department offers a choice of several tracks within the major, advisors have the added responsibility of providing students with ample information even when students do not ask many questions. This individualized approach to advising requires that no advisor be assigned too many advisees.[63]

Create a welcoming atmosphere and offer a co-curricular program of activities to encourage and support student interest in mathematics. Departments should provide space for informal student contact. Students who develop good working relationships with peers are more likely to succeed in their mathematics courses and to sustain interest and motivation. A student lounge equipped with work tables, writing boards and (comfortable) chairs should be available for student/student as well as student/faculty contact. Shelves containing current student-friendly mathematical journals as well as selected mathematics books would add to the atmosphere of such a room.

Departments should also offer opportunities for mathematics majors to interact with one another through a campus organization. This might be a mathematics club, a student chapter of the MAA or a chapter of Pi Mu Epsilon. Such an organization can serve as a catalyst for a number of retention activities — mentoring programs, attendance at local or national meetings, and sponsorship of movies and presentations. Such an organization also gives students a sense of ownership in their major.

D. Mathematical sciences majors with specific career goals

D.1. Majors preparing to be secondary school (9–12) teachers

In addition to acquiring the skills developed in programs for K–8 teachers, mathematical sciences majors preparing to teach secondary mathematics should

- ***Learn to make appropriate connections between the advanced mathematics they are learning and the secondary mathematics they will be teaching. They should be helped to reach this understanding in courses throughout the curriculum and through a senior-level experience that makes these connections explicit.***
- ***Fulfill the requirements for a mathematics major by including topics from abstract algebra and number theory, analysis (advanced calculus or real analysis), discrete mathematics, geometry, and statistics and probability with an emphasis on data analysis;***
- ***Learn about the history of mathematics and its applications, including recent work;***
- ***Experience many forms of mathematical modeling and a variety of technological tools, including graphing calculators and geometry software.***

The teacher preparation recommendations of this *Guide* have been informed by *The Mathematical Education of Teachers (MET),*[64] a recent CBMS report that presents detailed and carefully considered guidelines concerning the education of future teachers of mathematics. Mathematics faculty and departments are advised to study MET in its entirety.[65] Prospective teachers of secondary school mathematics

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[63] This recommendation and discussion are adapted from the 1991 CUPM report, which reappears as Appendix E of *MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences,* and from the recommendations in the *MAA Guidelines for Programs*.

[64] *The Mathematical Education of Teachers*, volume 11 of the Issues in Mathematics Education series of the Conference Board of the Mathematical Sciences, AMS and MAA, 2001, available at `www.cbmsweb.org`.

[65] The *MET* authors write, "This report is not aligned with a particular school mathematics curriculum, although it is consistent with the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* as well as other recent national reports on school mathematics."

should complete a major in the mathematical sciences.[66] Recommendation D.1 is thus a companion to the recommendations for all mathematical sciences majors in Section C. It does not, however, specifically address course work in education.

It bears repeating that the postsecondary education of future teachers of mathematics is an important responsibility of mathematicians. The need for qualified teachers of mathematics is great, and students potentially interested in teaching mathematics should be encouraged and supported, as well as prepared to be effective teachers. David Sánchez argues that mathematics faculty concerned about the quality of mathematics teaching in the schools should work to ensure

> …that the future teachers they produce have the best possible training and strong, conceptual understandings of mathematics. Hopefully, those teachers will be the leaders of reform by working within the educational system through mathematics and leadership workshops, becoming master teachers, and exemplifying the best in content mastery and pedagogy. Their undergraduate training will be a cornerstone of that effort.[67]

As its authors acknowledge, the *MET* recommendations "outline a challenging agenda for the education of future secondary school mathematics teachers," and each department will have to adjust its program, in ways that fit its mission and resources, to meet the challenge. Fortunately, these adjustments are likely to serve *most* majors well.

