Help students choose appropriate courses. For example, pre-calculus is frequently chosen inappropriately. Ordinarily, students not intending to study calculus should be discouraged from registering for pre-calculus and encouraged to choose courses more appropriate for their future mathematical needs. Carrying out this recommendation requires liaison work with other departments to improve placement and to clarify the true needs of their programs. Similarly, appropriate placement in other introductory courses requires conferring with departments whose students take those courses, educating academic advisors, and perhaps making use of placement tests. In institutions where students' academic plans are very fluid in the first two years, it is especially important that students take introductory mathematics courses that will flexibly equip them for future study of mathematics should their plans change.

B. Students majoring in partner disciplines

Partner disciplines vary by institution but usually include the physical sciences, the life sciences, computer science, engineering, economics, business, education, and often several social sciences.[37] It is especially important that departments offer appropriate programs of study for students preparing to teach elementary and middle school mathematics. Recommendation B.4 is specifically for these prospective teachers.

Many of the courses taken by students majoring in the partner disciplines are also taken by students who may choose to major in the mathematical sciences. (Sometimes they are the same students.) Obviously, the needs of these prospective mathematical sciences majors also warrant careful consideration.

The recommendations in this section are heavily influenced by the findings of the Curriculum Foundations project (see Appendix 2). The resulting reports, published in *The Curriculum Foundations Project*: *Voices of the Partner Disciplines*, can assist mathematical sciences faculty in discussions with faculty from other departments, serving both as a guide and a resource for collaboration.

B.1. Promote interdisciplinary collaboration

Mathematical sciences departments should establish ongoing collaborations with disciplines that require their majors to take one or more courses in the mathematical sciences. These collaborations should be used to

- ***Ensure that mathematical sciences faculty cooperate actively with faculty in partner disciplines to strengthen courses that primarily serve the needs of those disciplines;***
- ***Determine which computational techniques should be included in courses for students in partner disciplines;***
- ***Develop new courses to support student understanding of recent developments in partner disciplines;***
- ***Determine appropriate uses of technology in courses for students in partner disciplines;***
- ***Develop applications for mathematics classes and undergraduate research projects to help students transfer to their own disciplines the skills learned in mathematics courses;***
- ***Explore the creation of joint and interdisciplinary majors.***

The explosive growth and development of scientific disciplines over the past half century have resulted in unprecedented pressures on curriculum. Partner discipline faculty want to provide students with the necessary background and tools to understand current developments, but degree programs ordinarily limit

[37] See Appendix 2 for a list of the disciplines represented at the Curriculum Foundations workshops.

offerings to the same number of courses required fifty years ago. As a result, more topics and more sophisticated material are packed into courses of partner disciplines, magnifying the reluctance to increase allied field requirements in mathematics.

Yet many recent developments in partner disciplines are based on mathematics that is not presently included in the required courses. For example, the report of the National Research Council, *BIO 2010: Transforming Undergraduate Education for Future Research Biologists,* notes that most biology majors study calculus, and some may take a statistics course. However, the report goes on to say that these students would benefit greatly from also studying "discrete mathematics, linear algebra, probability, and modeling." The report notes, "While calculus remains an important topic for future biologists, the committee does not believe biology students should study calculus to the exclusion of other types of mathematics."[38]

When faculty teach a course in one of the partner disciplines that could use mathematics beyond that required for their majors, what do they do? Typically, either they incorporate their own brief introduction to the mathematical material or they decrease the mathematical content of their course. Both accommodations deprive students of the opportunity to deepen their scientific knowledge. Ultimately, such decisions adversely affect the health of all disciplines.

Programs for pre-service teachers are also under pressure. The increased technological sophistication of the world we live in has created a need for a more mathematically and technologically literate citizenry. Preparing future teachers to help address this need is a critical challenge for mathematics departments.

