REPORT OF THE COMMITTEE ON THE UNDERGRADUATE MATHEMATICAL PROGRAM

1. General statement. The Committee on the Undergraduate Mathematical Program was appointed by President E. J. McShane in January 1953. Since it was directed to consider the problems of making available in our society the values of modern mathematics, the word *program* was used in its name rather than *curriculum*. The Committee was instructed to attack the problem with broader scientific and cultural objectives than could be expressed through another mere study of curriculum revision.

At the Kingston meeting in September 1953, the Committee reported to the Board of Governors that there exists a widespread dissatisfaction with the existing undergraduate program in mathematics, complemented by a remarkable unanimity of feeling about the nature of the deficiencies in the present program and the general nature of the program which should replace it. Believing that the reformation of college mathematics cannot be accomplished by the adoption of a new curriculum emanating from any committee, we recommended a widespread program of "doing" to overcome the inertia of the enormously ponderous structure which carries onward the present program with all of its deficiencies. The findings of the Committee and its recommendation to get on with the "doing" was adopted by the Board of Governors. The Committee, reorganized to include C. V. Newsom, who is also chairman of the important Joint Committee on Teacher Education in Mathematics, was charged with organizing the "doing" phase of the work.

This charge to go ahead with the much-talked-about revolution in the mathematics program is one which we would not dare accept as our own private responsibility. In all fields we must learn the ways in which mathematics can contribute to scientific and social thinking and support the existing technology. To this end we must enlist the aid of the men in all these fields who have the greatest wisdom and scientific insight; for the history of previous attempts at curricular revision in mathematics strongly suggests that they failed because these attempts were made upon a narrowly pedagogical and organizational basis, without the participation of the real intellectual leaders in mathematics and in the fields which use mathematics. In particular, we must find the men with ideas, who, by experimental writing and teaching, can beat into shape new courses and their textbooks, so that the rest of us can teach new and (we hope) better courses in mathematics. The Committee must guard against the adoption of arbitrary opinions of its own or of others in order that any new program or movement which we foster shall be true to the development of mathematics itself.

In the existing state of affairs there are a large number of conditions which bear upon any movement to modify the traditional program, whether by natural

forces or by arbitrary effort. From the scientific standpoint, one must take into account the extended mathematical needs of modern engineering and physical science. At the same time such sciences as econometrics, physiology, sociology, and genetics seem to demand, in part at least, entirely new mathematics. Moreover, the emergence of new mathematical technologies based upon high-speed digital automata presents serious problems of policy. For the intriguing gadgetry of these machines tends to obscure the potentially great magnitude of their impact upon men. This current development based upon the old subjects of symbolic logic, numerical analysis, and combinatorial theory is another example of the fact that the work of the creative mathematician tends to be applied much less directly than that of the empirical scientist and, even so, largely through the mathematical knowledge which technical workers in other fields acquire in school and college.

