MAA FOCUS

NEWSMAGAZINE OF THE MATHEMATICAL ASSOCIATION OF AMERICA, VOL. 35, NO. 4, AUGUST/SEPTEMBER 2015

USAMO Winners Feted
Common Core Matters
New Writing Award

MAA FOCUS is published by the Mathematical Association of America in February/March, April/May, June/July, August/September, October/November, and December/January.

Advertising inquiries: *advertising@maa.org*

***MAA FOCUS* Staff**

Director of Publications: Jim Angelo

Editor: Jacqueline Jensen-Vallin, *maafocus@maa.org*

Managing Editor & Art Director: Lois M. Baron, *lbaron@maa.org*

Staff Writer: Alexandra Branscombe

Letters to the editor should be addressed to Jacqueline Jensen-Vallin, Mathematical Association of America, 1529 18th Street, NW, Washington, DC 20036, or by email to *maafocus@maa.org*.

Subscription and membership questions should be directed to the MAA Customer Service Center, 800-331-1622; email: *maaservice@maa.org*; 301-617-7800 (outside U.S. and Canada); fax: 301-206-9789. MAA Headquarters: 202-387-5200.

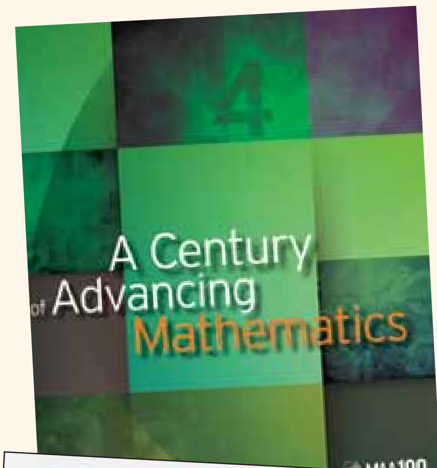
Periodicals postage paid at Washington, D.C., and additional mailing offices. Postmaster: Send address changes to *MAA FOCUS*, Mathematical Association of America, P.O. Box 90973, Washington, DC 20090-0973. ISSN: 0731-2040 (print). ISSN 2161-704X (online). Printed in the United States of America.

Visit Your Online Member Library

The new member library is now live online. This feature on your MAA profile page allows you to download select titles hand-picked by MAA. These titles will be added to your member profile on a regular basis, so please be sure to check in every month or so to review the fresh selections.

We are placing a free ebook copy of A Century of Advancing Mathematics *in every member's online library. This special commemorative volume, produced to mark MAA's centennial, includes sections on mathematical, historical, pedagogical, and computational developments, as well as a section on culture and communities.*

MAA members can access the new volume, as well as three other free titles, through the member library by logging in to their account on the MAA website. You'll see that a "Member Library" link now appears on the left side of the My Profile page. Click on that, and you'll be taken to the web page where you can download books.

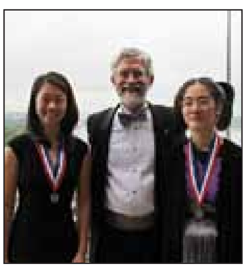


MAA FOCUS

NEWSMAGAZINE OF THE MATHEMATICAL ASSOCIATION OF AMERICA • VOL. 35, NO. 4 • AUGUST/SEPTEMBER 2015

Features

About the Cover



USAMO winners Celine Liang (left) and Danielle Wang, with John Holdren, director of the White House Office of Science and Technology Policy.

Photo: L. McHugh

Departments

News

Inreach Is the New Outreach

By William Yslas Vélez

When I started working (with Clark Benson) on retaining minority students in first-semester calculus, it was 1988 and we spent time every week talking with our 25 students. The rising number of such students each year got us to the point that such time-consuming interaction wasn't possible. In working out how to revamp my approach, I noticed that we had many entering students who started in second- or third-semester calculus—either transfer students or students who earned credit for calculus in high school.

With the help of a postdoc, I now send out emails inviting these students to meet with me for 20 minutes. The following is a typical conversation that I have with first-year students as part of my Minority Calculus Advising Program. These conversations begin about two weeks before the fall semester begins and go something like this (links are listed in the box, and a longer discussion is on the magazine's webpage: maa.org/maa-focus-supplements):

Javier timidly knocks on my door. "Dr. Vélez? You sent me an email asking me to stop by," he says nervously.

"Yes, yes, please sit down," I say. "Thanks for coming by. I see that you are enrolled in Calculus II, yet this is your first semester here. Where did you earn credit for Calculus I?"

Javier says he took calculus at a community college while in his senior year of high school because his own high school did not offer it. He couldn't take Calculus II because he couldn't afford it.