Mathematics courses, textbooks, and curricula changed dramatically during the twentieth century, often in response to the physics and engineering of the time. Further changes are needed in the twenty-first century to respond to the needs of the expanding set of disciplines for which mathematics now plays an increasingly important role. Strategies for initiating productive conversations with faculty in partner disciplines to foster such changes include:

1. visiting the courses of colleagues in partner disciplines, and discussing observations,
2. requiring a writing component within existing mathematics courses, and using this requirement as an opportunity to work with technical writing faculty,
3. inviting colleagues in partner disciplines to discuss curricular issues over lunch or at department social gatherings,
4. expanding the boundaries of faculty research to encourage collaboration with partner disciplines.

As noted in the discussion of Recommendation 4, constrained resources limit the capacity of departments to tailor separate courses for specific partners, so compromises are necessary. Nonetheless, the benefits of interdisciplinary cooperation far outweigh the potential difficulties.

At most institutions, only a portion of the mathematics faculty is interested in vigorously pursuing collaboration with faculty from other disciplines on course development and cooperative teaching arrangements. It is vital that departments support this activity with appropriate professional development (including opportunities for graduate student, part-time, and temporary teaching staff). As stated in the *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences,* "Participation in programs designed to assist college teachers is particularly important for members of a department who sometimes teach outside of their own mathematical sciences discipline." The *Guidelines* also say, under

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[38]The report of the National Research Council, *BIO 2010: Transforming Undergraduate Education for Future Research Biologists*, The National Academies Press, 2002, contains an excellent description of the need for interdisciplinary education for future life science researchers. The description of the type of mathematics that these students need to study is from page 5 of the Executive Summary. The report is available at `books.nap.edu/books/0309085357/html/5.html`.

Faculty Evaluation and Rewards, "In accordance with departmental mission and priorities, some consulting and other professional activities may advance the scholarship and teaching of faculty members and the department."[39] For some faculty, interdisciplinary collaboration on course development and cooperative teaching constitutes such a professional activity, and their departments should therefore consider this activity positively in recommendations for tenure, promotion, and financial compensation. It also is important to the future vitality of the department that interest in and aptitude for such collaborations be regarded as a plus when departments consider hiring new faculty.

Strengthen existing courses that primarily serve the needs of partner disciplines. Existing mathematics courses should be carefully rethought in order to better serve the current mathematical needs of partner disciplines. As an example, several representatives of partner disciplines at the Curriculum Foundations workshops expressed a desire for greater emphasis on differential equations in the first year, while others indicated a desire for matrix algebra through eigenvalues and eigenvectors. Most representatives indicated some desire for mathematics courses that incorporate appropriate technical tools, and all wanted greater emphasis on modeling and conceptual understanding of mathematical topics.

The high priority placed on modeling and conceptual understanding is not surprising. The partner disciplines need students who can apply mathematics to questions in their fields, reformulate such questions using the appropriate mathematical tools, and use appropriate technology to carry out actual computations. Students can perform these tasks accurately and confidently only if they understand the related mathematical structure.

Representatives from chemistry gave an urgent plea for an earlier introduction of multivariable calculus because multidimensional topics underlie concepts in introductory chemistry courses. Even more colleagues expressed dissatisfaction with the level of student understanding of concepts—geometric and otherwise—in three dimensions. Specifically, they stressed the need for early introduction of vectors in two and three dimensions, geometric and graphical reasoning, linear systems, and three-dimensional visualization skills.

An important lesson from the discussions with representatives of other disciplines at the Curriculum Foundations workshops is that different disciplines use language in different ways, and it is essential to be specific to avoid misunderstanding what the issues are. Moreover, constrained resources may force compromises. But experiences at several institutions, large and small, indicate that communication opens the door to progress.

At some institutions, mathematics courses to support the programs of partner disciplines are taught within those disciplines—for instance, statistics in biology departments and discrete mathematics in computer science departments. When this is the case, mathematical sciences departments can approach these departments to signal a willingness to be involved in the further development of the courses. Such an overture may be especially welcome when the partner departments are having difficulty staffing the courses. Often the reason the courses were originally developed was because mathematicians were not interested or were not teaching the topics in a way or at a level that was appropriate for the students in the partner discipline.

Unfortunately, students who take mathematics courses outside the mathematics department are unlikely to think of taking follow-up courses within mathematics, even though such a course might enhance their understanding of the other discipline. Regardless, introductory courses housed in partner departments should be carefully examined to determine whether they can reasonably be allowed as prerequisites for higher-level mathematics courses. Mathematics faculty should attempt to publicize the mathematics courses for which courses taught in other departments can serve as preparation.

