Schoenfeld likens this kind of effect to the effect that medical research has had on the health consciousness of the American public. The research itself is highly technical, and there's constant controversy, but over the course of time the more clear-cut, consistent findings have managed to sink in. Most notably, for example, the evidence linking tobacco with a whole slew of health problems has helped initiate changes in public attitudes toward smoking, to the point that smokers' "rights" are in serious jeopardy.

If the scientific crusade against smoking is an appropriate analogy, then educational researchers and reformers can expect a long, hard battle. The sheer weight of evidence won't tip any scales, but careful research can serve to inform the educational reform movement and help shape its future. The main benefit of research, say Hanna, is "the feeling that you operate from a position of knowledge rather than ignorance."

Open your mind to what I shall explain,
then close around it, for it is no learning
to understand what one does not retain.

—Canto V, 40–42.

Appendix A: Communicating Among Communities

A REPORT OF A CONFERENCE ON
Research in Collegiate Mathematical Education

The Mathematical Association of America (MAA), with support from the National Science Foundation, hosted an invitational conference on Research in Collegiate Mathematical Education in Washington, DC, November 8-10, 1991. Twenty-eight invited participants, representing the mathematics and mathematics education communities, candidly discussed issues relating to the growing interest among faculty and others concerning research in the teaching and learning of mathematics at the undergraduate level. The conference participants focused on four aspects of research in collegiate mathematics education:

- Communicating to college and university faculty the growing body of research in undergraduate mathematics education.
- Improving student learning by stimulating change in collegiate teaching based on the findings of this research.
- Encouraging high standards of research in undergraduate mathematics education.
- Supporting the increasing number of collegiate faculty who undertake research in undergraduate mathematics education.

It is no surprise that the views of those attending the conference varied considerably on these matters. There was general agreement on the urgency of seeking improvement in the teaching and learning of mathematics at the undergraduate level. A distinction was made

This Conference was supported by the National Science Foundation. Opinions expressed in this report are those of the authors, not necessarily those of the Foundation.

between general activity in support of improving undergraduate mathematics education and the role of research in the teaching and learning of mathematics at the undergraduate level.

There was some disagreement as to the role played by mathematics education research. Some felt that basic research in undergraduate mathematics education is an essential and continuing part of the process of change. Still others were skeptical that such research would have any effect at all, and cited the limited amount of persuasive evidence produced in a field which is perceived by many mathematicians to be jargon-laden. Another group argued that basic research is critically important but independent of attempts to improve undergraduate education. There was also a group who believed that there are some mathematicians in the teaching force who would not be motivated to change their habits regardless of how compelling the research results may be.

Those favoring more emphasis on mathematics education research argued that basic research in the learning and teaching of mathematics is an essential component of any endeavor that seeks to improve undergraduate mathematics. They cited as examples the many profound conceptual questions about students' understanding of function and the wide range of questions generated by the impact of technology for students' learning and doing of mathematics. They further argued that the growing number of individuals engaged in investigating these questions at the collegiate level requires scholarly support structures to ensure the vitality of their field.

Those opposed to placing more emphasis on mathematics education research argued that it is more critical to address current teaching practices and seek improvement. Furthermore, while many mathematicians are becoming more interested in improving the teaching of undergraduates, they do not have the inclination to learn the specialized vocabulary or understanding of techniques required to read or participate actively in research issues at the collegiate level. What would be beneficial, in this view, would be more access to survey or review articles, written in a more expository style, that convey the results of current research at the collegiate level.

Conference Statements and Recommendations

Many mathematicians tend to think of research in mathematics education as being concerned primarily with the improvement of teaching or the evaluation of some particular curricular innovation. For many attending the conference, the vast array of research topics being pursued by those working in pre-college and collegiate mathematics education was somewhat surprising. A sampling from that list includes:

- Students' understanding of the limit as a process and not a number;
- Stages of development in understanding the concept of function;
- Issues centering around problem solving;
- Issues addressing proof, logic, and reasoning;
- How students' learning styles may affect the context in which problems are posed;
- Students' use of visualization in "doing" mathematics;
- Translational difficulties as students move among graphical, symbolic, or numeric representations of ideas.

The well-established professional community doing research in school mathematics education has much to offer those investigating similar issues at the collegiate level. This is

particularly so since many colleges and universities teach school-level mathematics in their entry-level courses and are confronting the need for more careful attention to these courses. Efforts to make connections between those pursuing education research at the pre-college and collegiate level are already under way.