Links

Minority Calculus Advising Program: http://math.arizona.edu/~velez/MinorityCalculusAdvisingProgram.pdf

Link 1: Calculus webpage: http://math.arizona.edu/academics/undergrads/resources

Link 2: Math 100 UTA program: http://math.arizona.edu/academics/undergrads/employment/utas/m100

Link 3: Sample Resume: http://w3.math.arizona.edu/files/Sample%20Resume.pdf

Link 4: REU programs funded by NSF: http://www.nsf.gov/crssprgm/reu/reu_search.jsp

Link 5: Alumni Early Career Profiles: http://math.arizona.edu/academics/undergrads/careers/alumni

Vélez: "I am very impressed with you. I see that you are majoring in engineering. Great!"

Then comes that most difficult of questions.

Javier: "Should I stay in Calculus II or go back to Calculus I?"

V: "I would like for you to answer that question. I am sending you a link [Link 1, see box] for some advice. There you will find the calculus pages where the final exam review for Calculus I is posted. You have two weeks before classes start. Start working on those problems. If you can do most of them, you are ready for Calculus II."

I want to get to know the student.

V: "Tell me, what are your goals? What do you plan to do when you graduate?"

J: "Well, I don't exactly know what kind of engineering I want to specialize in. Perhaps mechanical. Then get a job."

V: "Have you given any thought to going to graduate school and earning a master's or PhD?"

Javier hasn't even started his undergraduate studies. His first college course is still two weeks away. Javier looks at me, confused. I have to explain the educational process, going from BS, MS to PhD, and how long each step takes.

J: "Well, sir, I barely have enough funds to pay for my tuition as it is . . ."

V: "If you go to graduate school, we pay you. Of course, you have to have good grades."

J: "WHAT? You mean if I get a job, then the company might send me to school?"

V: "No, the university that accepts you will pay your tuition and give you a stipend. As an example, suppose we accepted you into our graduate program in mathematics, then . . ."

I explain how we support graduate students and that some of his undergraduate courses will be taught by them.

V: "What do you plan to do next summer?"

Again a stunned look comes over Javier's face. Next summer? This summer isn't even over!

J: "Well, I will need to get a job to help pay for tuition. My uncle is a carpenter, and I think he can get me a summer job as a laborer."

V: "Do you have a job now?"

J: "When school starts, I will need to get a job."

V: "Our department has a program where we hire tutors for an elementary algebra class. I think you should apply. Here is a link [Link 2] to fill out an application. It looks much better on your résumé if you have academic positions."

I can see Javier thinking, "What is a résumé?"

V: "In case you don't have a résumé, here is a link [Link 3] to a sample résumé that I created."

I go over the résumé format, explaining each of the items.

What does a student get out of this conversation? I now insist that the student not leave my office until the student has recorded some part of our conversation.

V: "OK, so let's see where we are. I want you to write down what is to be done in the next couple of weeks. First of all, you are to start the calculus review. Second, I want you to apply for the tutor position. Then, third, I would like to see a copy of your résumé before classes start.

"Now I want to return to what you are going to do next summer. Here is a link to the National Science Foundation [Link 4] that shows summer research programs. Let's look at this one."

An REU comes up, and I ask Javier to look at the financial items.

J: "That's a lot of money. I don't think I can afford to pay the $3,000."

Now it's my turn to say, "What?"

V: "No, Javier. They pay you. Look more closely. If they accept you, they pay your round-trip airfare, give you free housing, and pay you $3,000!"

Javier is shocked, but it is about to get more interesting.

V: "Javier, I think that you are quite an amazing person. I think that you should add the mathematics major to your program of study. Graduating with degrees in engineering and mathematics will make you more competitive. Moreover, with another major, there is another department interested in your welfare. If you become a math major, I will be your adviser, and it is my responsibility to find you opportunities, as long as you work hard and earn good grades. How does that sound?"

J: "Well, I don't know. I had never thought of it. Maybe not. I don't really want to teach."

V: "Most of our math majors do not go into teaching. Let me show you the career paths of some of our math majors. Here is the link [Link 5]."

V: "Adding the math major is easy. So is dropping it. Let me take you across the hall to the Math Center, and Laurie—the coordinator of the Math Center—will sign you up."

Looking a little shell shocked, Javier walks over to the Math Center.

Our mathematics classes are not just for the transfer of knowledge. They are vehicles for motivating students to continue the study of mathematics. We need to show students how this vehicle can help them reach their goals. With a bit of information, some timely advice, and an invitation to become math majors, we can give the students an exciting and scenic ride.

William Yslas Vélez is a professor in the Department of Mathematics at the University of Arizona in Tucson. His most recent award was from the Association for Women in Mathematics for mentorship of undergraduate women in mathematics.