[39] *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences*, MAA, 2001; `www.maa.org/guidelines/guidelines.html`. See C.2.e.f and C.8.e.

Determine appropriate computational techniques. New topics and greater numbers of conceptual and modeling problems cannot be successfully added to a fixed-length course without eliminating or reformulating existing material. Curriculum Foundations workshop participants provided insight about the kinds of topics and problems that might be de-emphasized or eliminated.

It clearly is not appropriate for mathematics departments to be cavalier in reducing the teaching of computational skills. Students must develop a certain level of computational skill in order to understand mathematics and apply it effectively. But the specific skills that are important have become less clear since the advent of technology.[40] Careful analysis by mathematics faculty, working in concert with faculty from partner disciplines, can help determine how best to modify existing courses. Joint examination of problem sets, textbook assignments, and examinations—as well as cross-disciplinary classroom observations—can lead to agreements about essential mathematical material. Based on comments made by Curriculum Foundations workshop participants, mathematics faculty may well be surprised by the views of their colleagues about what topics are inessential.

Certainly the views of colleagues in partner disciplines should not overwhelm all other considerations. Specifically, mathematics faculty can speak to what is pedagogically feasible in teaching mathematics—especially concerning the material one can expect students to absorb in a fixed amount of time. Moreover, the content of these courses must also serve the needs of the prospective mathematics majors who are enrolled. The spirit of this recommendation is to involve both mathematics and non-mathematics faculty in discussions about modifying courses, keeping open minds and flexible attitudes about various possibilities.

Develop new courses. Reports from the Curriculum Foundations workshops emphasize the importance of an elementary, non-calculus-based **statistics and data analysis** course developed through careful consultation with faculty from partner disciplines. Comments from participants suggest that such a course would be an attractive addition to their major course requirements—even if it increased the total hours required for graduation. With today's computing tools, instructors can use real data sets from a variety of applications, websites, and information from popular newspapers and magazines to motivate topics and illustrate concepts. Use of real data helps students understand the need for random sampling in survey work and the principles of designing good experiments. Students can be introduced to the difficulties of collecting meaningful data and experience the "messiness" that often accompanies data analysis, such as that involved in decisions about whether to include an outlier in an analysis and how to measure the influence of an individual observation. Analyzing real data also serves as a constant reminder of the questions that led to the original data collection.

The report from computer science indicates that entry-level, non-calculus-based **discrete mathematics** —including a serious introduction to logic and proof, sets, relations, and functions—is essential for computer science majors. In 2001, the two major computer science societies (Association for Computing Machinery (ACM) and the Computer Science Division of the Institute of Electrical and Electronics Engineers (IEEE)) released recommendations that included discrete mathematics as part of computer science "core knowledge." These societies' report, *Computing Curriculum 2001*,[41] recommended at least one discrete mathematics course in the first year, and preferably a two-course sequence. Mathematical sciences departments that currently do not offer a course satisfying this recommendation should meet with colleagues in computer science to discuss options, utilizing team-teaching when possible. A discrete mathe-

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[40] For instance, physics workshop participants wrote: "…effective introductory physics instruction requires that students have complete confidence in their ability to understand and calculate simple derivatives and integrals. The more esoteric and complicated topics are not of use in the introductory courses. Furthermore, they are forgotten (and must be re-learned) by the time students reach upper-level courses." (*The Curriculum Foundations Project: Voices of the Partner Disciplines,* p. 116.)

[41] *Computing Curriculum 2001* is available at `www.computer.org/education/cc2001/final/`.

matics course or course sequence can be developed to serve mathematics majors, as well as computer science students, by smoothing the transition from calculus-level mathematics to more theoretical and abstract upper-division courses.

Course development in partnership with other disciplines is also important at the **intermediate and upper levels**. Especially in connection with joint and interdisciplinary majors, mathematics faculty may consider working with faculty in other areas to add courses such as mathematical biology, computational mathematics, the geometry of computer graphics, and financial mathematics.