Conferees generally agreed that a journal on research in collegiate mathematics education would be an important means of strengthening professional standards, of encouraging quality research, and of providing support for individuals doing this type of scholarly activity. However, to be effective in providing the scholarly support needed for workers in the field, such a journal would need to achieve sufficient stature to command the respect of the broad mathematical community—including both mathematicians and those doing research in mathematics education. Given the number of existing journals accepting papers in mathematics education research and the critical need for building awareness among the mathematical community as to the contribution that research can make to improve teaching and learning in mathematics at the undergraduate level, there was a clear feeling among the participants that launching such a journal at this time would not be the most appropriate first step. To deal with the present need for sharing the fruits of research in undergraduate mathematics education in as broad a way as possible, the following recommendations received consensus support:

Recommendation 1. *The Mathematical Association of America (MAA) and the American Mathematical Society (AMS), in cooperation with the National Council of Teachers of Mathematics (NCTM), should plan a series of annual volumes presenting exemplary research papers in collegiate mathematics education. These volumes would serve as precursors to the establishment of a journal.*

Conference participants believed that the reception of these annual volumes by the community would provide a measure of interest and of the need for further steps. To achieve maximum effectiveness of this recommendation, additional strategies will be needed to create awareness among the broader mathematical community of the issues being addressed by those working in this field of research. A full array of suggestions and strategies were noted by the conference participants. Many of these extend current means of communicating information at sectional, regional, and national meetings:

- More extensive use of mini-courses and contributed paper sessions;
- Developing appropriate panel presentations;
- Attempting to get topics on research in undergraduate mathematics on the agenda of Departmental Chairs meetings;
- Encouraging the program committee for national meetings to invite speakers to address these research topics;
- Encourage the professional societies to seek ways to develop summer faculty institutes for teaching and learning mathematics.

The final suggestion was for the societies to pursue vigorously the creation of a national network of centers for the teaching and learning of mathematics.

Recommendation 2. *Editors of MAA and AMS periodicals are encouraged to solicit substantial review or survey articles to appear simultaneously with publi-*

cation of the annual volumes to stimulate interest among mathematicians in issues addressed in these research volumes.

Most of the editors present supported the recommendation. The only concern expressed related to appropriateness for the mission of specific journals. In fact, some encouraged the submission of quality survey articles, written for an audience predominately consisting of mathematicians, for publication in their journals on an on-going basis.

Many participants noted that electronic linkages for sharing results among workers in the field are being established independently and through informal networks. It was strongly recommended that the MAA and AMS should investigate and develop as rapidly as possible appropriate mechanisms for more formal electronic exchange of information among individuals engaged in research on undergraduate mathematics education, and between members of this community of researchers and those who teach college and university mathematics. In particular, part of this investigation might include exploration of the possibility of establishing an electronic journal on undergraduate mathematics education. In the spirit of trying to enhance communication across the communities, the participants put forth two specific recommendations:

Recommendation 3. *Editors of journals and periodicals that are read by college and university mathematicians should provide regular information on research in undergraduate mathematics education through such means as brief "telegraphic" reviews of research papers and special survey articles or issue papers dealing with the application of educational research to the improvement of student learning.*

Recommendation 4. *The MAA and AMS, in cooperation with the NCTM, should plan special conferences or sessions at sectional, regional, or national meetings dealing with aspects of research in collegiate mathematics education. These activities should be designed to expand the interface between educational researchers and mathematicians.*

This recommendation further expands the current means of communicating information at sectional, regional, and national meetings of the professional associations. Other special meetings might be designed that would promote scholarly exchange among those actively engaged in research in collegiate mathematics education, for the purpose of setting a research agenda, or for the purpose of attracting newcomers to undertake investigation of issues important to the field.

There was considerable optimism about the perception that most mathematics departments are now more concerned about the teaching and learning of mathematics by their students, and that individual faculty frequently are discussing issues related to curriculum and teaching. There was concern, however, that some universities have addressed the issue by appointing special adjunct faculty to deal with instructional concerns of both students and faculty. This arrangement can lead to further isolation of the full-time faculty from activities in teaching and learning at the undergraduate level. Overall, there is much more faculty interest in looking for ways to improve student learning. Professional mathematics societies need to encourage this interest by identifying and publicizing in mainstream mathematics

journals effective methods for stimulating and institutionalizing improved mathematics instruction. When appropriate, ties with the results of research findings in undergraduate mathematics education should be directly indicated.

To spur the continued development of department-based activity regarding issues of curriculum and teaching, professional societies should identify lists of prominent individuals who would be prepared to speak on issues of research in undergraduate mathematics education. Once compiled, this list should be sent to departments as suggestions for colloquium speakers and to program planning committees for sectional, regional, and national meetings. Before distribution, this list of speakers would be reviewed by the MAA Council on Education and the AMS Committee on Education. The appropriate committee at NCTM would also be consulted.

Data from the CBMS Survey *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences and Computer Science 1990-1991* were cited to document the tremendous increase in the use of part-time faculty, particularly at two-year colleges. The Survey notes that there are far more part-time faculty than full-time faculty currently teaching at two-year colleges. Attempts to influence the teaching and learning of mathematics at these institutions must directly address this large instructional force. Conference participants expressed great concern about this becoming a serious barrier to creative change.