ALEJANDRO RIVERA/ISTOCK

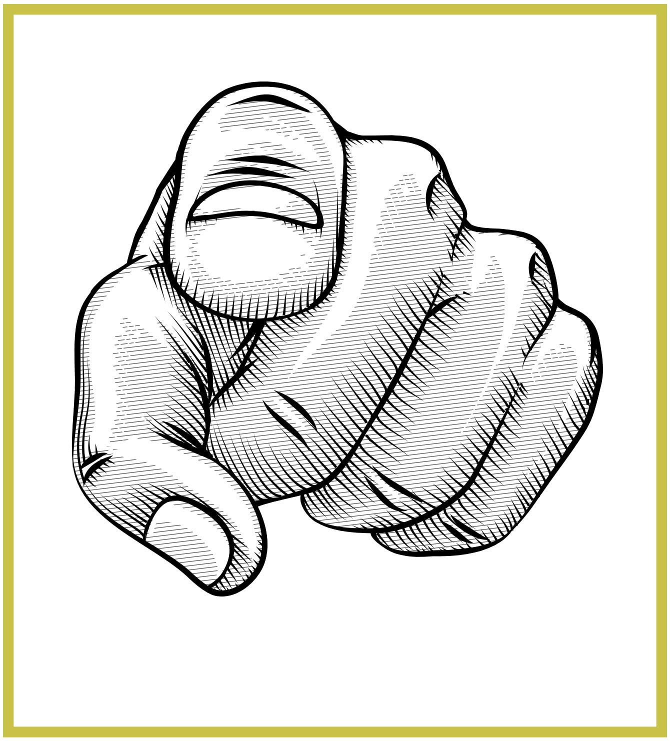
Common Core Standards: Why You Should Care

By Katharine Merow

If you've been ignoring the Common Core State Standards for Mathematics (CCSSM), which outline what K-12 students should know and be able to do at the end of each grade, you might want to reconsider.

In case you have no idea what it is, here's a rundown by Mike Steele, an associate professor of mathematics education at University of Wisconsin–Milwaukee:

> The Common Core State Standards for Mathematics outline the mathematics content that students should learn through their K-12 experiences as well as a set of eight Standards for Mathematical Practice that represent important habits of mind for knowing and doing mathematics. College faculty can use the content expectations, which are more specific than ever before, to get a stronger sense of what mathematical concepts and procedures we expect all students to have learned (as well as a specially designated set of "plus" standards for STEM-intending students).
>
> Moreover, the Standards for Mathematical Practice provide common language for describing the ways of thinking about and doing mathematics that transcend content foci. These eight standards in particular have been particularly useful in reconceptualizing and framing entry-level and remedial mathematics offerings for a number of institutions.



SHUTTERSTOCK

And here's why you, as a postsecondary-level mathematics educator, should pay attention:

You Teach Future Teachers

This one's a no-brainer. If your students will become teachers of primary or secondary mathematics, you should familiarize yourself with what they will be expected to teach. Even if your state has not adopted the Common Core, the future teachers you instruct will enter an educational landscape shaped by the new and nearly national standards.

Brian Katz, who works with preservice teachers in upper-division courses at Augustana College, says that the Common Core's Standards for Mathematical Practice (http://bit.ly/1izsZ8m) validate much of what he does in the classroom. He has adopted the language of the standards to make the connection explicit, and it's working.

Katz's former student Mary O'Malley is now responsible for making sure the Common Core is implemented in her school district. When she wrote a letter in support of Katz's tenure, she said that his inquiry-based approach to the classroom resembles many of the Common Core's key teaching criteria.

"Even though I did not grow up with many of the methodologies of the Common Core that I am currently teaching to my students," O'Malley wrote, "I feel fortunate that I was able to learn first-hand how to create a learning environment that fosters this critical mathematical thinking."

Your Students Come through the U.S. School System

Don't think, though, that if you're not involved in teacher education you can blithely disregard the CCSSM. Students who enter your institution having attended U.S. public schools will soon be coming to you with conceptions of mathematics colored by the new standards.

Both supporters and critics of the Common Core cite concern about the mathematical preparedness of incoming freshmen as a reason for mathematics faculty to educate themselves about the CCSSM.

"The promise of the standards is to reduce the gap between high school and college," says CCSSM lead writer William McCallum (University of Arizona). "College faculty have an interest in reducing that gap."

University of California, Berkeley, mathematician Marina Ratner, whose August 2014 op-ed in the *Wall Street Journal* (http://on.wsj.com/1GvAWqE) compared the CCSSM unfavorably with California's old standards, predicts that the college freshmen of tomorrow will arrive on campus ill prepared to succeed in mathematics.