Connect advanced and secondary mathematics. The assumption that the traditional curriculum for a mathematics major is adequate preparation for students preparing to teach secondary school is simply incorrect. It is not enough for secondary mathematics teachers to have an understanding of advanced mathematics—they must also be able to connect their advanced coursework to the material they will teach.[68] For example, secondary school teachers should

> …be able to answer the following typical high school students' questions, in ways that are both mathematically sound and also accessible and compelling to a 15 year-old:
> - Why does a negative times a negative equal a positive?
> - Why do I switch the direction of the less than symbol when I multiply both sides by a negative number?
> - In every triangle that I tried in *Sketchpad,* the angles add up to 180°. I don't need to do a proof, do I?
> - I am not convinced that .99999… = 1.
> - How do I know parallel lines never intersect?
> - I think that the number 1 has three different square roots: 1, –1, and .99999999.
> - I am sure that .99999999 is a square root of 1 because when I multiply it by itself on my calculator I get 1.00000000.[69]

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[66] As in Section C, this includes programs designed for prospective secondary mathematics teachers, whether they are called "mathematics" or "mathematics education" programs.

[67] "The Mathematics Major Overview," by D. Sánchez, in *CUPM Discussion Papers on Mathematics and the Mathematical sciences in 2010: What Should Students Know?*, MAA Reports 2001, p. 82.

[68] This point was made in 1996 in the presentation "On the education of mathematics teachers," by H. Wu at the Mathematical Sciences Research Institute. A later adaptation is Wu's "On the Education of Mathematics Majors," in *Contemporary Issues in Mathematics Education*, ed. by E. Gavosta, S.G. Krantz and W.G. McCallum, MSRI Publications, Volume 36, Cambridge University Press, 1999, pp. 9–23. Both papers (and related material) are available at `www.math.berkeley.edu/~wu`.

[69] "The Mathematical Education of Prospective Teachers of Secondary School Mathematics: Old Assumptions, New Challenges," by J. Ferrini-Mundy and B. Findell, in *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?*, MAA Reports, 2001, p. 34.

Highlighting these mathematical connections in upper-level courses is thus an important component of the education of future secondary mathematics teachers. The *MET* and other resources include many suggestions for modifying traditional mathematics courses to help prospective teachers make these connections. For example, a linear algebra course can include study of the rigid motions of the plane and relate this topic to Euclidean geometry. Assignments in algebra or number theory "might ask for the use of unique factorization and the Euclidean algorithm to justify familiar procedures for finding common multiples and common divisors of integers and polynomials" or "ask for each step in the solution of a linear or quadratic equation to be justified by a field property."[70] Attention to the connections between advanced material and topics studied in secondary school would be valuable for all majors. (See Illustrative Resources for additional ideas.)

Departments should require prospective secondary mathematics teachers, like all mathematical sciences majors, to complete a senior-level intensive project (Recommendation C.4). This project should help future teachers explore further the relationship between advanced mathematics and the mathematics they will be teaching.

Include specific topics in mathematics. Although many different choices of topics can provide strong preparation for mathematical sciences majors, prospective teachers need to study advanced material related to the topics they will likely teach.

- ***Abstract algebra and number theory***. "The algebra of polynomial and rational expressions, equations and inequalities has long been the core of high school mathematics."[71] Therefore, students preparing to teach secondary mathematics need a solid understanding of number systems, structures underlying rules for operations, and the use of algebra to model and solve real world problems. Indeed, all majors, not just prospective teachers, should know *why* our number systems behave as they do. They should be able to explain specific facts like $1/(1/5) = 5$. They should be able to explain the relationship between the algorithms of arithmetic for numbers in decimal representation and the arithmetic of polynomial algebra. An abstract algebra course for prospective teachers should include the study of rings and fields with explicit attention to the rational numbers—not just as an abstract set of equivalence classes of ordered pairs of integers, but as the rational numbers appear in the secondary school curriculum. The twin problems of integer factorization and primality testing can be used to explore the structure of the integers. A traditional 'groups, rings, fields' course can be adapted, with appropriate elaborations and emphases, but there are other possibilities too. For example, an algebra course might be organized around "the solutions of the three classical construction problems and the explanation of why the roots of some polynomials of degree greater than or equal to five cannot be extracted from the coefficients by use of radicals."[72]
- ***Analysis***. Analysis provides "a rigorous foundation for future teaching about functions and calculus. Informal notions about Euclidean space, functions and calculus that undergraduates have used for several years can be given sound formal definitions."[73] Although mathematics majors have a lot of exposure to functions, many develop only a formulaic approach to their use. Future secondary teachers should develop an understanding of the main characteristics of the basic functions in school mathematics. For example, they should know that the key features of polynomial functions are their zeroes