Recommendation 5. *The MAA and AMS, in conjunction with the American Mathematical Association of Two-Year Colleges (AMATYC), should undertake a study of the effects of the increasing reliance on part-time faculty for mathematics instruction, especially to determine in what ways part-time faculty may differ from full-time faculty in their approaches to teaching.*

A serious and frequently surfacing concern among conference participants was the need for research in collegiate mathematics education to become an "accepted" field of scholarly inquiry in mathematics departments. The field is relatively new and participation by creative, energetic individuals will be needed to enhance its vitality. There are some indications that as departments become more concerned about undergraduate education, they begin to broaden their definitions of scholarly activity. But this is by no means universal! In fact, several participants held the belief that mathematical research, and only that, should be the fundamental criterion for initial promotion and tenure in mathematics departments. Of course this must be carefully interpreted, for there are departments where individuals are specifically hired because of the contributions that they make in the field of mathematics education research and should be evaluated on that basis. However, given the growing number of individuals who are making significant contributions in the field of collegiate mathematics education, the issue cannot be ignored.

In addition, there are growing numbers of college and university faculty who are involved in highly creative curriculum projects or software development. These new directions for faculty, often recognized as valuable by the department (but sometimes not rewarded), require enormous amounts of time. If the definition of scholarship is broadened to include these types of activities, then there is still an issue of how contributions in the area can be adequately assessed. One clear suggestion was that the faculty need to write and publish results of their work. But even that task can be complex and suffers from differences in approach. There are those who pursue research using the methodologies inherent in the

field of mathematics education and there are others who deal with innovative practice in the teaching and learning of mathematics where the resulting articles are more anecdotal in style. In fact, broad discussion of the evaluation of alternative forms of mathematical scholarship is needed by the community. Fortunately, the Joint Policy Board for Mathematics (JPBM) has recently established the Committee on Professional Values, Recognition, and Rewards. The conference directs the next recommendation to this committee:

Recommendation 6. *The JPBM Committee on Professional Values, Recognition, and Rewards should seek to identify and disseminate effective evaluation and reward mechanisms that promote high standards in professional activities in mathematics education. In particular, the Committee's agenda should address the needs of those faculty whose professional work is devoted to research in mathematics education, as well as those whose work centers on curriculum development or educational practice.*

It will not be simple to implement these recommendations and suggestions. The final recommendation made by the conference is an effort to put in place a framework for monitoring progress on this report.

Recommendation 7. *The MAA Ad-Hoc Committee on Research in Undergraduate Mathematics Education should take the necessary steps to request that it become a permanent, joint committee of MAA and AMS. When such a joint committee is established, NCTM and AMATYC should be asked to appoint liaison representatives to this committee. The charge to this new permanent committee should include the monitoring of progress on all the recommendations contained in this report.*

Those participating in the conference did not agree at every juncture, but they did reach consensus that to achieve any objectives at all will require the visible and active leadership of both the MAA and the AMS.

Conference Organization

There was little question that the Conference participants knew they were attending a "working conference." The first general session convened after dinner Friday evening, November 8. In all, there were six general sessions and two structured writing sessions. The general sessions were focused on the four aspects of the Conference noted in the introduction, and two sessions were devoted to responding to the work of the writing groups. At each of the two writing sessions, three groups dedicated themselves to developing sets of strategies for addressing the concerns raised in the general discussions. The three writing groups were each asked to discuss and make recommendations regarding a pre-determined set of questions. On the basis of individual interest, the Conference participants self-selected the writing group in which they would participate. What emerged was consensus on a variety of statements and recommendations. Many of these address the urgent need to communicate across the mathematics and mathematics education communities. The efforts to strengthen and enhance this communication is a full community task—not one that can be done alone by individuals or by any single professional association.

To the hard working participants of the Conference we express our profound thanks. Sincere appreciation is extended to the National Science Foundation without whose support

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Organizing Committee

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GILBERT STRANG, *Massachusetts Institute of Technology,* and *Society for Industrial and Applied Mathematics.*

MARCIA P. SWARD, *Executive Director, The Mathematical Association of America.*

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Guests Attending Several of the Sessions:

BARRY CIPRA, *Northfield, Minnesota.*

MARGARET COZZENS, *Program Officer, National Science Foundation.*

RAY HANNAPEL, *Program Director, National Science Foundation.*

JOAN LEITZEL, *Division Director, Materials Development, Research, and Informal Education, National Science Foundation.*

JAMES LIGHTBOURNE, *Program Officer, National Science Foundation.*

Appendix B: Reading List

This list of recent publications related to educational research at the undergraduate level was compiled by Ed Dubinsky of Purdue University in order to illustrate the nature and scope of current research.

