Reaching for Quantitative Literacy

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The Focus Group on Quantitative Literacy addressed three key issues—why, what, and how. A lengthy introduction written by Donald Bushaw provides a response to the first question. The report of the Focus Group itself is devoted to a discussion of "what" and "how," based on experiences from many different institutions. Appendices provide information on research, commentary on course materials, and extracts from the Focus Group e-mail conversation.

Introduction: Why Quantitative Literacy?

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There seems to be wide agreement that a well-educated citizen should have some significant proficiency in mathematical thinking and in the most useful elementary techniques that go with it. In western civilization, the idea goes back at least to classical times, when four (the "quadrivium") of the seven liberal arts considered essential for the education of a free citizen were essentially mathematical. The role of mathematics was enlarged by the Enlightenment, by the Industrial Revolution, and by many events in modern science, technology, business, and the rapid intellectual evolution of humanity generally.

In recent years, amidst intense scrutiny and sometimes harsh criticism of the whole educational system in the United States, one group after another has expressed itself on the point. A representative statement (here considerably abbreviated) appears in the influential report *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (National Academy Press, 1989, pp. 7–8):

> To function in today's society, mathematical literacy—what the British call "numeracy" —is as essential as verbal literacy ... Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts—for example, change, logic, and graphs—that permeate daily news and routine decisions ... Functional literacy in all of its manifestations—mathematical, scientific, and cultural—provides a common fabric of communication indispensable for modern civilized society. Mathematical literacy is especially crucial because mathematics is the language of science and technology ...

An emphasis on the expanding importance of general education in mathematics beyond high school was made over twenty years earlier, in the COSRIMS report *The Mathematical Sciences: A Report* (1968), p. 56:

> The impact of science and technology has become so significant in our daily life that the well-educated citizen requires a background in the liberal sciences as well as the liberal arts. It has long been recognized that mathematical literacy is an important goal of all liberal education. But in current education this training often stops at the secondary-school level. With the increasing quantification of many of the newer sciences, the impact of high-speed computers, and the general expansion of the language of mathematics, it becomes increasingly important for the college graduate to have some post-secondary training in mathematics ...

Or consider the following words from *The Mathematics Report Card: Are We Measuring Up?* (Educational Testing Service, 1988, p. 9):

> Looking toward the year 2000, the fastest-growing occupations require employees to have much higher math, language, and reasoning capabilities than do current occupations. Too many students leave high school without the mathematical understanding that will allow them to participate fully as workers and citizens in contemporary society.

Those who have been pleading for more nearly universal quantitative or mathematical literacy have not all been mathematicians, by any means. Consider the words from *50 Hours: A Core Curriculum for College Students* (National Endowment for the Humanities, 1989, p. 35):

> To participate rationally in a world where discussions about everything from finance to the environment, from personal health to politics, are increasingly informed by mathematics, one must understand mathematical methods and concepts, their assumptions and implications.

These statements and many others like them add up to an interesting challenge, and since about half of American colleges and universities have no general mathematics requirement for graduation, the challenge is clearly not being met.

There have been encouraging signs of improvement in recent years, but optimism can be premature. As these words are being written, it was just announced by The College Board that the average quantitative score on the SAT has taken another downward turn, after more than a decade without any decrease.

We have been speaking of *mathematical* attainments. The term "quantitative literacy" has so far appeared only in the title. Whether there is a real difference between "quantitative literacy" and "some significant proficiency in mathematical thinking and in the most useful elementary techniques that go with it" is a matter of debate. Sometimes the term "quantitative literacy" is a virtual euphemism for some level, usually ill-defined, of accomplishment in mathematics. (How unfortunate that some people should consider it expedient to use a euphemism for "mathematics"!) At other times "quantitative literacy" is used much more broadly, to include logic, linguistics, and other subjects that have at least a relatively formal character, even if they are seldom or ever taught in mathematics departments.

Here we shall adopt the point of view that "quantitative literacy" primarily concerns mathematics, broadly understood. It is not an entirely fortunate term. For one thing, much of modern mathematics, even at elementary levels, is not distinctively quantitative; for another, "literacy" suggests both facility with *letters* and a possibly very low level of accomplishment. The term "numeracy" is shorter, at least. Most, if not all, of what will be said here will apply whichever reasonable interpretation of the term "quantitative literacy" is adopted.

It may be useful to enumerate some of the principal reasons for expecting quantitative literacy of educated people. The list that follows is surely not complete, and the items in it are not independent; but it directs attention to some of the major areas in the broad range of "Why study mathematics?"

- Mathematical thinking and skills are of great value in *everyday life*. "Other things being equal, a person who has studied mathematics should be able to live more intelligently than one who has not. And, up to a point at least, the more mathematics studied, the more intelligent the life should be" (NCTM, *A Sourcebook of Applications of School Mathematics*, 1980, Preface).

- One of the classic reasons for studying mathematics is that it strengthens *general reasoning powers*, for instance by developing problem-solving skills. While the research literature is ambiguous on this point, many thoughtful people are convinced that it is true in some sense.
- Quantitative literacy at varying levels is clearly needed in *preparation for further study* in many academic and professional fields. It is reliably estimated that the majority of undergraduates would be required to take a course or courses in the mathematical sciences for this purpose even in the absence of a general graduation requirement of this kind.
- Increasing amounts of mathematics are needed in an increasing number of *careers*. ... "More and more jobs—especially those involving the use of computers—require the capability to employ sophisticated quantitative skills. Although a working knowledge of arithmetic may have sufficed for jobs of the past, it is clearly not enough for today, for the next decade, or for the next century" (*Moving Beyond Myths: Revitalizing Undergraduate Mathematics* (National Academy Press, 1991, p. 11). And students, even college seniors, often do not know what careers they will enter, or where their career paths will lead them. A quantitative literacy requirement helps to hold some doors open.
- Many adults, and especially college graduates, are very likely to assume positions in their communities and in professional organizations where quantitative literacy (e.g., the ability to deal intelligently with statistics) will come into play and may even be essential for effectiveness. A quantitative literacy requirement can thus be expected to enhance the quality of *citizens*.
- Anyone who does not have a mature appreciation of mathematics misses out on *one of the finest and most important accomplishments of the human race*. A quantitative literacy requirement, sensibly defined, will contribute to the spread of that appreciation.
- Society can ill afford to under-develop *latent mathematical talent*. For many students the activities leading to satisfaction of a quantitative literacy requirement can be revelatory, inspiring them to consider for themselves careers in mathematics or mathematics-related fields.
- The fear of mathematics that is often called *math anxiety* or *mathophobia*, besides stunting the cognitive development of those who suffer from it, tends to communicate itself from one generation to the next, in the home, and elsewhere. It is usually learned, not in-born, and a quantitative literacy course or courses, if competently and compassionately taught, can be powerfully therapeutic against it. (Certain learning disabilities do seriously impede the learning of mathematics, but the number of people affected by these disabilities is small. Reasonable accommodations, for legal as well as humanitarian reasons, should be made for such students.)