[70] *The Mathematical Education of Teachers*, page 40.
[71] *The Mathematical Education of Teachers*, page 40.
[72] "On the Education of Mathematics Majors," H. Wu.
[73] *The Mathematical Education of Teachers*, page 134.

and the shapes of their graphs, that the exponential and logarithm functions owe their importance to modeling natural growth and decay as well as converting addition to multiplication and *vice versa*, and that the trigonometric functions are needed to model all periodic phenomena. Also, in practice school mathematics deals primarily with the rational numbers, not the real numbers, but the connection between the two number systems is rarely made clear. One goal of the study of analysis by prospective teachers should be to address this "missing link" to prepare them to handle this implicit but important issue in the classroom.[74]

- ***Discrete mathematics.*** The *MET* recommends that the program of prospective secondary mathematics teachers include work in discrete structures and their applications, design and analysis of algorithms, and the use of computer programming to solve problems. More specifically, the *MET* states that prospective secondary mathematics teachers should be exposed to graphs, trees and networks, enumerative combinatorics, finite difference equations, iteration and recursion, and models for social decision-making. Future teachers can encounter these topics in a variety of courses as well as in a discrete mathematics course. For example, iteration and recursion might appear in a calculus or differential equations course; finite difference equations and models for decision-making might be in a modeling course; graphs, trees and networks might be in a "bridge" course designed to help students make the transition from introductory to advanced courses.
- ***Geometry.*** Teachers of secondary school geometry need to understand the concepts of Euclidean geometry. In particular, they need to have well-developed geometric intuition—built up with hands-on experiences such as ruler and compass constructions, making three-dimensional models of polyhedra and curved surfaces, and/or creating tilings of the plane with polygons —so they can help their future students acquire it. Teachers should also know the logical structure of the subject. For example, they should know the relationship between what assumptions are made about parallelism of lines in a geometry and proving a theorem about the sum of the angles of a triangle. Teachers should be able "to use dynamic drawing tools to conduct geometric investigations emphasizing visualization, pattern recognition, conjecturing, and proof."[75] Teachers need to have facility with proof themselves in order to be able to lead their students from intuition and the drawing of pictures to logical reasoning. Prospective teachers should become fluent with "proofs by local axiomatics": starting with a fixed collection of (geometric) facts and deducing from them something interesting, and they should have some experience with the more subtle and difficult deductions from axioms.[76] Teachers should also be acquainted with recent developments in geometry and its uses.
- ***Statistics and probability with an emphasis on data analysis.*** Data analysis and probability form one of the core strands in the current K–12 curriculum. "Because statistics is first and foremost about using data to inform thinking about real-world situations, it is critical that prospective teachers have realistic problem-solving experiences with statistics."[77] In addition, the *MET* recommends that secondary mathematics teachers should have experience with exploring data, planning a study, anticipating patterns, using statistical inference, and applying appropriate technologies. A course meeting the ASA/MAA guidelines (as described in the discussion of Recommendation C.3 and elaborated in Illustrative Resources) would serve prospective teachers well.

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[74] Comment to CUPM by H.Wu.

[75] *The Mathematical Education of Teachers*, page 41.