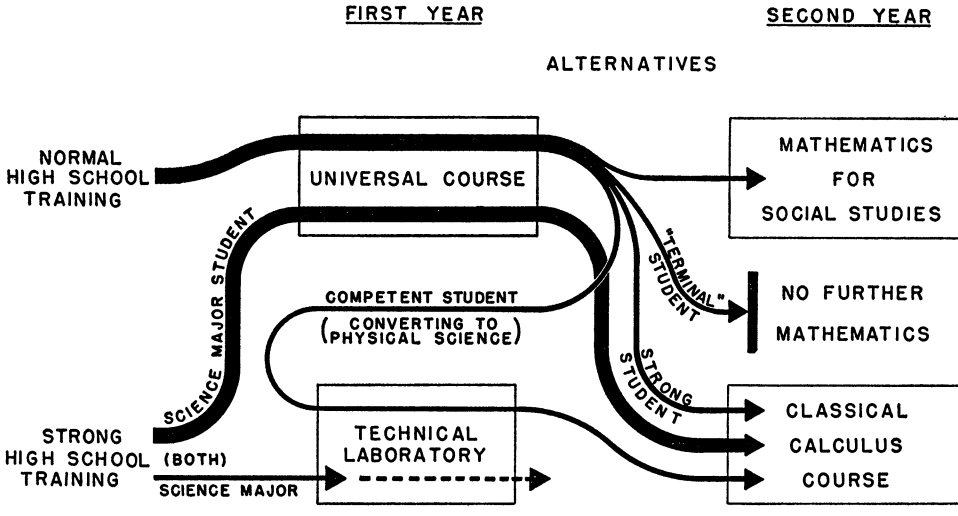
Thus, turning to the educational conditions affecting our problem, one finds that probably the central one is the desertion of elementary teaching by the best mathematicians, old and young. Besides this, there is the rigid sequential organization of the traditional program with each course depending heavily on technical prerequisites, the compartmentation in college programs with attendant difficulties for the student who transfers or changes program, the growing tendency to repeat high school courses in college, the terminal course idea, the emphasis upon cultural and liberal aspects in education today, the current popularity of logic, the recent studies of articulation of school and college, the incipient movement to train a special breed of college mathematics teachers by means of special education programs, and the complicated relationships of the policies of foundations, which might support the writing of new text material, to the publishing companies, and the royalty rights of authors. There are, of course, many mathematicians who still believe strongly in "the old time religion" and cannot see what this fuss is all about.

In view of this complex of inertial elements, it is the opinion of this Committee that, in mathematics, a broad, coordinated attack will be needed in contradistinction to the textbook revolution which was successful in the teaching of English literature. In elementary mathematics, new ideas introduced by local writing and teaching efforts have always turned out to be largely overpowered by the self-propagation of the traditional but retrogressive stock. Moreover, we believe that for very cogent reasons we must continue to seek *one, universal freshman course* for all reasonably qualified students, presupposing intermediate algebra and high school geometry, and ignoring the question of remedial courses for students who enter college without adequate mathematical training in school. Only by means of such a universal course can the best principles of liberal education be served. Only in this way can we avoid the error of forcing the immature student, upon entering college, to make choices which will seriously restrict his freedom of development in later years, and in this way we will attempt to make sure that the subjects taught are really the most valuable ones. We are confident that the mathematics which survives such a selection process from the common

intersection of all of demands of the various disciplines, including higher mathematics, will turn out to have the values of generality and abstractness which have always characterized good mathematics. Also we are confident that this selection process will naturally sift out essentially the same set of ideas which would be chosen as fundamental if one considered only the development of mathematics itself.

2. A universal freshman course and sequels. The proposed universal course would contain all of the theory prerequisite to any sophomore course in mathematics, covering this, with appropriate exercises and problem solving, in three hours per week for a year. In most schools of engineering, it should be supplemented by technical problem-solving work in additional "laboratory" class meetings.

To assure ourselves of the existence of a mathematics course for freshmen which meets these specifications, the Committee has attempted to construct one in outline. We do not propose our course as the solution but perhaps a schematic diagram might illustrate the general direction of our efforts. The "technical



laboratory supplement," which might be two hours per week, would presuppose a stronger high school training, perhaps three units, and this strong high school training, jointly with the universal freshman course, would form the prerequisite for the sophomore calculus. The laboratory itself would not be a prerequisite to sophomore calculus, though it could serve a remedial function which might fit many weakly prepared students. These supporting functions of the technical laboratory work are indicated by the dotted arrow pointing towards calculus. This scheme would continue to make essential use of strong mathematical training in high school without shutting the door to a good student who enters college without such training.

This Committee thought of the universal course as being made up of two subjects. (Arrange them as you will.) One subject would consist of functions, graphs and elementary calculus, covering differentiation and integration of polynomials, logarithms, exponential functions and general powers, with a minimum of formulas and with emphasis on ideas. The other subject would take off from set concepts, set up the fundamental language of theoretical mathematics and proceed, via some of the simpler abstract algebraic systems (but not the full real number system), to probability with emphasis upon the binomial distribution. If possible, this should lead to the sampling problem and here make a junction with statistics without actually getting into statistics. We believe that the universal course should contain little or no statistics but it should lay the foundation for this subject in a much more thorough way than the traditional course does. Engineers and physical scientists now need these probability ideas as well as the social scientists.