"Students in coming years will be by far weaker than they have ever been," says Ratner. "This, I believe, answers the question [of] why MAA members should pay attention to the CCSSM."

You Care about Math in America

You'll want to know whether you can assume freshmen foreknowledge of logarithms, sure, but Hung-Hsi Wu (University of California, Berkeley) sees engagement with the Common Core as a higher calling.

"There is a civic obligation that we should discharge," he says.

Wu, an advocate of the CCSSM who has worked with McCallum to write associated progressions documents (http://bit.ly/1FbrB7X), believes that mathematics education is in crisis in the United States and regards mathematicians as uniquely positioned to address the problem.

Wu has written critically about what he calls Textbook School Mathematics, or TSM. The mathematics laid out in textbooks has defects so deep and pervasive, Wu says, that the list he offers in a blog post, "Mathematical Education of Teachers, Part I" (http://bit.ly/1KZDwoF), cannot begin to capture them.

Definitions are absent or mangled, according to Wu, and reasoning is flawed. The purpose of studying anything is usually well hidden. Presentations of concepts lack precision and coherence.

As long as teachers learn TSM in elementary, middle, and high school—and until teacher educators provide a viable alternative—the "fundamentally and seriously flawed mathematics" of TSM will not be eradicated, Wu argues.

"Teachers do not shed the habits acquired over thirteen years of immersion in TSM without a protracted struggle and without a lot of help," Wu reminded his fellow mathematicians in a February blog post (http://bit.ly/1KZDwoF). "The help they need translates into sustained hard work on our part. This is hardly glamorous work, but if mathematicians don't do it, who will?"

Wu recommends that mathematicians keen to combat the education crisis let the CCSSM guide them; Ratner encourages colleagues to speak out against the new standards. Neither, however, could deny the centrality of the Common Core to the broader debate.

You Care about Mathematics

Now suppose, for the sake of argument, that you don't care about America's competitiveness on the world stage or whether students from public schools can compete with their peers from private schools or high-performing countries. You should still not dismiss the Common Core as irrelevant. If you care about mathematics, you should care about the CCSSM.

In her *Wall Street Journal* op-ed, Ratner mourned that, under the Common Core standards, math has been replaced "with some kind of illustrative counting saturated with pictures, diagrams and elaborate word problems." The swap doesn't square with Ratner's view of mathematics: "Mathematics is not about visual models and 'real world' stories," she wrote.

Lead CCSSM writer McCallum would agree that the reputation of mathematics hangs in the balance as national discussion of the CCSSM continues.

Speaking at the education forum before the National Math Festival in April (http://bit.ly/1JI9Jns), McCallum challenged his colleagues to expend expository energy illuminating what lies—too often poorly understood—at the center of the standards debate.

"If there's a call to action that I have for the mathematicians in the room," he said, "it's to expose that to the air, to bring some mathematical sensibility to describing the object itself called mathematics.

"Mathematics is not just a way station to get you someplace else, a sort of thicket that you have to struggle through and push out of the way and get scratched and hope that you finally make it through," McCallum preached to the choir. "It's a beautiful thing in its own right."

Shouldn't students have a chance to see that?

Stay Tuned

Readers should look to future issues of *MAA FOCUS* for coverage of CCSSM-related topics, ranging from equity issues to professional development efforts to the interpretation of the standards for mathematical practice. Got a suggestion for a topic or perspective to be included? Email it to *MAA FOCUS* editor Jacqueline Jensen-Vallin (maafocus@maa.org).

Katharine Merow is a freelancer writer. The article does not necessarily reflect MAA policy.

The Math Wars and the Common Core Standards

By Patty Anne Wagner

For decades, controversy has been the companion of school mathematics reforms. From the New Math of the 1960s to the current Common Core State Standards for Mathematics (CCSSM), heated debates and impassioned pronouncements have issued from all corners of U.S. society.

Much of the debate surrounding the Common Core State Standards (both math and language arts) has focused on general issues, such as federal versus local control of education, mandated assessments, and whether particular corporations or organizations are unfairly benefitting financially. However, lots of the criticism targeting the CCSSM evoke memories of the arguments—particularly heated in the 1990s—about how mathematics should be taught.

Given the enduring nature of the controversy, it is surprising that little research has focused on understanding how the critics interpret reform mathematics education and to what they so fiercely object. Without understanding critics' concerns in debates on mathematics education, how can we hope to penetrate the relatively more complex issues surrounding the CCSSM?