[76] Comment to CUPM by H.Wu.

[77] *The Mathematical Education of Teachers*, page 44.

Learn about the history of mathematics and its applications. Prospective secondary mathematics teachers need to know the history of the subjects they will teach. It gives them a better appreciation for the struggle that goes into mathematical advances. It enables them to identify conceptual difficulties and to see how they were overcome. And it enriches their own understanding of the mathematics they will teach and the role it has played in human history. This history of mathematics needs to include modern developments, as in Recommendation 3.

> Because of its enormous practical value, mathematics is frequently taught as a collection of technical skills that are applicable to specific tasks and often presented without reference to the intellectual struggles that led to contemporary understanding.… many non-European cultures have made sophisticated and significant contributions to mathematics.… [F]uture secondary mathematics teachers will be well-served by deeper knowledge of the historical and cultural roots of mathematical ideas and practices.[78]

While some departments may offer courses in the history of mathematics, others will explore these themes in a variety of classes, including perhaps the senior-level intensive project (C.4).

Experience mathematical modeling and a variety of technological tools. Prospective teachers should experience many forms of mathematical modeling, such as differential and difference equations, linear statistical models, probability models, linear programming, game theory, and graph theory. Work with a model should include attention to assessing how well models fit the observed facts. The *MET* discusses modeling real phenomena and using models to draw conclusions, largely in the context of stochastic models. The use of stochastic models is indeed important. Experience with deterministic models is also important for prospective secondary school teachers. Such models can illuminate and motivate the algebra and geometry in the secondary mathematics curriculum and help secondary students see and appreciate the broad usefulness of mathematics in the world. In addition, modeling is an area in which the importance of stating one's assumptions clearly comes to the front: it reinforces the point that clear thinking and precision are crucial for problem solving as well as deductive reasoning. As in B.2, learning to create, solve and interpret a variety of discrete and continuous models is important for prospective teachers.

Not only should prospective teachers use technology to build their own understanding of mathematical ideas and as a tool to solve problems in their own courses, as in Recommendations 5 and C.2, they also have to be prepared to use these and other tools appropriately and effectively as teachers. The discussion above of geometry and data analysis alluded to the value of technology for learning these topics and solving meaningful problems. The study of families of functions is found in many middle and secondary school curricula, often in a modeling context. The graphing and table-generating features of calculators or spreadsheets can be used effectively to study such functions. As recommended in the Curriculum Foundations workshop on teacher preparation, "Future teachers should be able to use tools such as tiles, cubes, spheres, rulers, compasses, and protractors to deepen their understanding of the mathematics they will have to teach, including 2-D and 3-D geometry and measurement. The use of electronic drawing tools such as the Geometer's Sketchpad and Cabri is also recommended for use in geometry courses for both elementary and secondary teachers. The dynamic capabilities of these tools allow both students and teachers to test conjectures relatively easily."[79]

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[78] *The Mathematical Education of Teachers*, p. 142.

[79] *The Curriculum Foundations Project: Voices of the Partner Disciplines*, p. 150.

D.2. Majors preparing for the nonacademic workforce

In addition to the general recommendations for majors, programs for students preparing to enter the nonacademic workforce should include

- ***A programming course, at least one data-oriented statistics course past the introductory level, and coursework in an appropriate cognate area; and***
- ***A project involving contemporary applications of mathematics or an internship in a related work area.***

More than 90% of mathematical sciences majors go directly into the workforce after graduation (see Appendix 4). Many of the recommendations in Part I for all mathematics students and in Section C for all mathematical sciences majors are of great importance for students entering the work force. The thinking skills described in Recommendations 2 and C.1 are vital. For example, the website `www.careers-in-business.com` lists the skills and talents required for work as a consultant, rating them in importance from medium (initiative, computer skills) to high (people skills, teamwork, creativity, ability to synthesize) to *extremely* high (just one: analytical skills). The Society for Industrial and Applied Mathematics (SIAM) lists as one of the most important traits of an effective employee outside academia "skill in formulating, modeling and solving problems" (`www.siam.org/mii`). A participant in a January 2000 CUPM focus group representing a variety of employers in business, government, and industry said his consulting firm "likes to hire math people" for the way they think, for their ability to formulate problems mathematically, and for their modeling skills.