Alibert, D., 1988. "Towards New Customs in the Classroom." *For the Learning of Mathematics*, 8(2) 31–35.

Artigue, M., 1987. "Ingenierie Didactique a propos D'Equations Differentielles." Proceedings of the 11^{th} Annual Conference of the International Group for the Psychology of Mathematics Education (J.C. Bergeron and N. Herscovitz, Eds.), Montreal, 236–242.

Ayres, T.; Davis, G.; Dubinsky, Ed; Lewin, P., 1988. "Computer Experiences in Learning Composition of Functions." *Journal for Research in Mathematics Education*, 19(3) 246–259.

Blum, Werner and Niss, Mogens, 1991. "Applied Mathematical Problem Solving, Modelling, Applications, and Links to Other Subjects—State, Trends, and Issues in Mathematics Instruction." *Educational Studies in Mathematics*, 22(1) 37–68.

Breidenbach, D.; Dubinsky, Ed; Hawks, J.; Nichols, D., 1992. "Development of the Process Conception of Function." *Educational Studies in Mathematics.*

Chevallard, Yves, 1990. "On Mathematics Education and Culture: Critical Afterthoughts." *Educational Studies in Mathematics*, 21(1) 3–27.

Clement, J. "The Concept of Variation and Misconceptions in Cartesian Graphing." *Focus on Learning Problems in Mathematics*, 11(1) 77–87.

Confrey, J., 1991. "The Concept of Exponential Function." In L.P. Steffe (Ed.), *Epistemological Foundations of Mathematical Experience.* Springer-Verlag: New York.

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Freudenthal, Hans, 1982. "Variables and Functions." *Proceedings of the Workshop on Functions.* Enschede.

Goldenberg, P., 1988. "Mathematics, Metaphors, and Human Factors: Mathematical, Technical, and Pedagogical Challenges in the Educational Use of Graphical Representations of Functions." *Journal of Mathematical Behavior*, 7, 135–173.

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Heid, K., 1988. "Resequencing Skills and Concepts in Applied Calculus Using the Computer as a Tool." *Journal for Research in Mathematics*, 19(1) 3–25.

Kaput, J., 1987. "Representation and Mathematics." In C. Janvier (Ed.), *Problems of Representation in Mathematics Learning and Problem Solving.* Erlbaum: Hillsdale.

Leron, U., 1983. "Structuring Mathematical Proofs." *American Mathematical Monthly*, 90, 174–185.

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Monk, G.S. "Students' Understanding of a Function Given by a Physical Situation." In G. Harel and Ed Dubinsky (Eds.), *The Development of the Concept of Function.* Mathematical Association of America (forthcoming).

Orton, A., 1983. "Students' Understanding of Integration." *Educational Studies in Mathematics*, 14, 1–18.

Orton, A., 1983. "Students' Understanding of Differentiation." *Educational Studies in Mathematics*, 15, 235–250.

Owen, E. and Sweller, J., 1989. "Should Problem Solving Be Used as a Learning Device in Mathematics?" *Journal for Research in Mathematics Education*, 20, 322–328.

Schoenfeld, A., 1983. "Beyond the Purely Cognitive: Belief Systems, Social Cognitions, and Metacognitions as Driving Forces in Intellectual Performance." *Cognitive Science*, 7, 329–363.

Schoenfeld, A.H.; Smith III, J.P.; Arcavi A., 1990. "Learning—the Microgenetic Analysis of One Student's Understanding of a Complex Subject Matter Domain." In R. Glaser (Ed.), *Advances in Instructional Psychology*, 4. Erlbaum: Hillsdale.

Selden, A. and Selden, J., 1987. "Errors and Misconceptions in College Level Theorem Proving." In J.D. Novak (Ed.), *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*, Cornell University.

Sfard, A., 1991. "On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin." *Educational Studies in Mathematics*, 22, 1–36.

Shaughnessy, J.M., 1977. "Misconceptions of Probability: An Experiment with a Small-Group, Activity-Based Model Building Approach to Introductory Probability at the College Level." *Educational Studies in Mathematics*, 8, 295–316.

Tall, D. and Vinner, S., 1981. "Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity." *Educational Studies in Mathematics*, 12, 151–169.

Thompson, P., 1985. "Experience, Problem Solving and Learning Mathematics: Considerations in Developing Mathematics Curricula." In E. Silver (Ed.), *Teaching and Learning Mathematical Problem Solving*. Erlbaum: Hillsdale, 189–236.

Treisman, U., 1985. "A Survey of the Mathematics Performance of Black Students at the University of California, Berkeley." (Unpublished thesis available from the author at the Dana Center, Department of Mathematics, University of Texas at Austin, Austin, TX 78712-1082.)

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