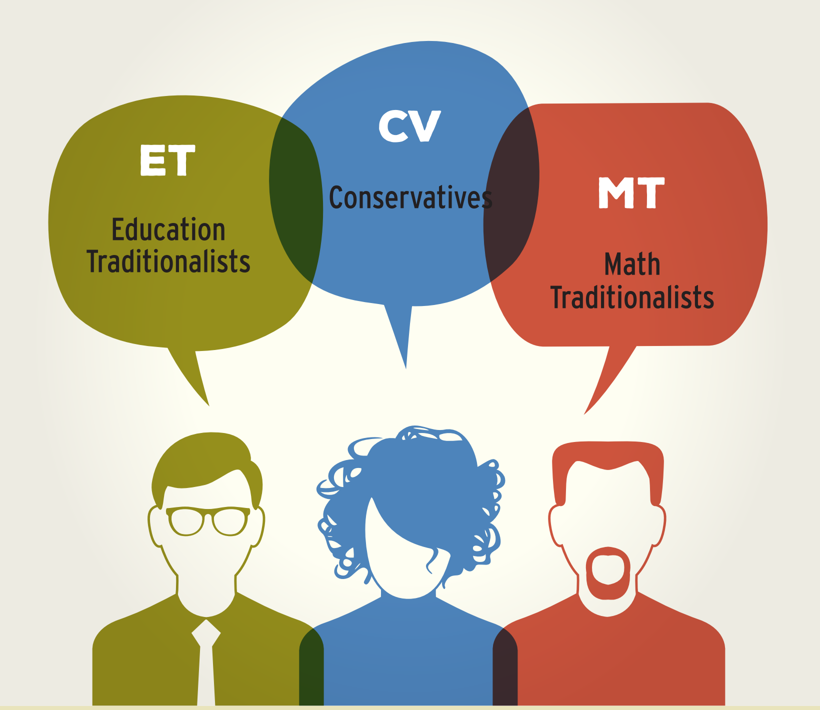
This question motivated my dissertational study to determine the fundamental concerns and motivations of critics of past reform mathematics education. I wanted to develop a theoretical description of critics' beliefs and interpretations, so I studied only critics whose objections transcended local reform efforts and used a grounded theory qualitative approach.

My initial data were collected indiscriminately from articles populating the websites *Mathematically Correct* and *New York City HOLD National,* but later extended to written documents posted to other websites, general web pages, video, commentaries, PowerPoint documents, books, and journal articles. Most of the documents were produced between 2000 and 2010; however, some were written as early as 1996 and as late as 2013. In all, I analyzed 99 documents written by 41 authors, producing 1,800 coded references.

Three Core Groups

My analysis revealed three communities within the critics of mathematics education reforms. Although some concerns overlapped, the communities fundamentally differed in how they interpreted mathematics education reforms and their motivations for objecting to them. I differentiated between the groups, Education Traditionalists (ETs), Conservatives (CVs), and Math Traditionalists (MTs), by inferring the primary lens through which these critics viewed mathematics, mathematics education, and associated reforms.

The ETs and the CVs both had fairly simplistic views of mathematics, which they described only as "hierarchical." They spent little time talking about mathematics, focusing their criticisms on other elements of reform mathematics education. The ETs were primarily driven by



their beliefs about teaching and learning, which corresponded to their views of the inherent nature of humankind and the purpose of education. This group's objections largely hinged on their assumption that students cannot learn in a nontraditional environment.

The CVs, on the other hand, held a worldview that they saw as inconsistent with that embedded in reform mathematics education. Individuals in this group believed that a primary motivation driving the reforms was developers' liberal ideology. Many suspected the reforms were designed, intentionally or not, for liberal indoctrination of students.

Both the ETs and the CVs were highly influenced by what they perceived as the prevailing opinion of mathematicians, viewing them as experts in determining what and how mathematics should be taught.

Math Traditionalists

All the MTs had at one time been academic mathematicians, and their primary lens was their view of mathematics. The MTs valued the coherence of mathematics and its internal structure. Because they viewed mathematics as a sensible system, the MTs believed that children progressing through a well-designed program would advance according to their capability and form the conceptual understanding inherent to mathematics.

Noting that increasing students' conceptual understanding is one of the goals of reform mathematics education, MTs argued that it is truly demonstrated when a student can supply mathematical reasons for a method.

For this to occur, the MTs believed students must commit previously learned material to memory; in doing so, they free their working memory from trivial concerns. The MTs worried that reform instruction overemphasized mathematical processes at the expense of attending to important algorithms and procedures.

IF MOST MATHEMATICIANS ARE SILENT, THEN AUTHORITY IS GIVEN TO THE FEW WHO ARE NOT.

The MTs felt strongly about the need for mathematical precision, both in definitions and in mathematical discourse itself. To the MTs, mathematics is unambiguous; that is, real-world problems are mathematical only when they are well posed. The MTs interpreted reform mathematics education as introducing unacceptable levels of ambiguity in the name of "doing mathematics."