Also mentioned on these two web pages and in virtually every discussion of preparation for the workforce is the importance of oral and written communication skills, as well as the ability to work well as a member of a team. Experience with a project of the kind recommended in C.4 is also highly valued, especially if the project requires working as a member of a team.

Programming, statistics, and cognate coursework. The 1993 NSF survey of recent graduates found nearly half of mathematical sciences bachelor's degree recipients in nonacademic positions spent substantial time working with computer applications. This is certainly no less true today. Programming skills enable a user to take advantage of the full power of many software packages and are a valuable selling point with potential employers. In "Business View on Math in 2010," Patrick McCray writes, "When evaluating a resume to decide whether or not to extend an invitation for a job interview to an applicant, … I look at an inventory of skills, such as fluency with programming languages, computer environments.… Applicants who do not have the potential to be productive within two weeks of being hired are not even considered for an interview." [80]

The view of programming as consisting only of if-then, do-while, and a few other structures is several decades behind the current state of the art. Object-oriented languages such as *C* were a jump up in abstraction and complexity from *FORTRAN*, and *Pascal*, *Java*, and *.NET* are another jump up. If a person needs to learn a programming language in the future, the best basis is to know one of the state-of-the-art languages of today.

In the list of mathematics used in different occupations on the careers webpage assembled by three of the professional societies in the mathematical societies,[81] statistics and probability appear repeatedly. The 1997 and 1993 national surveys of recent recipients of bachelor's degrees in mathematical sciences find large numbers of students employed in sectors where data analysis and probabilistic modeling are impor-

[80] Paper by P.D. McCray of Pharmacia Corporation in *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?*, MAA Report, 2001, p. 75.

[81] The careers page of the MAA, AMS, and SIAM is at `www.ams.org/careers/mathapps.html`.

tant. (Also, see the ASA web page at `www.amstat.org/careers/`.) Courses in statistics and probability are valuable assets to students seeking to enter the workforce after graduation, especially if these courses have involved work with substantial data sets and have required reports explaining and interpreting the results of the data analysis. Having cognate coursework that makes significant use of statistics or modeling further enhances a student's chances of finding employment.

Numerous institutions have developed combinations of courses that give majors access to a wide range of interesting and well-paid jobs. In addition, many courses beyond those listed above are valuable to students seeking non-teaching jobs after graduation (see Illustrative Resources).

Projects and Internships. Working in business, industry, or government helps students make good decisions about the kind of work they might enjoy and do well. Sometimes an internship can even open the door to specific employment opportunities. Internships teach students valuable lessons about how the knowledge and skills they gain in school can be deployed in the workplace. And, internships look good to prospective employers. McCray writes, "My advice to prospective applicants: While a student, get work experience in the same line of business in which you wish to seek employment after graduation…It is easier to gauge how a person will fit into a specific work environment if that person has already taken advantage of opportunities to explore similar ones."[82]

Departments often need the assistance of their institution's careers office to help students find good internships. Even institutions far from urban settings should have access to data bases of opportunities for undergraduates. Departments need to work cooperatively with careers offices to build resources and to take maximum advantage of resources that exist. Students also can help each other by reporting on their internship experiences at mathematics club meetings, brown bag seminars, or other informal sessions (see Illustrative Resources for examples).

A project based on an applied problem of contemporary interest is also valuable. Analyzing the problem, choosing appropriate mathematical or statistical tools, collecting relevant data, writing a clear description of the analysis and solution of the problem, and interpreting the results for a non-mathematical audience all build and reinforce skills needed in the workplace. Collaboration with colleagues in other departments and, where possible, with employers in the area can strengthen the project and add to the authenticity of the experience.