Even if, as many thoughtful people believe, the educational process that finally produces college graduates should be regarded as seamless, practical considerations require that some line should be drawn between the pre-college part and the college part, or in other words between the secondary part and the tertiary part. The present report is sponsored by the Mathematical Association of America, which by its charter is concerned with "collegiate mathematics," so is concerned mainly with the college part.

The term "remedial" (or "developmental"), as applied to a college mathematics course, has a definite meaning only where there is a clear understanding of where pre-college mathematics leaves off and collegiate mathematics begins. There are various opinions about where this line may be. However "remedial" is defined, the volume of remedial instruction to college students has certainly increased in the past several decades. According to *A Challenge of Numbers: People in the Mathematical Sciences* by Bernard L. Madison and Therese A. Hart (National Academy Press, 1990, p. 29):

> In fall 1970, college enrollments in remedial courses constituted 33% of the mathematical sciences enrollments in two-year colleges and by 1985 had increased to 47%. In four-year colleges and universities, remedial enrollments constituted 9% of the mathematical sciences enrollments in 1970 and had increased to 15% by 1985.

In spite of the volume of resources being poured into the teaching of such courses, there is widespread skepticism, backed up by some empirical studies, about their effectiveness, especially in preparing students for genuinely college-level mathematics courses. One should expect more from a quantitative literacy program for undergraduates.

But *is* there an intrinsically "college" part for all students? If agreement can be reached on what "mathematical methods and concepts, their assumptions and implications" every college graduate should understand, does it really matter whether that understanding is acquired before or after matriculation in a college or university? Is it not imaginable that, for example, the goals set for secondary mathematics in the NCTM *Curriculum and Evaluation Standards in School Mathematics* (1989) define an acceptable concept of quantitative literacy? And if so, and if the *Standards* are widely adopted, will there be anything left for the colleges and universities to do in this area beyond supplying suitable remedial experiences for those students who slip through the cracks? To put the matter another way, is it not imaginable that any quantitative literacy appropriately required for a bachelor's degree should in fact be regarded as an appropriate requirement for admission to a college or university?

There are several very large "ifs" in the preceding paragraph. They relate to difficult questions of definition, curricular diversity and inertia, a great lack of homogeneity in the student population, and other inconveniences. A more important consideration, perhaps, relates to the nature of the post-secondary experience.

College students, on the average, are more mature, more experienced, and more thoughtful about their personal goals than they were before they became college students. One does not need to invoke William Perry's scheme to justify a belief that college students should be better able to acquire, and to acquire more deeply, quantitative literacy in any reasonable sense. Indeed, because of the pervasiveness of mathematical ideas in the careers that college graduates usually enter, they should be *expected* to have acquired them more thoroughly and meaningfully than if they had not gone to college.

These ruminations are leading relentlessly to the conclusion that it might be a mistake to speak of "quantitative literacy" as if it were a single, monolithic idea. Surely there are meaningful *degrees* of quantitative literacy, and perhaps it would be useful to identify some of them. Here, we speak of only one—the degree of quantitative literacy appropriately expected of all *college* graduates. As we have suggested, we do not believe that this is identical with the degree of quantitative literacy appropriately expected of all *high school* graduates, even as implied in such a forward-looking statement as the NCTM *Standards*.

Thus we take the stance that, for many reasons, some significant level of quantitative literacy is desirable in all adults; that the amount appropriate for college graduates is greater than that to be expected at the time of graduation from high school; and that the difference is not merely a matter of "remediation."

Cultivation of quantitative literacy at any level is, of course, a matter of teaching and learning. And teaching and learning involve far more than mere identification and communication of appropriate content. There is ample evidence that the traditional "lecture-and-listen" mode of instruction, still probably far more the rule than the exception in American higher education, does not work as well as some other modes—certainly not as well as it should. Particularly for those students who are studying in the mathematical sciences not by their own choice, teaching and learning styles that include active involvement, cooperation, and the personal touch are much to be preferred over those that do not.

So while the emphasis in this report will be on what the elements of quantitative literacy are, we also implore those who are responsible for providing students with classes and other opportunities for developing quantitative literacy to give a great deal of attention to the form those opportunities should take and the manner in which they should be delivered.

What Is Quantitative Literacy?

The Focus Group on Quantitative Literacy was summoned to its conference tasks by words from the National Academy Press' publication *Everybody Counts* (1989):

> Functional literacy in all of its manifestations—verbal, mathematical, scientific, and cultural—provides a common fabric of communication indispensable for modern civilized society. Mathematical literacy is especially crucial because mathematics is the language of science and technology. Discussion of important health and environmental issues (acid rain, waste management, greenhouse effect) is impossible without using the language of mathematics; solutions to these problems will require a public consensus built on the social fabric of literacy.

The conference invitation noted that the issues of quantitative literacy are of critical importance for the lives of all of us, since they shape the future of our democracy as we would like to see it. We are confronted daily with conflicting quantitative information and need to be aware of both the power and limitations of mathematics.

Faced with these weighty statements, conference participants attempted to sift and sort their thoughts into coherent responses to questions posed by the conference moderator. The difficulty of the questions posed was readily acknowledged. One participant who promised a "good answer" soon observed, "The trouble is, I keep changing my answers every few days; this is a tough nut to crack!" Yet another said, "I really am overwhelmed by your questions."

Despite difficult questions, the group uncovered a number of factors that need to be addressed in a quest for quantitative literacy—some arising in a straightforward manner and others arising more subtly.

At first, conference participants struggled with a response to the question of determining the key issues in quantitative literacy for the next five years. The issues were expressed, in one participant's words, in a series of questions:

> Is there a valid concept of quantitative literacy which makes sense for all U.S. citizens? If so, how might it be identified? Is it the NCTM *Standards*? If not, what can serve as a guide to educators? A set of options? A set of components which might be mixed in various proportions? What evidence should be considered in answering these questions? If opinions, whose?

Quantitative literacy certainly has different meanings to different people, so in order to work with this concept, some group must say just what we will mean by quantitative literacy. Just what skills, concepts, and behaviors in this general area should students be expected to learn?

Beyond determining a set of objectives (or behaviors) for quantitative literacy, there are attitudinal problems which must be dealt with in our society and in the mathematics community. For many decades it has been "okay" within our culture to dislike mathematics and to seek to "get by" with a level of literacy which would probably have not even been acceptable in a society far less complex than our current one. How can the mathematics community turn around the negative attitudes of our citizens towards quantitative literacy? And how can the resolve of the collegiate mathematics community be heightened to face the enormous remedial task in areas such as problem interpretation and formulation, probability models, and estimation—"areas where students show so little understanding of the basic characteristics of simple operations." It must no longer be acceptable for "educated" citizens to lack quantitative literacy. It must no longer be acceptable for the mathematics community to be willing to see graduated from our colleges and universities students who lack quantitative literacy. But how do we change these attitudes?

The focus group made several suggestions about what quantitative literacy study should include. One idea that was reiterated by several people was that we need to look at concepts in context (i.e., applications areas). "The key is to have the contexts relate to student interest, daily life, and likely work settings." As another put it, "People must be able to absorb quantitative relationships that are expressed in daily life. I see this as essential to an educated citizenry." Among concepts which might be put in context were dimension, distance (or area or volume), model, and data analysis. One participant particularly lauded the study of statistics because it "can generate a need to know basic geometry, algebra, and of course arithmetic." These ideas provide an opportunity to turn negative attitudes around, for they all suggest relevance to the student's life as the student perceives it.