As an example, MTs objected to the popular use of pattern tasks in reform instruction, arguing that patterns can be continued in any way one wishes unless a rule for the pattern has been established. They complained that reform mathematics education was riddled with similar problems in which students, lacking precise parameters, are encouraged to fill in missing information with their own assumptions.

The MTs' overarching concern was that reform mathematics education was redefining mathematics as a domain. They worried that left unchecked, school mathematics reforms would fundamentally change the subject with which they strongly identified.

The present controversy over school mathematics targets the CCSSM. To some, the CCSSM establishes only what mathematics content should be taught; others have argued that the CCSSM also prescribes teaching practices supported by the National Council of Teachers of Mathematics (NCTM). Differing interpretations of what the CCSSM mandates might explain why critics of NCTM recommendations are found on both sides of the CCSSM debate.

Regardless, if we listen to the MTs, the ability to determine what and how mathematics should be taught concerns a set of more fundamental questions: What *is* mathematics, what does it mean to do it, and how can we know one has learned it?

So What?

I assert that mathematicians, particularly those who teach in the academic setting, should concern themselves with addressing these questions for the following reasons:

First, as evidenced by the ETs and CVs, mathematicians are seen by the public as authorities. If most mathematicians are silent, then authority is given to the few who are not.

Second, if the reforms cause mathematics to be portrayed inaccurately, this could have implications for the future of the field that should concern mathematicians.

On the other hand, if as advocates suggest, NCTM's recommendations for teaching mathematics result in greater student achievement, more diversity in the field, and more favorable dispositions toward mathematics, then should not *all* mathematics instructors, including those in higher education, employ these practices?

The fate of the CCSSM will not settle this matter. History shows us the debate over what and how mathematics should be taught will simply reemerge in the (re)form of the next movement to improve mathematics instruction. At the risk of inflaming another "war," I encourage mathematicians to take a stand.

Patty Anne Wagner is an assistant professor of mathematics at the University of North Georgia. Her opinions do not necessarily reflect those of the MAA.

Seattle Skyline via photopin (license)

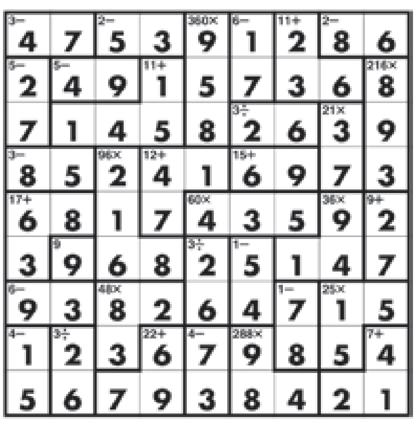
Modern Math Workshop

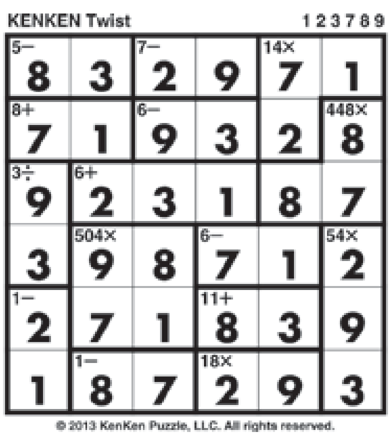
October 28–29

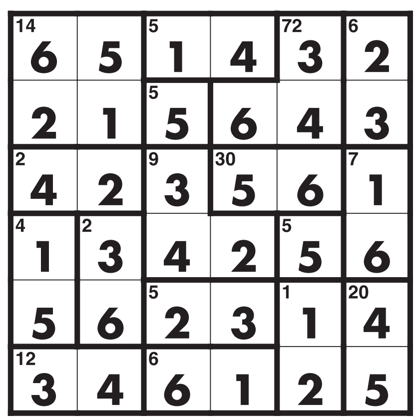
The nine NSF-funded U.S.-based math institutes present the annual Modern Math Workshop (MMW) on October 28–29 (Wednesday-Thursday) in the Washington, DC, area. The MMW is part of the institutes' Mathematical Sciences Diversity Initiatives and the workshop is a pre-conference activity of the SACNAS National Conference (Society for Advancement of Hispanics/Chicanos and Native Americans in Science, see http://sacnas.org/events/national-conf). The MMW includes two minicourses for undergraduates and talks related to the research programs at the math institutes that would be of interest to graduate students and early career researchers. The workshop is intended to encourage minority undergraduates to pursue careers in the mathematical sciences and to assist undergrads, graduate students and recent PhDs in building their research

Puzzle Page Solutions

In the June/July issue, we presented three KenKen® puzzles. The solutions are below.