Now in this scheme, what becomes of the traditional college algebra, trigonometry, and analytic geometry? The essential algebra in college algebra would be covered in the universal course. Numerical trigonometry might be handled in the technical supplement. Analytic trigonometry might form the first subject of the sophomore calculus, which would be time enough for its uses in mechanics. Euclidean analytic geometry is well known to be not a proper prerequisite to calculus, which requires the more general geometry, the geometry of the cartesian product of the real line and the real line, *i.e.*, "graphs." This includes straight lines, slopes and the (weak) area of a rectangle. Thus the first real treatment of *Euclidean* analytic geometry would be in the sophomore course. (This beautiful old subject needs a proper revival in American college mathematics in a way which neither the hurried freshman nor sophomore courses permit!) Moreover, the graphs, or affine geometry, of the Universal Course, is not a negligible geometry.

The schematic diagram indicated a branching in the second year with a course called mathematics for social sciences and statistics appearing parallel to the traditional calculus. This is a long-needed development. For the calculus has not been a suitable universal second course. Actually, however, this alternate second course will probably turn out to meet many needs other than those of the social science student and potential statistician. For example, the prospective high school teacher might find it better for his needs than calculus. What the syllabus of such a new course should contain is an open question. Statistics? Multivariable algebra? Postulational models? Scales of measurement? Maximum problems in several variables?

Returning to the technical laboratory of the first year, we think of the first semester being devoted to numerical trigonometry and to numerical methods, with considerable practice in graphical methods and the use of tables of logarithms and exponentials. Worded problems would form an important part of it, with calculus drill beginning after mid-semester. Since relatively little calculus

would appear in the second semester of the universal course, the technical laboratory for the second semester would become largely a calculus drill course. Thus engineering students would enter the sophomore course in analytic geometry and calculus with almost a year's study of elementary calculus behind them, a good part of which would have been based upon a strong high school training.

By these means and by providing at minimum cost an appropriate outlet for those who want and need less technical mathematics, we hope to strengthen the foundations of the classical sophomore calculus course and enable it to be started at a relatively advanced point and to proceed into the methods and applications of calculus. This should clear the way for more vector methods, at least in the second semester, and perhaps permit the calculus to include more differential equations, mechanics, and numerical integration than has been possible in the past. Finally, we point out that the principle of the Universal Course provides for an efficient conversion of those students who discover, after entering college, that they want to study engineering or physical science. The so-called "terminal course" often makes this change of plan prohibitive in cost.

The Committee has nothing of its own to report on courses beyond the sophomore year. It has been seeking out interesting new courses which are being offered in various institutions, some of which are mentioned in Section 3 of this report. A number of the discussions of the Committee have centered on the revival of geometry.

3. Summary of known activities. For the answers to the questions about the proposed sophomore course in mathematics for social scientists, and other questions on mathematics basic to social science, our Committee is relying upon another group, *viz.*, the Committee on the Mathematical Training of Social Scientists which developed into the 1953 Summer Institute on Mathematics for Social Scientists at Dartmouth, Professor William G. Madow, Chairman. This very significant project was described at the Association Christmas Meeting in Baltimore by several members of the Institute: W. G. Madow, R. M. Thrall, R. R. Bush, and Howard Raiffa.* In addition to this group, K. O. May of Carleton College is working on an interesting book on mathematics for social scientists which has much new material. Also W. G. Madow is individually writing an elementary book on mathematics for social scientists. This is significant in view of the author's experience in the Dartmouth Summer Institute. Another body of material which might go into the Mathematics for Social Science is the semester course in statistics as constructed by S. S. Wilks for Princeton freshmen to follow a semester of calculus.

Since there existed no such Committee on the important questions relating to the mathematical training for the engineering of today and of the future, a joint committee was appointed by the Association and the American Society for

* See this MONTHLY, vol. 61, pp. 550–561.