Determine appropriate uses of technology. Participants in the Curriculum Foundations workshops understood the importance of technology, and took for granted that mathematics courses would incorporate technology to some degree. Mathematics courses should teach students how to choose the appropriate method of calculation for a given task—whether mental, paper-and-pencil, or technological—and should stress intelligent and careful interpretation of results.

Perhaps more surprising is that spreadsheets are the most utilized technology for a large number of partner disciplines. Although individual workshop reports stopped short of recommending spreadsheets as the primary technological tool in mathematics instruction, their widespread use is relevant to the technology choices made in mathematics courses that primarily serve other disciplines. A related observation was that very few workshop participants reported the use of graphing calculators in their own disciplines. Therefore, the use of calculators in mathematics courses should be for pedagogical, logistical, or budgetary reasons, not to support the use of that particular hardware in other disciplines. The bottom line: mathematics faculty need to be aware of the preferred tools of the partner disciplines in general and at their own institutions.

Develop applications and undergraduate research projects. Students often have difficulty seeing the relationships between problems in non-mathematical disciplines and material studied in mathematics courses. Interdisciplinary cooperation can help students begin to overcome this transfer problem from mathematics courses to partner discipline courses. Most faculty in partner disciplines believe that exposing students in mathematics courses to discipline-specific contexts for various mathematical topics has a positive effect on their ability to transfer knowledge between courses.

Indeed, the participants in the Curriculum Foundations workshops were so excited by the possibility of increasing the use of real models in mathematics courses that many volunteered to help develop such models. This enthusiasm suggests that if mathematicians at individual institutions invite colleagues to help develop disciplinary applications for mathematics courses, there is a good chance that they will receive a positive response. However, it is the responsibility of mathematics faculty to take the initiative in seeking such collaborative arrangements.

Interdisciplinary lectures—in both mathematics and partner discipline courses—also can serve as a means for increasing students' abilities to transfer knowledge between disciplines. An engineer or economist presenting specific applications to a mathematics class increases the sense of relevance for students. Conversely, a mathematics professor coming into an engineering or economics course when specific mathematics topics are being reintroduced reinforces earlier mathematics instruction (see Illustrative Resources for examples).

Explore creation of joint and interdisciplinary majors. Many students in fields such as biology, computer science, or economics do not think of pursuing a double major with mathematics, either because the course requirement total is too great or because they think of a mathematics major as not relevant to their career path. Joint and interdisciplinary majors are increasingly important avenues for such students to prepare themselves for advanced work in the newer, more mathematically intensive parts of their disciplines.

By working together, mathematics faculty and faculty in partner disciplines can develop programs that are specially tailored to appeal to students whose primary interests lie outside mathematics but who enjoy mathematics and can benefit from more mathematics than is required in their major. Such programs often require more courses than the combination of a major in the other discipline and a minor in mathematics, but fewer courses than a double major.

Joint and interdisciplinary majors have the double benefit of improving the mathematical knowledge of students in partner disciplines and increasing enrollment in intermediate and upper-level mathematics courses. In some cases, these programs utilize existing mathematics courses, without any alteration. In other cases, modifications may be considered, such as adding examples and applications. Mathematics faculty should be open to these kinds of modifications, recognizing that mathematics majors also benefit from learning more about the relationships between mathematical structures and real-world models. Occasionally, the introduction of joint or interdisciplinary majors may require development of entirely new mathematics courses. Such new courses are a positive sign of the vitality of the field, as well as an opportunity for professional growth (see Illustrative Resources for examples).

B.2. Develop mathematical thinking and communication

Courses that primarily serve students in partner disciplines should incorporate activities designed to advance students' progress in

- ***Creating, solving, and interpreting basic mathematical models;***
- ***Making sound arguments based on mathematical reasoning and/or careful analysis of data;***
- ***Effectively communicating the substance and meaning of mathematical problems and solutions.***

Although mathematical thinking and communication have already been discussed (see Recommendation 2), students majoring in the partner disciplines have additional needs in this area. The partner disciplines value the precise, logical thinking that is an integral part of mathematics. Faculty who participated in the Curriculum Foundations workshops commented frequently that all disciplines look to mathematics courses to enhance students' abilities to reason logically and deductively, but that they want this ability developed in a context that increases understanding of underlying concepts. While faculty from disciplines other than computer science were wary about the use of formal proof, especially in early courses, the computer scientists specifically requested that lower-division discrete mathematics courses include an introduction to formal proof.