Dear MAA

Organizing Sessions, New Departmental Membership

Get Involved in a National Meeting

Dear MAA,

I was so excited by the list of contributed paper sessions happening at the Centennial Celebration at MAA MathFest 2015. I would love to be involved—a colleague and I would like to organize a poster session or a themed contributed paper session for either the Joint Mathematics Meetings in 2016 or MAA MathFest 2016. How can we get involved?

Wanting to Help

Dear Wanting,

MAA relies heavily on the involvement of its members—both as organizers and speakers—so we love it when more people want to be involved! People interested in organizing a session can submit their proposals online at *http://www.maa.org/meetings/meeting-guidelines*. Unfortunately, the deadline for the 2016 JMM has passed (those proposals were due by January 31, 2015. Proposals for the 2017 JMM are not due until January 31, 2016, however, so you might consider that!). Proposals for MAA MathFest 2016 are due by October 15, 2015, so you should definitely consider submitting for next summer!

When you submit your proposal, you will be asked for the following information:

- Names and contact information for organizers, as well as a biographical sketch for each organizer or panelist.
- The title/abstract of the session, which must:
    - ☐ Define and describe the topic of the session clearly in the first three to five sentences. This introduction is meant to succinctly describe what the session is about in order to attract potential audience members and speakers. You may also briefly mention why this topic is timely, recent changes in this field, or other reasons this session would interest meeting participants.
    - ☐ Specifically describe the types of presentations sought for the session (e.g., original research, expositions, innovative ideas, demonstrations, problems, applications, projects, curricular materials, single class descriptions, whole course outlines, assessment methods, accessible to certain groups of people).
    - ☐ Emphasize (either explicitly or implicitly) that presentations are expected to be scholarly in nature.
    - ☐ Include sponsoring organizations if any (e.g., MAA committees, SIGMAAs).
- Requests for any special equipment or logistical needs (subject to approval);
- Any additional information that would be helpful when evaluating proposals, such as previous related sessions and their attendance.

MAA is also interested in people who wish to propose and organize poster sessions, panel discussions, and even minicourses. For more information about those types of sessions, please see the website above.

We look forward to your involvement. If you have any further questions, please let us know!

Choose a Departmental Membership

Dear MAA,

I was hoping to get my students involved with the mathematics community during the coming year. What's the best way to get my students enrolled as members of MAA?

Excited Faculty Member

Dear EFM,

I'm so glad you asked. We have a new option for departments that want to sponsor students in order to get them involved in the MAA. Our departmental membership has many perks (see maa.org/departmental-membership-benefits for a full list) including

- Unlimited student memberships for any nominated full-time enrolled mathematics students
- Membership for one faculty member who will serve as the Departmental Membership administrator

Departmental members (Departmental Membership administrator and nominated students) have access to all the usual MAA membership perks, including →

Pricing for a Departmental Membership

	Total Enrollment at Institution? (academic full-time equivalents)		
Does Institution Grant PhD in Mathematics?	< 7,000	7,000–19,999	20,000+
No	$500	$700	$900
Yes	$750	$1,000	$1,500

- Online subscriptions to

  The *American Mathematical Monthly* (10 issues per year)

  The *College Mathematics Journal* (5 issues per year)

  Mathematics Magazine (5 issues per year) →

  Math Horizons (4 issues per year)

  MAA FOCUS (6 issues per year)

- Discounts on MAA books.

- Discounted registration for the Joint Mathematics Meetings and MAA MathFest.

This is the MAA's new version of student chapters.

I know what you're thinking—"This sounds too good to be true! It must be expensive!" However, the dues structure is completely reasonable and based on your undergraduate enrollment and highest mathematics degree awarded at your campus. See the table, previous page.

This is the MAA's new version of student chapters, but instead of having each student pay for a membership, the institution sponsors the students, and we include a membership for the adviser. This allows your students access to all the perks of the MAA.

The enrollment period is August 1 through September 30, so recruit your students and ask your department chair and dean to sponsor a departmental membership in the MAA during the 2015–2016 year!

"Dear MAA" is our regular column offering advice and information. Please send us questions, large or small, regarding the MAA and life as a mathematician. "Dear MAA" will answer as honestly as possible. Address questions to the attention of DearMAA@maa.org.

Nominations Wanted for Awards

HENRY L. ALDER AWARD for Distinguished Teaching by a Beginning College or University Faculty Member

The Alder Award honors beginning college or university faculty whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. Each year at most three college or university teachers are honored with this national award. The nominee must be a member of MAA. Details on the award are given at the MAA website. Nominations for the award may be made by any member of the MAA or by any section of the MAA and should be sent to Barbara Faires, secretary of MAA, *secretary@maa.org*, by October 1.