Engineering Education. Professor G. B. Thomas of M.I.T. is Chairman. This Committee will search out the engineering leaders who have the knowledge and foresight to tell us what forms of mathematical training are likely to be most valuable in the engineering of the next twenty years. To assist this committee in its work it is hoped that some conferences on modern mathematics and modern engineering will be arranged, possibly with the sponsorship of the National Science Foundation. R. S. Burington, who is a member of the joint committee, and also president of the Mathematics Section of A.S.E.E., arranged an extremely interesting program on this subject at the 1954 summer meeting of A.S.E.E. in Urbana, Illinois. It is hoped that the proceedings will soon be available in print.*

At a somewhat higher level, training in applied mathematics has been the subject of a study directed by F. J. Weyl for the Division of Mathematics of the National Research Council. Two symposia were held in connection with meetings of the Society. Proceedings of these symposia will be available in print soon.** This work was supported by the National Science Foundation.

The idea of early introduction of calculus, inherent in the proposed Universal Course, is by no means new or radical. This is the form of the first college course implied by both of the studies of articulation of school and college supported by the Fund for the Advancement of Education. The first report appeared in *General Education in School and College*, Harvard University Press (1952). The mathematical part was reprinted in AMERICAN MATHEMATICAL MONTHLY.§ The mathematical part of the second study is known as the Brinkmann Committee, whose results were reported to the Association at the Baltimore meeting, December 1953.§§ The older textbooks which form prototypes for at least the first half of the Universal Course include those of F. L. Griffin, *Mathematical Analysis;* and Milne-Davis, *Introductory College Mathematics*, the latter being interesting also for its sections on numerical methods. The use of such books with an early introduction to calculus dispensing with the traditional "preparation" is increasing and meeting with good success, notably in the California schools, and in Minnesota, following the University of Minnesota. Other recent books on freshman calculus of special interest are those of D. E. Richmond of Williams College, E. G. Begle of Yale, and Karl Menger of Illinois Institute of Technology. Begle's book has for this level perhaps the most modern treatment of the formal theory available in print. Menger's book contains a radical attack on the conceptual ideas of the variable and proposes a solution using the identity function as a variable. This book may have great influence, for notable simplifications are achieved.

A unique text in many ways is the preliminary edition of Allendoerfer-Oakley

* See this MONTHLY, vol. 62, 1955, pp. 385–392.

** See this MONTHLY, vol. 61, No. 7, Part II, Slaught Memorial Paper, No. 3.

§ See this MONTHLY, vol. 60, 1953, pp. 380–383.

§§ See this MONTHLY, vol. 61, 1954, pp. 319–323.

Principles of Mathematics.* This text contains the best material on sets, logic, and Boolean algebra of any informal general mathematics text.

A number of books have been written or are in writing which seek to introduce mathematics in a more formal and logical manner than is conventional. These include *Fundamental Mathematics*, written by the College Mathematics Staff of the University of Chicago. It aims at an organization and explication of basic mathematical concepts, built up from elements of logic and set theory. Also at Chicago, the Department of Mathematics is preparing texts for undergraduate courses. In some respects Herman Meyer of the University of Miami, in his *Mathematical Analysis*, pushes the postulational development of the traditional freshman material farther and more consistently than any other text. Also in this field, the Johnson, McCoy and O'Neill *Fundamentals of College Mathematics* has been used experimentally in a number of places.

At the University of Kansas this summer, the Kansas Summer Writing Group under the direction of G. B. Price is writing a preliminary edition of text material for the Universal Course as designed by the Committee. This text material will be tried out on a full scale at Tulane and on a smaller scale at several other institutions next year. This non-commercial writing was supported by the University of Kansas and the Social Science Research Council.