These suggestions immediately pose another problem: how could they be presented in the college curriculum? The mathematics community finds it very difficult to think in any terms other than those courses that have become traditional at the college level. Focus group participants wrestled with thoughts of an "extended" definition of "statistics" and bemoaned the disfavor into which once popular mathematics appreciation courses have fallen. At the college level, new courses must be devised and old courses modified to foster the objectives of quantitative literacy. Courses must be developed on the basis of the determined meaning of quantitative literacy rather than just by asking which existing courses determine quantitative literacy. Just as the Sloan Foundation has conducted some interesting experiments for enticing talented students into mathematics and the Annenberg Foundation supported development of *For All Practical Purposes* to foster a new view of mathematics by the public, so new courses in quantitative literacy must be developed and resource materials made available for their teaching.

The development of new courses is a costly effort, often taking away time from other tasks the mathematician would rather do. Further, producing relevant materials to daily life means materials must, at least in part, be of a "throw away" nature—that is, they will have a shorter life span than text material would often be expected to have. Hence there is need to support the development of a range of both innovative courses that have quantitative literacy as their focus and materials for use in teaching such courses. In addition, once such courses are developed, those that have the greatest success should be disseminated to the broader mathematics community as models. The important question of how quantitative literacy courses relate to other courses in the curriculum will still need to be addressed, of course.

Pre-college Numeracy

Links with pre-college mathematics are also important considerations in the development of quantitative literacy in college students. Driven by efforts to implement the NCTM *Standards*, the pre-college curriculum in mathematics is in a stage of flux. More and more, number sense is being developed in children by consideration of real problems—problems that capture student interest and that arise readily in daily life. One conference participant said, "I think one intent of the new NCTM curriculum is to give students both an appreciation of the power of mathematics and some essential mathematical skills. We can only hope that this curriculum succeeds, but what do we do in the meantime?"

Quantitative literacy at the college level must not wait for the *Standards* to take hold, but it must be approached in a manner that is cognizant and supportive of the changes in the pre-college curriculum which will lead to greater strength in quantitative literacy for everyone.

Another issue to consider is the effect on teacher training programs of the establishment of a quantitative literacy requirement for all college students. Here the recent publication *A Call For Change* may well lead the way to the problem's solution, since it couches its recommendations in terms of *experiences* rather than *courses* a student should have. Surely quantitative literacy courses could offer some valued experiences with mathematics and even some models of ways to teach mathematics.

On the other hand, college professors must right now participate in the teaching of what one participant called "remedial areas"—whether teaching the least or the best-prepared students. Many mathematics instructors do not want to be involved with such teaching—partly because they feel ill-prepared to teach remedial work, but also because they feel students should be quantitatively literate by the time they reach college. In their view those students who are not already quantitatively literate should not be admitted to college. These views lead to another important issue: Is there a difference between quantitative literacy for high school graduates and that for college graduates? If so, what difference in expectations should be made?

One participant felt that the average college-educated person should have a reasonable appreciation for the power and importance of mathematics in our society and culture along with a certain level of mathematical skills.

> I think these two goals are often confused, and that they often work at cross purposes. For example, we make virtually every undergraduate pass a course in college algebra before they graduate (I am not defending this requirement, so boo if you want to). This assures a certain level of

mathematical skills, but these students have absolutely no appreciation of the role of mathematics in contemporary society (or if they do, it is not because of anything we teach them in the mathematics department). In fact, I would wager that the opposite effect is the case: students here regard mathematics as a boring collection of useless tricks for solving dull problems.

On the other hand, another conferee argued that a distinction between high school and college-level quantitative literacy arises, in part, from the form of communication expected: "College implies that one could understand relationships expressed symbolically—whereas high school implies an understanding of everyday level vocabulary." Another suggested that the key word for the distinction between the two levels of schooling is "deeper." "The reasoning and examples can become more sophisticated and the students may allow a broader set of applications at the college level." These comments offer at least two conclusions from the conferees—namely that there is a distinct notion of quantitative literacy for college graduates versus high school graduates, and that algebra does play a role in college-level quantitative literacy.

Establishing Goals

Just what is involved in establishing goals and objectives for quantitative literacy at colleges and universities? At the elementary and secondary school levels, the NCTM *Standards* speak of students acquiring "mathematical power." The term denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. This notion is based on the recognition of mathematics "as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual, mathematical power involves the development of personal self-confidence." If this standard determines quantitative literacy for high school graduates and students at the college level are required to take a mathematics course in this spirit which goes deeper, will such students be quantitatively literate as college graduates?

Most conference participants argued for some further elements in the baccalaureate experience.

I also expect quantitative literacy to exhibit a set of behaviors ... (but) all this will fail unless students see that what we are trying to do is really important in their majors.

On my campus we are gearing up for 'writing across the curriculum' ... and I contend that one can simply replace 'writing' with 'mathematics' in virtually everything I have read about writing across the curriculum and get a true statement.

To me mathematics is a very broad topic ... I see mathematics in politics, sociology, history, etc.

I think there is a general move to writing in mathematics—as well as mathematics across the curriculum.

Thus the participants argued not only for a mathematics course that fosters quantitative literacy for college students, but also that quantitative literacy goals be connected with the broader curriculum and also involve writing.

Two other connections with the broader curriculum surfaced in the discussions. Although they are not new, in the current context they elicit deeper thought. These are "helping people to be life-long learners" and the issue of "appreciation" of mathematics. Regarding life-long learning and quantitative literacy, one conferee said:

> I'm not sure how one goes about doing this, but part of it must involve inciting curiosity about numerical phenomena. I think people tend to be curious about numerical things (what are my chances of winning the lottery?), but think the solutions are somehow mysterious and beyond them. Thus, we need to provide simple ideas for handling numbers that do not involve messy algorithms and can be seen as widely applicable building blocks to number sense (a jackknife, not a surgeon's scalpel?).

As for "appreciation" of mathematics, it was argued, hopefully, that appreciation would come with the acquisition of skills (where skills are intended in the broadest sense as terminology). Another voiced the opinion that students might profit from the study of some mathematical models: "Perhaps students can learn to appreciate the power of mathematics by understanding the simpler components well enough to envision the role of a more detailed model, even if they have no interest in constructing the model themselves."

Yet another connection which the group brought up is the role that the ability to use hand calculators and personal computers should play in quantitative literacy. One conferee exclaimed, "I must share my support for a hands-on technologically enriched curriculum in which a person becomes quantitatively literate—a person *becomes.*" Technology is used, among other things, to clarify, to depict, and to manipulate data; implied in its use are the needs for estimation skills and skills at analysis of error. Quantitative literacy must include the effective use of common machines in order to bring about increased understanding of mathematical situations and to solve problems either precisely or in an approximate form. Since such use beyond the sciences and mathematics is still in a developmental stage, this idea requires special wisdom on the part of those who teach with calculators or computers. College teachers in every discipline need ideas and experience in how to use these machines appropriately and well.