Different disciplines may require different levels and types of precision and logical thinking. For instance, business and economics often require qualitative analysis and a broad overview, while engineering may require more detailed and formal analysis. The natural sciences may require heuristic arguments and drawing sound conclusions from data, while computer science and software engineering require an ability to use logic to do simple proofs.

Logical, deductive reasoning and rigorous proof are at the heart of mathematics. It is important that these fundamental aspects of mathematics are embedded in the undergraduate curriculum, while at the same time not burdening the courses that serve partner disciplines with an overemphasis on formal proof. The correct balance can and should be determined through consultation with colleagues in partner disciplines.

B.3. Critically examine course prerequisites

Mathematical topics and courses should be offered with as few prerequisites as feasible so that they are accessible to students majoring in other disciplines or who have not yet chosen majors. This may require modifying existing courses or creating new ones. In particular,

- ***Some courses in statistics and discrete mathematics should be offered without a calculus prerequisite;***
- ***Three-dimensional topics should be included in first-year courses;***

- ***Prerequisites other than calculus should be considered for intermediate and advanced non-calculus-based mathematics courses.***

As much as possible, prerequisite requirements for mathematics courses should be designed to make the courses accessible to students majoring in other disciplines and to students who have not yet chosen majors. For instance, many institutions now offer a version of calculus that incorporates review of precalculus along with calculus topics. Other institutions have found that material developed through the calculus renewal movement enables students to master the basic concepts and application of calculus, despite limitations of algebra skills that might have impeded their success in a more traditional course.

Statistics and discrete mathematics. Faculty in partner disciplines do not want a calculus prerequisite for introductory statistics. The fundamental ideas of statistics, such as the omnipresence of variability and the ability to quantify and predict it, are important subjects that can be studied without sophisticated mathematical formulations. In particular, the notion of sampling distribution—which underlies the concepts of significance testing and confidence interval—is challenging enough on its own to justify a first course in statistics. Similarly, computer scientists have requested that the introductory course in discrete mathematics be accessible to computer science students in their first year, and a number of mathematics departments now offer such a course or course sequence.

Three-dimensional topics. As noted earlier, faculty in several partner disciplines want greater emphasis on vectors in two and three dimensions, geometric and graphical reasoning, linear systems, and three-dimensional visualization skills. None of these topics require a full year of calculus to be understood, and all can be embedded in first-year courses in a variety of ways. Some institutions have structured their calculus sequence so that multivariable calculus can be reached in the second semester rather than having to wait until the third. This approach is particularly useful to students intending to major in chemistry or economics.

Advanced courses. One of the most serious consequences of overly rigid prerequisite structures is that they unnecessarily deprive mathematical sciences departments of potential students for intermediate and advanced mathematics courses. Some prerequisites are necessary, but generally not as many as are typically required. For example, if the only real requirement for a certain course is "mathematical maturity," then courses other than calculus may provide it. Many versions of discrete mathematics courses stress development of students' ability to prove and disprove mathematical statements, and some even contain brief introductions to number theory, combinatorics, or matrix theory. Such a course could be accepted as a prerequisite for linear algebra, number theory, abstract algebra, or combinatorics. Mathematics faculty should also explore allowing mathematics courses taught in other departments to serve as prerequisites for their own courses (see section B.1).

