbon dioxide emission from deforestation.
7. Total carbon dioxide emission functions.
- III. Accumulation of carbon dioxide.
  8. Increases in atmospheric carbon dioxide concentration.
  9. Factors contributing to atmospheric carbon dioxide concentration.
- IV. Social factors.
  10. People.
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- V. Save the planet!
  12. Changing energy demand.
  13. Cost and efficiency of alternative energy sources.
  14. What can you do to save the planet?

### Environmental Mathematics (Ben Fusaro)

The emphasis is on computational, qualitative, and visual mathematics. All modelling is done by a seven-step process, moving from the visual and qualitative to the computational (calculators and BASIC). The students solve differential equations but they are called "Flow Equations." There is a major project that is done (preferably) by teams of two students. Course outline:

- Systems and Diagrammatics
- Energy and Entropy
- Energy and Growth
- Simulation of Models
- Energy Flow and Money Flow
- Production and Diversity

## Appendix B: Mathematicians Develop New Tools to Tackle Environmental Problems

by David L. Wheeler, *THE CHRONICLE OF HIGHER EDUCATION*

Idling cars spewing fumes, northern spotted owls seeking nesting sites in diminishing plots of old-growth forest, and molecules of sulfur dioxide settling through the branches of the human lung: Such events would not strike most scientists as inherently mathematical. But mathematicians using graphs, equations, and their own brand of abstract thinking have been involved in each of those problems and are seeking a larger role in other environmental research.

"Environmental mathematics is an attempt to get mathematicians to connect again with the natural world," says Ben A. Fusaro, a professor at Salisbury State University and the chairman of the Mathematical Association of America's new committee on mathematics and

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