Research

Certainly whatever efforts are exerted to make college students and society more quantitatively literate must be handled so as to see that those groups that have been underrepresented among our educated citizenry are not treated adversely. Research needs to be done to see how quantitative literacy programs impact not only the attainment of the desired education, but also to see if various groups of people are being affected differently, or if different strategies must be adopted for certain groups of learners based on learning styles. If an important part of quantitative literacy programs is to be study-in-context of problem situations, the contexts must not be selected so as to be understood only by a subgroup of the students in a program. In the past, women and minority groups may have lacked the "key to opportunity" which being quantitatively literate provides, but this must not continue inadvertently in a design of new programs for quantitative literacy for all college graduates.

Implied in the call for research in the previous paragraph is a strategy of assessment. Colleges need to know where their students are on the road to "becoming" quantitatively literate in order to know what the students must yet attain. Some portion of the assessment by the National Center for Education Statistics on "The State of Mathematics Achievement" might be helpful in establishing baseline information for aspects of quantitative literacy. (See Appendix A for a more thorough discussion of an appropriate research data base.) Colleges or universities might also consider some entrance level assessment in quantitative literacy—a task which may serve a dual role of information for the school to use in advising the individual student and of political force. As one conferee said, "...entrance level assessment in quantitative literacy at the university level would help impress on high school students a certain minimal competency level."

However, beyond these thoughts the conference members did not come to grips with means for assessing whether the goals for quantitative literacy have been achieved for an individual student or whether it is desirable to have a "rising junior" examination as in the state of Florida where a student may not become a junior without exhibiting some minimal skills in mathematical thinking on a standardized test. Perhaps some new instruments are needed to truly validate levels of quantitative literacy, especially when consideration is given to the goals and objectives which the group discussed as being part of quantitative literacy.

If at the college level quantitative literacy includes the establishment of connections between mathematics and other disciplines or is fostered by mathematics across the curriculum, then mathematical sciences faculty members must reach out to colleagues in other departments on campus and to administrators to effect change which will enable students to become quantitatively literate. Ideas on how to make the connections need to be developed and disseminated. (The only group of courses developed so far of this nature appears to be those courses on quantitative reasoning put together with the support of the Sloan Foundation in the New Liberal Arts Program; a description of one such course, from Mount Holyoke College, appears in Appendix C.)

Outreach

Along the lines of what is working, it should be pointed out that the NCTM *Standards* have alerted the teaching population at the elementary and secondary levels to the need for change in the teaching of mathematics and serve in part as a model for change. The NSF project on statistics which was organized through a joint committee between the NCTM and the American Statistical Association "with the goal of providing curriculum materials and in-service training so that mathematics teachers in the secondary schools could effectively incorporate basic concepts of statistics and probability into their teaching of mathematics" has met with outstanding success. These have given excellent support for cooperative learning, provided a basis for writing in a mathematics class, and engendered positive attitudes by students toward mathematics. Workshops and courses have been taught so as to model how these materials should be used. Similar kinds of activities will likely be necessary at the college level, although materials need not all be statistical in nature. (See Appendix B for examples of other related course materials.)

Besides students and colleagues in our colleges and universities, who else do we need to reach to effect change in the area of quantitative literacy? Groups advanced were: the press, parents, school boards, PTA's, politicians, and CEO's. One participant observed that many states have state mathematics coalitions supported through federal funds and the Mathematical Sciences Education Board. The people we want to reach to bring about change should be represented on these coalitions. "A plan could be developed that works with the mathematics coalitions of every state to begin to change the quantitative literacy level of all people."

Part of such a plan might be an idea advanced by another conferee who felt that we had to try to educate parents so they and their student children can work together on improvement of reasoning skills. He noted, "Two projects in this direction are the *Math Kits* program of the MSEB and the recent *Math Power* series for middle schools from AAAS, one of which is entitled *Math Power at Home*. These publications are at the school level, but I think we should understand and improve that level so that we might achieve some carry-over to the college level."

Further parts of a plan must be to acknowledge that politicians are the decision-makers in our society; they must have some quantitative literacy skills of their own and understand the needs for education in quantitative literacy if they are to support funding for programs in mathematics education. It was also noted by one participant that "my experiences suggest that politicians have relatively low regard for educators but high regard for successful business people," so "we must use business and industry connections" to reach them.

Another idea advanced regarding how to reach the public is one which could also have the effect of heightening the awareness at colleges and universities of the need for improved quantitative literacy.

> We as educators, and our students, should be more forceful in making the media do a better job of handling quantitative information, and calling them on it when they do a poor job. Critical reading of such articles is one of the things we should teach, just as English teaches critical reading from the standpoint of grammar and style. We should simply not stand for some of the things we see in advertising, or the misuse of polls, or the incorrect interpretation of medical studies. ... We must show people the importance and correct interpretation of good uses of mathematics (the Physician's Health Study, tracking the CPI and unemployment rates), and point out the problems and disasters that occur from no or incorrect mathematical modelling (the Challenger disaster, the flap over AIDS testing).

The College Algebra Debate

In an editorial in the Washington *Post* on April 20, 1991, Colman McCarthy gave his response to "Who Needs Algebra?"

> Would millions of high school students trudge into their algebra classes if it weren't a gate through which they were forced to pass to enter college? And in a few years would they submit to college algebra if it weren't a requirement to graduate? Not likely. Algebra is more loathed than learned, more endured than embraced. It is more memorized to pass tests than understood to comprehend problems.

Is McCarthy right? Many colleges and universities require college algebra of their graduates, but why do they do it? There are a number of factors involved, at least one of which is that it is said to assure that students have had the type of intellectual challenge which the study of college mathematics represents. But there are other reasons too.

College algebra fits neatly with the standard curriculum in high school and with the lower division courses taught in college which are required in majors outside mathematics. Placement tests can be easily written, administered, and evaluated to see if a student needs college algebra. Proficiency examinations (or CLEP tests) are also easy to give and evaluate. Further, mastery of the computational skills of college algebra is essential for continued study in mathematics for most students. Thus, if students take college algebra upon entering college, they may leave open choices for majors which require additional mathematics. In an editorial in a recent issue of the *UMAP Journal* (Vol. 11, No. 4, 1990), Joe Malkevitch argues that preserving access to study of mathematics, science, and engineering remains *the* reason students are pressured into studying algebra.

While it is true that mathematics is the "language of science," it is increasingly true that social and management sciences are also using algebraic techniques and computational skills. However, other disciplines are using much mathematics that is not algebra as well.

At most colleges and universities the focus of study in a college algebra course is on those computational skills needed to do calculus (although the course is also used as prerequisite material for courses in finite mathematics, statistics, etc.). With this focus exclusively, college algebra does not make a good general education course—a course which should convey how mathematicians think and develop their ideas. Further, a college algebra course with this focus does *not* meet the objectives of quantitative literacy as suggested by our e-mail conference discussion.