B.4. Pre-service elementary (K–4) and middle school (5–8) teachers

Mathematical sciences departments should create programs of study for pre-service elementary and middle school teachers that help students develop

- ***A solid knowledge—at a level above the highest grade certified—of the following mathematical topics: number and operations, algebra and functions, geometry and measurement, data analysis and statistics and probability;***
- ***Mathematical thinking and communication skills, including knowledge of a broad range of explanations and examples, good logical and quantitative reasoning skills, and facility in separating and reconnecting the component parts of concepts and methods;***
- ***An understanding of and experience with the uses of mathematics in a variety of areas;***
- ***The knowledge, confidence, and motivation to pursue career-long professional mathematical growth.***

The teacher preparation recommendations of this *Guide* have been informed by *The Mathematical Education of Teachers (MET),* [42] a recent CBMS report that presents detailed and carefully considered guidelines concerning the education of future teachers of mathematics. Mathematics faculty and departments are advised to study *MET* in its entirety.[43]

The need for sound mathematical preparation of prospective teachers is acute.

> The predicted huge shortage of mathematically trained teachers at all grade levels, the growing levels of state-mandated assessment and testing, and the perceived weak preparation in mathematics of entering college freshmen all point to the fact that teacher preparation must be one of the highest priorities of all institutions of higher education, and especially the state-supported ones.[44]

Mathematical sciences departments in institutions that offer programs leading to elementary and/or middle school certification face a substantial challenge, and the stakes could not be higher. These departments bear the major responsibility to provide future teachers with the solid knowledge of mathematics they require. But every mathematical sciences department also has a role to play. In a recent report, the Committee on Science and Mathematics Teacher Preparation of the National Research Council reminds higher education faculty that:

> ... all colleges and universities, including those which do not have formal teacher education programs, should become more involved with improving teacher education because the nation's teacher workforce consists of many individuals who have matriculated at all types of two- and four-year institutions of higher education. Although many of these schools do not offer formal teacher education programs, virtually every institution of higher education, through the kinds of courses it offers, the teaching it models, and the advising it provides to students, has the potential to influence whether or not its graduates will pursue careers in teaching.... Science, mathematics, and engineering departments at two- and four-year colleges and universities should assume greater responsibility for offering college-level courses that provide teachers with strong exposure to appropriate content and that model the kinds of pedagogical approaches appropriate for teaching that content.[45]

The phrase "solid knowledge of mathematics" means mathematical training that promotes real understanding of mathematical concepts at a level needed to be able to teach them effectively. This statement presents a risk and a challenge.

First, the phrase "solid knowledge of mathematics" does not mean the same thing as preparation for the further study of college-level mathematics. For example, while prospective teachers need knowledge of algebra, as detailed in the next subsection, the traditional college algebra course with a primary emphasis on developing algebraic skills does *not* meet the needs of elementary teachers as described in B.4 and *MET*.

Second, much work remains to be done in specifying exactly *what* underlying mathematics is required. Examples are needed of problems for pre-service teachers that can be used to improve students' ability to

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[42] *The Mathematical Education of Teachers*, volume 11 of the Issues in Mathematics Education series of the Conference Board of the Mathematical Sciences, AMS and MAA, 2001, available at `www.cbmsweb.org`.

[43] The *MET* authors write, "This report is not aligned with a particular school mathematics curriculum, although it is consistent with the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* as well as other recent national reports on school mathematics."

[44] "The Mathematics Major Overview," by D. Sánchez, in *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?,* MAA Reports, 2001, p. 82.

[45] National Research Council. *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium*. Washington, DC: National Academy Press, 2001, pp. 9–12.

reason mathematically and to evaluate children's thinking. It is necessary to know what foundational knowledge pre-service teachers need to learn in their content courses to equip them to judge the validity of their students' arguments/justifications and work. And determining what algebraic concepts and skills are essential for elementary teachers to know and be able to use is still an issue not resolved—but is one that urgently needs to be addressed.

Mathematics faculty should collaborate with mathematics educators in the school (or department) of education to maximize the effectiveness of their efforts. Two-year college faculty are important in the preparation of teachers; CBMS 2000 data (pp. 125–126) show that about half of all two-year colleges offer a special course for pre-service K–8 teachers, and enrollments rose 13% in these courses between the 1995 and 2000 CBMS reports. Given the diversity in content, number of hours, and number of courses offered at four-year and at two-year colleges, collaboration is needed to effect smooth articulation, as well as coherence, in the mathematical preparation of prospective teachers. "Greater communication and cooperation is necessary among all stakeholders in the mathematics preparation of teachers."[46]

According to *MET*, adequate development of the abilities specified in Recommendation B.4 requires at least the equivalent of 9 semester hours of appropriate mathematics courses for pre-service elementary teachers, and 21 semester hours for students who will (or may at some time) teach mathematics at the middle school level. Special mathematics courses for pre-service elementary school teachers have a long-established history, and several innovative approaches are currently being developed to improve them. In addition, many institutions are now creating courses and minors for pre-service middle school mathematics specialists because courses designed primarily for mathematics majors are not likely to reach or effectively serve these students.