Courses which show interesting developments, and of which we are aware, include Artin's honors course for specially selected freshmen at Princeton. This goes further in calculus than conventional courses do in two years. The Stanford University department is constructing an experimental honors course in calculus for social scientists, having a course in logic taught in the philosophy department as a prerequisite. Most mathematicians are familiar with Pólya's work, also at Stanford, on heuristic methods in mathematics. His forthcoming book on *Mathematics and Plausible Reasoning*** is awaited with much interest. New courses in appreciation and understanding of mathematics are being offered, but the returns are not yet in on the support for them. Besides courses based upon the books of R. L. Wilder, E. R. Stabler, Courant-Robbins, and Kershner-Wilcox, we know that W. Feller at Princeton and A. M. Gleason at Harvard are developing courses of this character. At Brown University, H. Federer, Jonsson, and K. G. Lister have been developing a four-semester course in elementary mathematical analysis from a modern point of view, based on text material written by Federer and Jonsson. A significant feature is the exercises for students in mathematical writing, with complete quantification and logical detail. Among the more advanced undergraduate courses, A. W. Tucker has a well developed undergraduate course in combinatorial topology and R. L. Bing at Wisconsin has one on elementary set topology.

New courses and programs for training teachers of mathematics, either for high school or college teaching, are too numerous to mention in detail here,

* Printed edition, McGraw-Hill Book Company, 1955.

** Princeton University Press, 1954.

especially since they are in the province of the Newsom Joint Committee of M.A.A. and N.C.T.M. Briefly we may indicate the internship program for prospective college teachers, sponsored by the Fund for the Advancement of Education at nine universities, and new graduate programs, such as that at Yale. Also the special emphases of the University of Wisconsin and Notre Dame departments in this direction are noteworthy. The first Summer Conference for College Mathematics Teachers last year at Boulder was organized by B. W. Jones. New ones are being directed this summer by Ivan Niven at Oregon, E. A. Cameron at North Carolina, and for high school teachers by C. B. Allendoerfer in Seattle, all sponsored by the National Science Foundation. A college lectureship program for the coming year is being set up with a grant from N.S.F. The Internationale Matematische Unterrichts Kommission, founded by Felix Klein, is being revived at the Amsterdam International Congress. Saunders MacLane, S. S. Cairns, and A. M. Gleason represent the United States in that activity.

4. An appeal to all mathematicians. We here appeal to mathematicians, teachers, scientists, and engineers to give us the benefit of your counsel. Tell us what you are doing and what you think is needed. Tell us also specifically what this "dead wood" is which everyone says we can cut out of the present program. Give us copies of your notes or syllabi for new courses and tell us about your experiments in teaching or writing. We are particularly concerned to have the results of self-study analyses which departments of mathematics have conducted. Ultimately any complete job of writing must be in the form of writing for the classroom text with exercises. Anything short of that is likely to be of very transient value. If possible, send us seven copies of any such material you have which you consider to be significant for a reformulation of undergraduate mathematics, especially now the freshman program, but ultimately reaching on to the advanced work and back into the schools. This information will be directly helpful in carrying out the work itself and indirectly it will provide us with a basis upon which we can assess the amount of financial assistance which will be needed in furthering the program on a national basis.

Direct approaches to the physiologists, physicists, and earth scientists have been initiated with a view to getting counsel from them, analogous to that which we sought from the engineers and social scientists. Scientists in other fields will be approached later, and the results will be presented in another report.

Meanwhile, in mathematics itself there is the most urgent need for the active participation of the very best living mathematicians in the work of reformulation of our school and college programs. The last successful reformulation of the program in mathematics was accomplished in the early part of this century and was founded upon the ideas of such outstanding mathematicians and teachers as Felix Klein, Poincaré, Boltzmann, E. H. Moore, J. W. A. Young, John Perry, and many others. That a number of subsequent reform efforts have been failures or at most minor successes may be attributed to their lack of mathematical

leadership; for the teaching organizations tried to go it alone. This Committee is seeking to enlist the active participation of our able mathematicians in this work but, independently of our efforts, many established and young mathematicians who formerly thought primarily in terms of research are turning their attention to the teaching of mathematics. For it is realized that the economic basis of the mathematical profession and its relation to the general welfare is still largely founded on teaching. If graduate departments of mathematics do not furnish the teachers of college mathematics, and these teachers are qualified in some other way, we may expect that the values of modern mathematics will be lost to all but a few specialists. Moreover, the economic basis of the profession will be destroyed, as it was in the profession of classics. Unfortunately, many of the leaders of the mathematical profession are in universities where they are insulated from the signs of deterioration which are apparent in the average college and hence do not have much direct experience with the problem.