Many in the mathematics community view the standard college algebra course as "in trouble." In an article "What's Wrong with College Algebra?" appearing in the *UMAP Journal* (Vol. 12, No. 2, 1991), Peter Lindstrom lists eight areas of disagreement among mathematics teachers about the course. Among these areas are three which were brought out in a debate titled "Resolved—All College Graduates Should Know College Algebra" that was sponsored by the CUPM Subcommittee on Quantitative Literacy at the January 1991 AMS/MAA meeting in San Francisco. First, many students taking the course are not prepared for it (so the course is often compromised). Secondly, a college algebra course which does not focus on computational skills tries to do too many things to succeed in conveying essential skills for the calculus or other courses for which it is prerequisite. And thirdly, a course which stresses problem solving (not routine problems and skills development) is a more appropriate course for *all* college graduates. Implied in the discussion was the thought that a modified college algebra course which stresses problem solving with problems connected with daily life and which uses hand calculators as a natural tool in problem solving may serve neither of the two objectives it seeks to meet.

The proper balance of elementary algebra and problem solving is yet to be determined in the composition of appropriate quantitative literacy for all college students. Yet to be determined also is the effective use of hand calculators in the teaching, learning, and use of college algebra taught as prerequisite to the further study of mathematics. But now is the time to determine both.

Challenges

These latter thoughts show what consequences can occur, and may occur more and more, if the key issues in quantitative literacy are not addressed. Quantitative literacy is essential as a foundation for democracy in a technological age. Without it our citizens are ill-equipped to make the responsible decisions a democracy places in their hands, and we can expect more and more disasters—if not extinction from poor management of the environment. Citizens need to be able to understand research results which are often expressed only in quantitative terms.

The time has come for professional groups such as the MAA to speak out on what quantitative literacy for college students should mean. As one participant put it, "Part of the reason, in my opinion, that a state ... would try to define quantitative literacy as a college algebra course is because the mathematics professional groups are leaving such a void there." Vague generalities are not good guidelines, and good guidelines are greatly needed as to what quantitative literacy entails and how it can be accomplished or its level improved. Such guidelines could be helpful to faculty and deans of colleges as well as to others.

There is also need for research. As one participant said, "If we are to improve teaching and learning, we must know something about how people learn, especially about their intuitive understanding of the world." The more we understand about how students learn those skills, concepts, and behaviors we choose to call quantitative literacy, the better we should be able to teach them.

These were the issues the Focus Group on Quantitative Literacy discussed. Interestingly, missing from the discussion was any mention of "math anxiety" or "mathophobia." Participants seemed to be following the thought pattern of what should a college educated person be able to do. Quantitative literacy was not viewed as a spectator sport in these discussions. It was a vital need for everyone!

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Appendix A: Assessment and a Research Data Base

For many years the National Assessment of Educational Progress (NAEP) has conducted a mathematics assessment of fourth, eighth, and twelfth graders. The assessment is designed to measure mathematics proficiency in six content areas: (1) numbers and operations; (2) estimation; (3) measurement; (4) geometry; (5) data analysis, statistics, and probability; and (6) algebra and functions. A new aspect of the 1990 program was that states could participate on a voluntary basis in assessment of eighth graders enabling state comparisons with each other and the nation. Thus the U.S. Department of Education issues "The State of Mathematics Achievement: NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States (Executive Summary)" as well as for those participating states a separate report available through that state's Department of Education.

The purposes of the NAEP data gathering are expressed as: "...to provide a detailed portrait that can be used in examining where the nation is in relation to its overarching goals for mathematics education, and how far mathematics educators have moved toward meeting their standards ... This information can be used to monitor students' progress in achieving what has been recommended for reform in school mathematics, to explore issues of equity in opportunity to learn mathematics, and to examine both school and home contexts for educational support."

A perhaps lesser-known study is the Longitudinal Study of American Youth (LSAY) initiated in 1986 by a grant from the National Science Foundation. The primary objectives of the LSAY were and continue to be improvement in our understanding of:

- The development of student skills in science and mathematics.
- The role of student attitudes toward science and mathematics in the development of skills in those areas.

- The development of a student preference for or against a career in science, mathematics, engineering, and medicine.
- The influence of parents and other family members on the development of student attitudes toward and skills in science and mathematics and the emergence of career preferences.
- The influence of school curriculum and environment on the development of student attitudes toward and skills in science and mathematics and the emergence of career preferences.
- The influence of teacher training and classroom practice on the development of student attitudes toward and skills in science and mathematics and the emergence of career preferences.
- The influence of informal science education on the development of student attitudes toward and skills in science and mathematics and the emergence of career preferences.
- The influence of scientific literacy on the development of a sense of citizenship efficacy in regard to public policy issues involving science and technology.

The LSAY follows students through the years and has mined some models on predicting mathematics achievement, effects of ability grouping in middle school science and mathematics on student achievement, and more. For example, analysis of young adult data showed that the extent of formal education in science and mathematics is the most important influence on attentiveness to and interest in science. The research data base generated by this study should enable some researchers to obtain a better understanding of how students learn. Data is collected so as to trace students into their college years.

The LSAY has been directed by Jon Miller and Robert W. Suchner of the Public Opinion Laboratory and Social Science Research Institute at Northern Illinois University, DeKalb, Illinois. Available now is the LSAY Codebook: *Student, Parent, and Teacher Data for Cohort One for Longitudinal Years One, Two, and Three (1987-1990), Volume 1* (March 1991). Some reports of work with the data have been given at meetings of the American Educational Research Association. Among these are "Some Models to Predict Mathematics Achievement," and "Modeling NAEP Items by Cognitive Process" by Robert W. Suchner, and "The Longitudinal Study of American Youth and the National Education Longitudinal Study of 1988: A Comparison" by Thomas B. Hoffer.

Appendix B: Course Materials

Billstein, Rick and Lott, Johnny W. *Mathematics for Liberal Arts: A Problem Solving Approach.* Benjamin Cummings Publishing Company, 1986.

This book involves a survey of some elementary mathematics—topics which have been standard in mathematics appreciation courses—but it presents and uses the heuristics of problem solving as an integral part of the mathematics. The book does not presuppose specific high school mathematics. Problems emphasizing calculator usage are included in problem sets, and "Computer Corners" present programs in BASIC and Logo.

Caruth, J. Harvey. *Algebraic Reasoning Motivated by Actual Problems in Personal Finance.* The University of Tennessee, Knoxville, 1990.

"Traditional treatments of most mathematical concepts covered in this text may be found in virtually any high school or college algebra text." The book features actual problems in personal finance to motivate algebraic concepts—problems relevant to most college students' experience such as borrowing money for the first year of college, or paying for a car or house with monthly payments. Emphasis is placed on reasonableness of results and the development of "number sense." The use of calculators and computers is included in text treatment of problem situations.

For All Practical Purposes: Introduction to Contemporary Mathematics, Second Edition. W.H. Freeman and Company, 1991.

This book is a revised version of the book written by some 14 authors and edited by Lynn Steen. The original version was a project of COMAP, Inc., and was intended to be used in conjunction with the telecourse available for PBS through funding by the Annenberg Foundation. The book is intended for a one-term course in liberal arts mathematics or a course that surveys mathematical ideas. Presupposes some ability in arithmetic, geometry, and elementary algebra. Emphasis is placed on connections between contemporary mathematics and modern society rather than on the capacity of the student to *do* mathematics.