Middle school mathematics (grades 5–8). Nationwide there is a shortage of teachers prepared to teach middle school mathematics. As a result school systems have been forced to assign individuals to middle school mathematics classes who originally prepared to teach at the elementary level or who prepared to teach another subject in middle school. According to the National Center for Education Statistics,[47] of those middle school teachers for whom mathematics is their *main assignment*, 53.1% do not have the equivalent of either a major *or minor* in mathematics or mathematics education.

Many of these teachers therefore do not have a sufficient understanding of what they are teaching and compensate by teaching mathematics as a set of rules or procedures not as a way of thinking (a hazard at any level of instruction). The *MET* discussion of middle grade mathematics highlights the following conclusion of L. Resnick, "Good mathematics learners expect to make sense of the rules they are taught…"[48] It is precisely this habit and desire to "make sense" that future middle school teachers need to develop and subsequently to share with their students.

While no department can meet every challenge there is a massive need for well-prepared middle school teachers. This *Guide* urges departments, particularly those in colleges that prepare substantial numbers of teachers, to develop and offer programs that will attract large numbers of potential middle school mathematics teachers. Some departments have designed minors in mathematics to prepare these students appropriately. See the Illustrative Resources for details.

Elementary mathematics (grades K–4). The mathematical preparation of elementary school teachers poses its own unique challenges. Since elementary school teachers are usually generalists, teaching all sub-

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[46] *The Curriculum Foundations Project: Voices of the Partner Disciplines*, p. 145.

[47] National Center for Education Statistics NCES 2002-603.

[48] Resnick, L. B. (1986). The development of mathematical intuition. In M. Perlmutter (Ed.), *Perspectives on intellectual development: The Minnesota Symposia on Child Psychology* (Vol. 19, pp. 159–194). Hillsdale, NJ: Erlbaum.

jects in the elementary curriculum, many pre-service elementary teachers do not approach their future career with any particular interest in mathematics. Indeed, "... many teachers were convinced by their own schooling that mathematics is a succession of disparate facts, definitions, and computational procedures to be memorized piecemeal. As a consequence, they are ill-equipped to offer a different, more thoughtful kind of mathematics instruction to their students." (*MET*, p. 17)

There needs to be greater awareness that elementary mathematics is rich in important ideas and that its instruction requires far more than simply knowing the "math facts" and a handful of algorithms. "It is during their elementary years that young children begin to lay down those habits of reasoning upon which later achievement in mathematics will crucially depend. ....When the goal of instruction is to help children attain both computational proficiency and conceptual understanding, teaching elementary school mathematics can be intellectually challenging." (*MET*, p. 15) Mathematics programs for prospective K–4 teachers must emphasize the intellectual depth of the elementary mathematics curriculum and provide the pedagogical tools to effectively teach this critical material to elementary school children.

A solid knowledge of mathematical topics at a level encompassing much more than computational or symbolic fluency is essential for teachers of mathematics at any level. Teachers must be "able to represent concepts in multiple ways, explain why procedures work, or recognize how two ideas are related." They must also be "able to solve problems and to make connections among mathematical topics or between mathematics and other disciplines."[49]

