

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 132: Selecting digits

By Dr. James Tanton, MAA Mathematician at Large

Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$ and Sergio randomly selects a number from the set $\{1, 2, \dots, 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Probability

COMMON CORE STATE STANDARDS

S-CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY.](#)

SOURCE: This is question # 24 from the 2002 MAA AMC 10A Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Probability questions always make me nervous. They tend to be subtle and tricky to think through.

So there are ten options for the two numbers Tina will choose, each summing to something. I am going to list them all out!

$$1 + 2 = 3$$

$$1 + 3 = 4 \quad 2 + 3 = 5$$

$$1 + 4 = 5 \quad 2 + 4 = 6 \quad 3 + 4 = 7$$

$$1 + 5 = 6 \quad 2 + 5 = 7 \quad 3 + 5 = 8 \quad 4 + 5 = 9$$

and there are ten options for the number Sergio will choose.

One-tenth of time he will choose “1” and won’t have a figure bigger than Tina’s sum. One-tenth of the time he’ll choose “10” and will, for sure, have a figure than Tina’s sum. One tenth of the time he’ll choose a “5” and might or might not have a figure bigger than Tina’s sum. These middle cases are the tricky ones.

Well, in this “5” case, there are ten options for Tina, two of which give a sum for her smaller than 5. So two-tenths of that one-tenth time Sergio wins.

Maybe I can just plow my way through that kind of thinking!

One-tenth of the time Sergio gets 1:

He loses.

One-tenth of the time Sergio gets 2:

He loses.

One-tenth of the time Sergio gets 3:

He loses.

One-tenth of the time Sergio gets 4:

He wins one-tenth of those times (when Tina gets a 3).

One-tenth of the time Sergio gets 5:

He wins two-tenths of those times.

One-tenth of the time Sergio gets 6:

He wins four-tenths of those times.

One-tenth of the time Sergio gets 7:

He wins six-tenths of those times.

One-tenth of the time Sergio gets 8:

He wins eight-tenths of those times.

One-tenth of the time Sergio gets 9:

He wins nine-tenths of those times.

One-tenth of the time Sergio gets 10:

He wins every time.

Okay, tedious, but we have it! Sergio wins:

$$\begin{aligned} 0 + 0 + 0 + \frac{1}{10} \times \frac{1}{10} + \frac{1}{10} \times \frac{2}{10} + \frac{1}{10} \times \frac{4}{10} \\ + \frac{1}{10} \times \frac{6}{10} + \frac{1}{10} \times \frac{8}{10} + \frac{1}{10} \times \frac{9}{10} + \frac{1}{10} \\ = \frac{1 + 2 + 4 + 6 + 8 + 9 + 10}{100} = \frac{40}{100}, \end{aligned}$$

that is, 40% , of the time.

Extension 1: How does the answer change if Tina can select two numbers from her set, with a repeat choice allowed?

Extension 2: Are there integers k and N so that if Tina chooses two distinct integers from the set $\{1, 2, \dots, k\}$ at random and Sergio an integer from $\{1, 2, \dots, N\}$ at random the probability that Sergio has a figure greater than the sum of Tina’s two figures is exactly 50% ?

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