Growney, Joanne Simpson. *Mathematics in Daily Life: Making Decisions and Solving Problems.* McGraw-Hill Book Company, 1986.

This book is intended for use in college or university general education mathematics courses that are focused on developing abilities in quantitative and logical reasoning. It presupposes and emphasizes mainly arithmetic in decision making and problem solving. Use of mathematics (not the mathematical topics) is stressed. The text, its topics, and its exercises are quite nonroutine in nature. Students are expected to use hand calculators in completing exercises.

Pollatsek, Harriet and Schwartz, Robert. "Case Studies in Quantitative Reasoning: An Interdisciplinary Course." Extended Syllabi Series of the New Liberal Arts Program, Alfred P. Sloan Foundation, 1990.

This is a syllabus for a course developed at Mount Holyoke College which is open to all students at that college (as a general education course). The course teaches quantitative methods in the context of how they are used. A Macintosh computing facility is available for student use. The goals of the course are "to help students strengthen their analytical skills and acquire a more confident understanding of the meaning of numbers, graphs, and the other quantitative materials that they will encounter in many subsequent courses." The three main units are: I. Narrative and Numbers: Salem Village Witchcraft; II. Measurement and Prediction: SAT Scores and GPA; III. Rates of Change: Modeling Population and Resources.

Schwartz, Richard H. *Mathematics and Global Survival, Second Edition.* Ginn Press, 1990.

This book, "written to make students aware of critical issues facing the world today, and to help them respond effectively," might be termed a developmental mathematics book—students are only assumed to have elementary computational skills. Mathematical concepts used are some arithmetic ideas (percents, ratios, etc.) and some ideas in elementary probability and statistics. Almost every problem is related to an issue of global survival.

Sons, Linda R. and Nicholls, Peter J. *Mathematical Thinking in a Quantitative World.* Preliminary Edition. Kendall/Hunt Publishing Company, 1990.

This book is written for a college course in quantitative reasoning which presupposes two years of college preparatory high school mathematics including one year of high school algebra. The intent of the material is to help "develop in the student a competency in problem solving and analysis which is helpful in personal decision-making; in evaluating concerns in the community, state, and nation; in setting and achieving career goals; and in continued learning." It is assumed students will use elementary hand calculators. Emphasis is placed on *use* of the mathematics students have studied through facing them with problem-solving situations which are relevant to their daily lives. Both text

and exercises seek to provide the bridge students need from their previous more computational-skills oriented mathematical experiences to higher levels of mathematical thinking.

Wattenberg, Frank. *Personal Mathematics and Computing: Tools for the Liberal Arts.* McGraw-Hill Publishing Company, 1990.

This book is intended as a computer literacy tool but also as a tool to teach a student to use mathematics to reason about a variety of important real problems. The text teaches True BASIC programming but emphasizes applications rather than mathematics and programming. It presupposes college preparatory algebra and geometry. Chapter topics are probability and statistics, economic models, optics, local aid distribution, and population models. The book is a text for one of the courses in the New Liberal Arts Program which was developed through funding by the Sloan Foundation.

Appendix C: Case Studies in Quantitative Reasoning

by Harriet Pollatsek and *Robert Schwartz,* MOUNT HOLYOKE COLLEGE

Case Studies in Quantitative Reasoning teaches quantitative methods in the context of how they are used. The course is distinctive in a number of ways. For instance, it has been designed and is taught by members of many departments: Biological Sciences, Economics, History, Mathematics, Statistics and Computation, Physics, Psychology, and Education. The course has been developed in discussions among faculty over the past few years. Its structure differs from that of a "regular mathematics course" in that it includes lectures, labs, and small discussion sections (of about fifteen students). A Macintosh computer facility has been created especially for Quantitative Reasoning (QR) students. The Macs are used for writing papers, doing data analyses, making graphical displays, and creating models of changing systems.

The primary difference, though, is one of approach. The course, which has no prerequisites, is designed to appeal to students with a broad range of academic interests and widely differing mathematical backgrounds—from math-phobes to calculus-philes. It is not a simple presentation of technical methods followed by practice problems. Instead, case studies from a variety of disciplines form the subject matter of the course. Different quantitative methods are introduced and used in the attempt to develop understanding of these examples. The emphasis is not on rote computation, but on reasoning; not on formulas, but on ways to construct and evaluate arguments. The goals are to help students strengthen their analytical skills and acquire a more confident understanding of the meaning of numbers, graphs, and the other quantitative materials that they will encounter in many subsequent courses, no matter what their majors.

Each semester three sections are offered with room for about 45 students. Distribution credit in mathematics is given for successful completion of the course. However, it is recommended that students work for a second semester to reinforce and extend their mastery of quantitative arguments. Continuations that particularly emphasize quantitative reasoning skills include an interdisciplinary Topics in Quantitative Reasoning, Elementary Data Analysis and Experimental Design, Psychology 201 (Statistics), and Calculus I.

Here is an outline of the three major units of the course.

I. NARRATIVE AND NUMBERS: SALEM VILLAGE WITCHCRAFT

Witchcraft in seventeenth-century New England forms the central problem for investigation. The major project is to write a paper formulating and discussing a hypothesis about the relationship between wealth and power as reflected in the historical records for Salem Village during the 17th century.

This first section concentrates on what can be called "exploratory data analysis," that is, the search for meaningful patterns in numerical data. There is heavy emphasis on graphical and other methods as tools for finding and presenting patterns. The overall goal is to facilitate the assignment of meaning to a set of data, stressing the process of translation between quantitative patterns and plausible explanations. Hypothesis testing is one method studied to safeguard against building up a theory on the basis of numerical coincidence or mere chance.

Skills:	Concepts:
contingency tables	cross-classification
percents	standardizing comparisons
bar graphs	defining meaningful categories
mean, median	choosing variables
percentiles	constructing an argument
expected value	simulation
chi-square	probability as a ratio (relative frequency)
using statistical software	bias
probability as a measure of surprise	logic of hypothesis testing
	null and alternate hypotheses

II. MEASUREMENT AND PREDICTION: SAT SCORES AND GRP

Aptitude and achievement test scores, grade point average, choice of major and other background information for recent Mount Holyoke graduates form a rich data set for investigation. Students formulate and test hypotheses and study the relationships between hypotheses, formal models, predictions, and actual results.

This section concentrates on hypothesis testing, experimental design, and modelling. The ideas and techniques of measuring differences, the identification of critical and missing variables, and the role of disconfirming evidence in the construction of an argument are emphasized. Important statistical concepts such as scales of measurement, distributions, averages, variation, correlation, test reliability, and test validity are introduced.