- The study of **number and operations** provides opportunities for prospective elementary teachers "to create meaning for what many had only committed to memory but never really understood." (*MET*, p. l8) Middle school teachers need to understand the algorithms for multiplication and division of integers, rational numbers and decimals well enough to teach them "in ways that help their students remember them without resorting to thoughtless, rote techniques and that serve as a foundation for later learning." (*MET*, p. 29)
- Although formal study of **algebra and functions** is not part of the elementary curriculum, state and national standards and the National Assessment of Educational Progress (NAEP) tests require algebraic thinking from the early grades on. Prospective elementary teachers should be able to represent and justify general arithmetic claims, recognize properties of operations, appreciate that a small set of rules governs arithmetic, and be able to represent and interpret functions by graphs, formulas and tables. (*MET*, p. 20) Middle school teachers need to be able to work with algebra as a symbolic language, a problem-solving tool, and as generalized arithmetic. They need to understand and use variables and functions, especially linear, quadratic, and exponential functions. They should be able to give a rationale for common algebraic procedures. (*MET*, p. 30)
- **Geometry and measurement** have become more prominent in the elementary grades, and prospective elementary teachers need competence in visualization skill, understanding of basic shapes and their properties, and the process of measurement. Teachers in the middle grades also need to understand congruence and similarity, be able to make conjectures and prove or disprove them, understand, derive and use measurement techniques and formulas, and connect geometry to other topics. (*MET*, p. 32)
- **Data analysis and statistics and probability** are new to most prospective elementary teachers. They need to learn to pose questions that can be addressed by data, design and conduct data investigations, represent data in multiple ways, interpret results, and make judgments under conditions of uncertain-

[49] *The Curriculum Foundations Project: Voices of the Partner Disciplines*, p. 147.

ty. (*MET*, p.23) Middle school mathematics teachers need to understand random sampling or random assignment to treatments, explore and interpret data by observing patterns and departures from patterns, and understand what it means to draw conclusions with measures of uncertainty.

Mathematical thinking and communication skills are vital for teachers. Their needs are well served by the guidelines given in Recommendation 2, but in some respects they must go farther to acquire the abilities necessary to recognize and shape the mathematical thinking of their students. In order for prospective teachers to move beyond the attitudes and strategies they often bring from their own experiences, "they need to have classroom experiences in which they become reasoners, conjecturers, and problem solvers." (*MET*, p. 56)

The uses of mathematics should be included in the teaching of mathematics at every level. Recommendation 3 applies to courses for prospective teachers, as it does to all mathematics courses, and, suitably interpreted, it also applies to the teaching of school mathematics. Teachers need a wide repertoire of examples that illustrate the power of mathematics. Without exposure to significant applications a teacher will be effectively crippled, unable to understand accurately—and hence unable to convey—the primary motivations for many K–8 mathematics topics. It is a challenge to develop the solid knowledge of mathematical topics specified in B.4 in just three courses focused on the K–8 curriculum. However, employing applications to motivate and illustrate the material helps sustain students' interest and assists them in building that solid knowledge. Experiences with mathematical modeling afford excellent opportunities for translating between mathematical and verbal description, clarifying assumptions, and interpreting results.

Career-long professional mathematical growth is necessary to achieve and maintain excellence in the mathematics classroom. "In some countries where student achievement is high, teachers, alone and in groups, spend time refining their lessons and studying the underlying mathematics." (*MET*, p. l0) College mathematics faculty have little power to create these opportunities for professional development, but faculty can ensure that prospective teachers' college experiences in learning mathematics prepare and motivate them to take advantage of future opportunities to strengthen their understanding of mathematics.

C. Students majoring in the mathematical sciences

The recommendations in this section refer to all major programs in the mathematical sciences, including programs in mathematics, applied mathematics, and various tracks within the mathematical sciences such as operations research or statistics. Also included are programs designed for prospective mathematics teachers, whether they are "mathematics" or "mathematics education" programs, although requirements in education are not specified in this section.

Although these recommendations do not specifically address minors in the mathematical sciences, departments should be alert to opportunities to meet student needs by creating minor programs—for example, for students preparing to teach mathematics in the middle grades.

These recommendations also provide a basis for discussion with colleagues in other departments about possible joint majors with any of the physical, life, social or applied sciences.

Two premises underlie the recommendations in this section: the number of bachelor's degrees granted in the mathematical sciences—including joint majors— should be increased, and the population of potential majors has changed.

The number of bachelor's degrees granted in the mathematical sciences should be increased.

- Many other disciplines, including computer science, business, economics, the life sciences, medicine, the physical sciences, and engineering, have much greater mathematical content now than in