5. Suggested Special Undertakings. Finally, we list a few possible subjects for special projects which will contribute to the general objectives. We hope that individual mathematicians and departments will undertake them. If financial assistance is needed, this Committee will undertake to be of such assistance as it can to those who plan to submit a proposal asking for it. The necessary coordination of these financial appeals is the business of a committee under the chairmanship of Dean Mina Rees of Hunter College. A partial list of special problems on which work is needed is: (1) The writing of several manuscripts for a universal freshman course and classroom trials of these text materials. (2) The simplification of calculus so that it can be taught to freshmen generally. (3) Mathematics for genetics and physiology. (4) Mathematics for the earth sciences. (5) Theory for first year students. For example, how far should one go into logic? How well do rigorous postulational treatments get across? (6) Correct and teachable set theory and probability ideas for the first year. (7) The role, if any, of the normal probability distribution. (8) Dimensional analysis and measurement. (9) Vectors and linear operators as early as possible and in calculus. (10) Heuristic methods and problem solving to teach students to *make* the abstraction rather than to teach them previously abstracted theory. (11) The resetting of geometry in the curriculum. (12) Calculus technique making simplest use of numerical methods, galleries of graphs and tables of integrals. (13) Assembling and preparing readings, other supplementary materials for use in elementary courses, and guides which will make the mathematics library more accessible to students. (Here H. L. Meyer and I. Wirszup of the Chicago College Staff are doing good work.) (14) Undergraduate "research." (15) New courses in pure mathematics for undergraduates (such as R. L. Bing's topology course). (16) Combinatorial analysis and digital theory. (17) Boolean algebra. (18) Courses in computation. (19) Number theory for undergraduates and perhaps upper high school students. (20) Mathematics in the arts. (21) Complex numbers and functions in the undergraduate curriculum. (22) Trigonometry. What

is its place and value? (23) Grassman algebras and exterior products. Shall the 3-dimensional vector product be retained in courses? (24) School-college articulation effecting the advancement of well-trained students. With college credits? (25) Training for applied mathematics. (26) Morale building in high school mathematics. (27) Training for actuarial science. (28) Tukey's "mathematical engineering." (29) Reformulation of standard undergraduate programs.

6. Publication Policy. In conclusion, we state our policy on copyrights and relations to commercial publishers. This Committee will not get into the textbook business and will not approve or disapprove any textbook. Writers of experimental text materials, when subsidized by grants under the jurisdiction of this Committee will assign all royalty interests to the Association or the foundation making the grant until the expiration of the period of the grant is reached or the repayment of the grant has been effected. Ultimately we expect that any new textbooks, which may result wholly or in part from our efforts, will be published by normal methods. We must treat whatever ideas come to us as belonging to the public domain, though we hope we will always acknowledge credit where it is due. With these understandings we invite every mathematician to contribute his own ideas to the general effort.

Along with the undersigned members of the Committee, President E. J. McShane has worked as hard as any member of it, and R. C. Yates contributed greatly as an active member in 1953.

August 16, 1954

W. L. DUREN, JR., *Chairman*
C. V. NEWSOM
G. B. PRICE
A. L. PUTNAM
A. W. TUCKER

ADDENDUM

The Committee on the Undergraduate Program is being reorganized with a new Chairman, E. J. McShane, who originally appointed it. The new Committee will also include John G. Kemeny of Dartmouth. W. L. Duren will remain an ex officio member as President of the Association.

The grant of $25,000 made in March 1955 by the Ford Foundation, and announced elsewhere in this MONTHLY, gives the Committee its first dependable financial support. This amount of money is generous for a start but the magnitude of the general effort is so great that these funds are small in comparison.

The Committee met in Lawrence in January 1955 with further support from the University of Kansas. Here a week's work was done on Universal Mathematics, Part II, particularly on applications of the Boolean set operations. A new Summer Writing Group is now being planned for 1956. Its location has not been determined. A critical account of the Tulane experience with Universal Mathematics, Part I, has been prepared for publication in this MONTHLY.

April, 1955

W. L. DUREN, JR.