All skills and concepts from Unit I are reinforced, plus:

Skills:	Concepts:
xy plots	measurement
standard deviation	variability
equation of line	correlation: strength of relationship
slope	correlation: spread about best-fitting line
correlation	prediction
best fitting line	$(\text{correlation})^2$: predictive advantage
$(\text{correlation})^2$	correlation vs. causation
linear models	missing variables and confounding factors
	correlation and hypothesis testing
	reliability and validity

III. Rates of Change: Modelling Population and Resources

The rate of change of a quantity is the focus of this section. Here case studies are drawn from the sciences, particularly ecology. The effect on a population of birth and death rates, the spread of disease, and the decay of radioactive waste are but a few of the examples in which rate of change is a natural quantity to measure. Laboratories help develop the concept of rate of change by modelling changing systems over time.

Rate of change is a concept that is usually covered in calculus courses, but its usefulness in understanding a variety of situations makes it a natural topic of study. As in the two earlier sections of the course, the formulation of hypotheses, the translation of data into argument, and the construction of models are used to further understanding.

Skills:	Concepts:
slope as rate of change	rates of change
arithmetic growth	linear models
geometric growth	exponential models
net change	limited growth (logistic)
graph reading	rate equations
simple algebra	mathematical modelling
using *Stella*	simulation

The Laboratory

The laboratory is a crucial element in the teaching of Case Studies in Quantitative Reasoning. Each week students attend a three-hour computer laboratory in addition to the lecture and two discussions. A special laboratory equipped with Macintosh Pluses is set aside for their use. Laboratories are taught (and laboratory reports are graded) by a laboratory instructor. The laboratory instructor also holds some office hours (supplementing faculty office hours), participates in weekly staff meetings, and attends lectures and discussions. After many variants, this arrangement has proved to be very satisfactory. Students have enough time and support to do the work we expect of them, the load on faculty teaching the course is manageable, and the laboratory instructor gives continuity as the faculty involvement changes from semester to semester.

The Macintoshes were chosen because they are extremely user-friendly and also because excellent software is available for them. For the first two units of the course a spreadsheet-style statistical package called *StatView 512+* by Abacus Concepts, Inc. is used. Its attractive features include the ability to handle both categorical and numerical variables, to permit the use of meaningful verbal labels for variables, and to create varied visual displays of data.

For the third unit, students use *Stella* by High Performance Systems. By the use of clever visual icons, *Stella* permits the student to model dynamical systems of considerable complexity without the explicit use of calculus. The central visual metaphor is plumbing: quantities that grow or shrink appear as the contents of "bathtubs." "Pipes" feed into and out of the tubs, each with a "valve" which determines the rate of increase or decrease. These rates may, in turn, depend on other quantities in the system. For example, a population's birth rate (the valve on the in-flow pipe) may be a constant (arithmetic growth) or may be a constant multiple of the current size of the population (geometric growth). In the latter case, an arrow is drawn from the population tub to the valve, and the valve's flow rate may then be defined as the desired constant times the population size.

The typical pattern in each unit is that the first two laboratory assignments are highly structured and require formal laboratory reports. The third laboratory assignment is a somewhat less structured preparation for that unit's paper. An informal written report on the third lab—in effect, a very rough draft of the paper—is brought to the faculty instructor for a conference before the final version of the paper is prepared. In the last week of the unit, the laboratory is simply open for students to drop in as needed. Only the first unit deviates from this pattern; it lasts five weeks, so the first laboratory is an introduction to the machines themselves.

Over the three years since the course was first offered, more and more students have had some prior computer experience. However, many students still begin with no experience—and often a great fear of computers. Even these most fearful students become confident computer users by the courses' end.

Appendix D: Voices from the Focus Group Conversation

You pose extremely difficult questions. First, let me emphasize that I do not think Sloan has found the answers or even acceptable answers. Sloan has generated some interesting experiments (Mount Holyoke, Wellesley, Grinnell, for example), but these are designed for talented students who need to be enticed into mathematics. The examples are probably not useful with average college students (or those below average) who dislike math.

While the key problem is, of course, in the pre-college arena, we in colleges must face the remedial task in areas such as problem interpretation and formulation, probability models, and estimation—areas where students show so little understanding of the basic characteristics of simple operations.

—John Truxal, SUNY at Stony Brook

I need to address ideas within a context—I guess I am a "situationalist." Thus, I have to define the situation from which I am responding before I respond. For example,

1. What would be outcomes of schooling (K–4, 5–8, 9–12, 13+) in the area of quantitative literacy?
2. What populations would we be addressing?

As scientists, we are used to responding to "data" (i.e., facts) but here we don't have them. What is known about quantitative literacy at this time? What does research tell us in this area? For example, is it reasonable to assume that the general public perceives quantitative literacy to be the province of the select few? Do we have any measurement on quantitative skills as they are interpreted by the general public?

In order to formulate needs to be addressed over the next five years we need to know where we are relative to quantitative literacy. Thus, my first "need" for the next five years is a well-defined research program that describes where we are.

To continue in this vein, we need an oversight committee that endures for sufficient time to gather data and to make sure that the research is helping to define where we are as a quantitatively literate nation. As I continue on this thought, I realize that in order to define the research needed, we need some commonly accepted definition of quantitative literacy. Thus, we need a definition of quantitative literacy, or at least some measurable entities associated with this concept.

So now I need to describe the behaviors that I would expect of a quantitatively literate person—or society. From these behaviors we can decide whether or not people have them, we can formulate means to "educate" people to have them, and we will be on our way to a five-year (or 500-year?) plan.

1. What is the effect of technology on proliferation of data?
2. What is the effect of technology on understanding concepts involved in data analysis?
3. How can we educate our population to the sophistication needed to understand decision making in the context of uncertainty?
4. How much is ignorance of quantitative skills a matter of beliefs?
5. What would be a core curriculum for quantitative literacy, K–13+?
6. How could quantitative literacy concepts be used as the environment in which to teach (re-teach?, develop?) arithmetic skills and concepts?
7. How can we educate the public to understand "modelling" as a legitimate problem-solving technique?
8. What computer technology will be needed to answer the above questions?
9. What mathematics research must be supported to answer the above questions?
10. Are women and minorities affected (differently?) by lack of quantitative literacy skills?
11. What effect does a decision relative to quantitative literacy have on the preparation of all teachers? of mathematics teachers? of elementary teachers ? of secondary teachers? of university teachers?
12. What research agenda do we set up to answer these questions?

These are my questions—from all contexts!

—Katherine Pedersen, Southern Illinois University

On my campus, we are gearing up for "writing across the curriculum," so that is on my mind. And I contend that one can simply replace "writing" with "math" in virtually everything I have read about writing across the curriculum, and get a true statement.

—Robert Bernhardt, East Carolina University

It seems to me that it would be more useful to decide *what we want to mean* by quantitative literacy than to try to discover *what is meant* by it, e.g., in comparison with mathematical literacy. The main question, I think, is: what things (skills, concepts, behaviors) in this general area should students be expected to learn? The total answer might serve as an "extensive" definition of quantitative literacy. After we have that answer, it will be time to talk about courses, curricula, etc.