D.3. Majors preparing for post-baccalaureate study in the mathematical sciences and allied disciplines

Mathematical sciences departments should ensure that

- ***A core set of faculty members are familiar with the master's, doctoral and professional programs open to mathematical sciences majors, the employment opportunities to which they can lead, and the realities of preparing for them;***
- ***Majors intending to pursue doctoral work in the mathematical sciences are aware of the advanced mathematics courses and the degree of mastery of this mathematics that will be required for admission to universities to which they might apply. Departments that cannot provide this coursework or prepare their students for this degree of mastery should direct students to programs that can supplement their own offerings.***

There are many graduate and professional degree programs that actively solicit mathematics majors. In addition to the traditional master's and doctoral programs in pure and applied mathematics, many disci-

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[82] *CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know*, page 75.

plines with rich mathematical content offer professional master's degrees. These include statistics, financial mathematics, operations research and industrial engineering, biomedical imaging, molecular structure and pharmaceutical sciences, natural resource management, and cryptography. There also are doctoral programs in many of these fields. Medical schools, law schools and business schools also recognize mathematics as strong undergraduate preparation for their programs. Departments need to be aware of the varied opportunities for further study that are available to their students.

The professional (terminal) master's degree in the mathematical sciences is gaining both visibility and importance. As reported in *SIAM News*, there is "a new class of professional science master's programs, many offering rich interaction with business, industry, and government."[83] The Alfred P. Sloan Foundation and the William M. Keck Foundation have supported the establishment of many new multidisciplinary master's programs with strong mathematical components, including applied financial mathematics, applied statistics, financial mathematics, industrial mathematics, mathematical sciences, mathematics for entrepreneurship, quantitative computational finance, and statistics for entrepreneurship.[84] The Society for Industrial and Applied Mathematics and the Commission on Professionals in Science and Technology are tracking the performance of these programs. More information will be available on their websites.

A core set of faculty members should be identified to advise students interested in the growing number of cross-disciplinary graduate programs that combine mathematics with another subject. Students in these programs will need fluency in both mathematics and the partner subject. They will need to be able to handle both precision and approximation, modeling and analysis, problem solving and careful reasoning, communication with computers and with people. Because these fields are evolving rapidly, a strong intellectual foundation is essential to facilitate career-long learning.

Departments should be able to prepare students for the wide variety of post-baccalaureate programs within the mathematical sciences. These include mathematics, applied mathematics, statistics, applied statistics, actuarial science, operations research, and mathematics education. Requirements vary, but facility with the tools of calculus, statistics, differential equations, and linear algebra is basic to all of these programs. Facility implies more than having taken a course in that topic. It means that the student understands the ideas that lie behind the tools and is adept at solving problems that require those tools. It also means that the student knows when and how a particular mathematical idea is likely to be most useful.

The requirement that all mathematics majors should have some familiarity with at least one programming language applies as much to those intending to pursue graduate work as to those headed directly for employment outside academia. This might be programming in *Maple, Mathematica*, *MatLab*, *Java* or *C++*; programming in *SPlus*, *R*, or *SAS* is valuable for students planning graduate study in statistics. Departments should know which languages and professional software will be most useful for students heading into the various graduate programs.

Because these graduate programs and their requirements are so varied, it is important that certain faculty be designated to keep current on the requirements for different programs. These faculty can serve as liaisons with programs that have the potential to attract graduates from their institution. They should arrange to bring in representatives from different graduate programs including some of their own alumni who can talk about a variety of graduate experiences

***For majors intending to pursue doctoral work in the mathematical sciences*,** faculty must communicate the fact that facility with the tools of calculus, differential equations, and abstract linear algebra is funda-

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[83]"SIAM Survey to Target Professional Master's Programs," *SIAM News*, Volume 36, Number 1, January/February 2003.