—Donald Bushaw, Washington State University

My feeling is that we need to look at "concepts in context." Contexts, i.e., application areas, will change with time and fashion. Today, we might do space, robotics, etc., but tomorrow (even five years from now) this will change. The key is to have the contexts relate to student interest, daily life, and likely work settings. The concepts, likely, will have more staying power. Undoubtedly we'd all have different lists, but mine would include: chance, model, cardinality, dimension, distance (area and volume), and data, among others. I find this "concepts in context" a useful way to approach quantitative literacy.

—*Sol Garfunkel, COMAP*

I see two problems concerning quantitative literacy:

1. That the average college-educated person have a "reasonable" appreciation for the power and importance of mathematics in our society and culture; and
2. That the average college-educated person have a certain level of mathematical skills.

I think these two goals are often confused, and often work at cross purposes.

—*Robert Bernhardt*

I do feel that there is some "truth" in defining quantitative literacy as understanding and communicating relationships—and other such words. Some questions and comments:

1. Are we discussing quantitative literacy or mathematical literacy?
2. Are we creating a mathematics course? Is this a means rather than an end?
3. To me quantitative literacy and mathematical literacy are *not* the same.
4. If we are looking at how to develop quantitative literacy in a classroom situation then we are looking at technology and communication. We are looking at data collection and analysis—maybe even more than "statistics." We are looking at *all* estimation skills—we are looking at communication of data and interpretation of this data, making inferences, justifying results, communicating results—and using technology to clarify, depict, manipulate, etc.
5. There is no question that deciding whether or not information communicated by quantitative expressions is reasonable and relevant is an important part of quantitative literacy.
6. I must share my support for a hands-on technologically enriched curriculum in which a person *becomes* quantitatively literate.
7. Must we not, however, specify some behaviors of the person who is quantitatively literate?

—*Katherine Pedersen*

I begin with a little history of my involvement with quantitative literacy issues—all from the perspective of a statistician, not a mathematician.

I have been involved with an NSF-funded project called Quantitative Literacy for about ten years now, and I thought it interesting that this Focus Group has the same title. Our quantitative literacy project was organized through a joint committee between NCTM and the American Statistical Association (ASA) with the goal of providing curriculum materials and in-service training so that mathematics teachers in the secondary schools could effectively incorporate basic concepts of statistics and probability into their teaching of mathematics. The project has been successful beyond our wildest imaginings and formed much of the background for the NCTM *Standards* committees as they wrestled with the problem of how to enhance mathematics education. Thus the *Standards* does, indeed, reflect a data analysis perspective not only in the statistics strand but also in other strands such as algebra, measurement, and functions. By beginning with a real problem of interest to the students, collecting data pertinent to the problem, and then analyzing the data as objectively as possible, one can motivate and illustrate many of the basic concepts in secondary school mathematics. (On a more creative day, not a Monday morning, I might say "all of the basic concepts.") The numbers with which students work must have a context (they then become data) which they can understand and view as relevant to their lives.

You might be interested in our working definition of quantitative literacy: *A complex of skills and knowledge associated with the collection, display, and analysis of data.* This definition is much broader than what most would take as a definition of statistics, but it attempts to include all of the ways that students may interact with data throughout their lives.

I might add, at this point, that American industry has finally caught on to the importance of statistical process control to maintain and improve quality and productivity. SPC methods use elementary data collection and analysis techniques much in the spirit of quantitative literacy. Industries teach these techniques to all levels of employees by inhouse courses that involve much hands-on work with real data. The techniques are largely graphical and are used by many with poor mathematics backgrounds. So, I think we have something to learn from the experiences of industry and can, in turn, develop some ideas that would be beneficial to the long term goals of industry. Many industries would like to find ways to work cooperatively with colleges and universities.

So, what about my broader thoughts on quantitative literacy? To be literate means to be able to read and write, to understand what others are trying to convey, and to convey information to others. Thus, quantitative literacy must mean to be able to understand what others are attempting to convey in numerical form and to convey information to others in numerical form. I used the term "numerical form" rather than "mathematical form" intentionally. I think our basic goal is to build an understanding of numerical information—number sense, if you will—and not an understanding of abstract symbolism. The latter would be nice, but is a luxury at this stage.

To combine the ideas expressed above, I think we can and should teach people "number sense" by emphasizing applications with real data that come from their own experiences in the world around them. This approach makes use of mathematics, but in the context of application—real application. The goal is to help people to think correctly about numbers and use data to help make intelligent decisions in life. All of standard secondary school mathematics can be incorporated here, as well as most introductory college mathematics.

Arithmetic: Constructing and interpreting data tables involves fractions, percent, ratio and proportion, averages of different types, etc. Also, estimation is important to the understanding of computations. (Did my calculator give me the correct value of the mean? Is this the mean I want?)

Algebra: I view algebra as an understanding of relationships, most of which can be understood without memorization of an algorithm or appeal to abstract notation. I know some college students who cannot figure out the gasoline mileage they are getting on their cars. I once had a secretary who often asked me something like the following: "If the fringe benefits rate is 25% and we are paid a total of $10,000 for the summer, how much of that is actually salary?" I believe students can be taught how to reason through such problems by concentrating on experiences with real data. So what if ten different students reason out the solution in ten different ways?

Geometry: Many of the measurement problems with which we are confronted in everyday life have a geometric implication (how much fence to buy to enclose my lawn, how much fertilizer to buy to cover my lawn, how to find the shortest distance from home to work, etc.). At another level, the graphs we use to convey information have a geometric interpretation. In fact, geometric skills are much under-utilized in practical approaches to problem solving.

So, all the basic quantitative ideas can involve data. I would even suggest that we go one step further and try to teach people how to deal with data that comes from a sample. (Sampling variability, sampling distribution, sampling error for proportions and means are some key examples.) Polls and the interpretation of experimental data (especially health-related data) are so common in the media that it behooves us as educators to try to help people deal with the problem. (Did the recent report from the Physician's Health Study actually say everyone should take aspirin to reduce the risk of heart attack?) These ideas can also be conveyed without the formal structure of confidence intervals and hypothesis tests.

A related goal should be in the area of "Helping people to be life-long learners." I'm not sure how one goes about doing this, but part of it must involve inciting curiosity about numerical phenomena. I think people tend to be curious about numerical things (what are my chances of winning the lottery?) but think the solutions are somehow mysterious and beyond them. Thus, we need to provide simple ideas for handling numbers that do not involve messy algorithms and can be seen as widely applicable building blocks to number sense. (A jackknife, not a surgeons scalpel?)

I have not addressed, except indirectly, the issue of appreciation of mathematics. I believe this is important so that parents, legislators, and others can evaluate the importance of mathematics education. Although I have not thought through this idea real carefully, I think appreciation can be taught through data as well. One way to extend the elementary notions of data analysis mentioned above is through the building of models. We can see that aspirin reduces the risk of heart attack in some people, but a more comprehensive view of how the risk is affected by age, weight, race, and a myriad of other variables can only be obtained through a statistical model. We can see data on the reliability of a component of a space shuttle but the reliability of the entire shuttle can be estimated only through a complex mathematical model. Perhaps students can learn to appreciate the power of mathematics by understanding the simpler components well enough to envision the role of a more detailed model, even if they have no interest in constructing the model themselves.

—Richard Scheaffer, University of Florida