[84]A website with links to all these programs can be found at `www.sciencemasters.com`.

mental, as is some knowledge of programming. It is not possible to prescribe exactly what preparation students will need; much depends on the program they wish to enter. For a doctoral program in statistics (and for some master's programs), an analysis course is essential; students taking analysis as undergraduates have an advantage for admittance to doctoral programs in statistics. For both master's and PhD statistics programs, a strong cognate area (i.e., depth through several undergraduate courses or a second major) is valuable (e.g., biology, economics, physical science).

Doctoral programs in pure mathematics almost always require their students to take graduate courses in analysis and algebra and to pass the demanding qualifying exams based on these courses. Departments that prepare students for doctoral work in mathematics should ensure that their students arrive in graduate school prepared to take such courses.

Doctoral programs in pure mathematics assume that incoming students are already familiar with the fundamental definitions, concepts, and theorems of analysis and algebra. They often presuppose additional knowledge of topics in complex analysis, geometry, and topology. There may be flexibility in that a student may be able to rely instead on knowledge of numerical analysis, logic, or another mathematical subject, but algebra and analysis are almost always central to the first year of graduate study. For example, the departmental website at the University of Illinois at Urbana-Champaign has a list of the basic topics with which they assume familiarity. For real analysis and abstract algebra, these consist of:

Real analysis: *Completeness properties of the real number system; basic topological properties of n-dimensional space; convergence of numerical sequences and series of functions; properties of continuous functions; and basic theorems concerning differentiation and Riemann integration.*

Abstract Algebra: *Modular arithmetic, permutations, group theory through the isomorphism theorems, ring theory through the notions of prime and maximal ideals; additional topics such as unique factorization domains and classification of groups of small order.*[85]

Some doctoral programs require additional topics in real analysis and abstract algebra or specify that preparation for their programs requires an undergraduate analysis class at the level of Rudin's *Principles of Mathematical Analysis* and an undergraduate algebra class at the level of Herstein's *Topics in Algebra*, while others will allow entering students to start with a preparatory course at the level of Rudin or Herstein.

Some programs have a wide variety of qualifying topics and students may have some choice of which qualifying examinations to take. In addition, some programs permit the student without the required background to catch up. Department advisors should advise students accordingly, including realistic appraisals of how much preparatory work will be needed before the student is ready to take required graduate courses.

Preparing students for doctoral work in mathematics requires more than exposing them to topics in real analysis and abstract algebra. Students need to be able to read and critique proofs. This skill includes the ability to determine how assumptions are used and to find counterexamples when any of the hypotheses are weakened. Students need to be familiar with the common techniques employed to prove results in analysis and in algebra, and they should be able to use these techniques to prove theorems they have not seen before. Above all, students heading into doctoral programs in mathematics need to be able to read with understanding the textbooks they will encounter written in the languages of analysis and algebra. This is a skill that departments should ensure that their students learn.

Participation in an REU[86] program is not a replacement for subject matter courses, but it can be good

[85] `www.math.uiuc.edu/GraduateProgram/intro.html#Background`. This should not be interpreted as an endorsement of these topics as the proper undergraduate preparation for a doctoral program in mathematics. It is simply an example of what one respected graduate program expects.

[86] NSF Research Experiences for Undergraduates.

supplemental preparation that introduces students to the excitement and hard work involved in doing mathematical research. There are other summer programs aimed at students considering doctoral study in mathematics and also semester-long programs for visiting students; see Illustrative Resources for details. Departments should be aware of these programs and direct promising students toward them.

Departments that cannot afford to provide the full preparation needed for the doctoral program to which a student aspires should consider providing options such as reading courses or cross-registration for courses at a nearby institution. They might recommend a transitional fifth year of study or transfer to another institution.

For some students, a master's degree in mathematics is an attractive option that permits them to "try out" graduate school as well as improve their preparation for doctoral study if they choose to continue. However, funding opportunities may be more limited for master's programs than for doctoral programs.