ANNIE AND JOHN SELDEN PRIZE for Research in Undergraduate Mathematics Education Call for Nominations

Annie and John Selden Prize for Research in Undergraduate Mathematics Education honors a researcher who has established a significant record of published research in undergraduate mathematics education and who has been in the field at most 10 years. The prize is designed to be an encouragement to such researchers; at most one is awarded every other year. The recipient should have a significant record of published work in *RUME,* normally several articles in reputable journals.

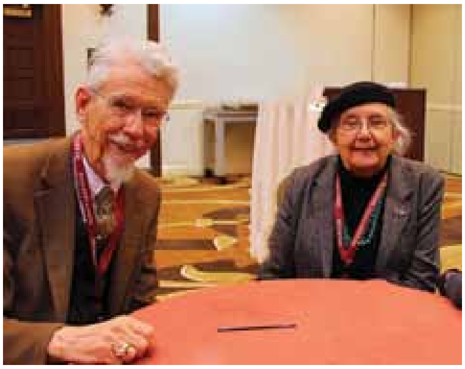
A nomination form can be found on the MAA website at *maa.org/selden-award-nomination-form*. This completed form, along with a narrative that describes the qualities of the research that make it significant, should be sent to Barbara Faires, MAA secretary, *secretary@maa.org*, by October 1.

YUEH-GIN GUNG AND DR. CHARLES Y. HU AWARD for Distinguished Service to Mathematics

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award for service offered by the MAA. It recognizes service to mathematics that has been widely recognized as extraordinarily successful on a national scale. The service may comprise one or more activities, and the period of service may have been short or long.

The nature of the service has varied widely among past awardees, and includes mathematical writing and editing, advancing diversity, leading professional organizations, and educating those who will teach mathematics.

Please send the names of those who should be considered for the 2017 Award to the chair of the Gung and Hu Award Committee, Jennifer Quinn (*jjquinn@u.washington.edu*), by December 1. Include a short discussion of the reasons behind your nomination, highlighting the most important aspects of the person's contributions and impact. Lengthy documentation and letters of recommendation are not necessary. More information about the award, including a list of past awardees, can be found at *maa.org/awards/gunghu.html*.



John and Annie Seldon.

MAA FOCUS Welcomes New Editor

Jacqueline Jensen-Vallin has been named the new editor for *MAA FOCUS.* Jensen-Vallin is beginning her role in the MAA's centennial year, an exciting milestone for a new editor.

"I want to focus on how we transform the magazine as we move into the next century of the MAA," says Jensen-Vallin, an associate professor at Lamar University in Beaumont, Texas.

Her main goal is to make sure the magazine reflects the array of voices within the mathematics world. "I hope to remind people that MAA is a living, breathing organization that is evolving," she says.

Jacqueline Jensen-Vallin, new editor of the MAA's newsmagazine.

Part of the Community

Involved in different groups within the MAA membership, Jensen-Vallin understands the diversity in the mathematics community. Her experiences include a Project NExT fellowship (Forest Dot), serving on the Committee on Undergraduate Student Activities and Chapters (CUSAC), the Committee on Sections (COS), and co-organizing events for the Association for Women in Mathematics (AWM).

As an undergraduate student, she was not even registered for a mathematics degree until her final semester at the University of Connecticut as a sociology major. She enrolled in Calculus III, which inspired her to stay and get a second degree in mathematics.

Looking back, Jensen-Vallin sees many things she missed that would have helped her studies. At that time she was not aware of a broader mathematics community, or that there were many women in mathematics, she says. "I was always one of two or three women in my math classes as an undergraduate." Fortunately this changed in graduate school when she was invited to sit on a panel at an MAA Pacific Northwest Section meeting.

"I had never been to an MAA meeting before, and it hooked me," she says. This was her introduction to a group that got together just to talk about mathematics. It was also where she met prominent women in leadership roles.

"It was a cool moment to see women in mathematics involved in the MAA," she says. "It made me feel like I was part of something bigger."

A New Role

Ever since that first meeting, Jensen-Vallin has jumped at opportunities to participate in the MAA community and draw more people into the world of mathematics. As an editor, she hopes to draw from her connections with the MAA sections, SIGMAAs, AWM, Pi Mu Epsilon, and MAA committees to bring new and current stories to the member magazine.

"I hope to use *MAA FOCUS* to give voice to different people: new graduates, the SIGMAAs, new and senior faculty, those dedicated to research, pure math, teaching, and more," she says. "I am excited and humbled to be serving MAA in a new way."

—Alexandra Branscombe

Send letters to the editor to Jacqueline Jensen-Vallin at Mathematics Department, 200F Lucas Bldg., P.O. Box 10047, Lamar University, Beaumont, TX 77710. Or send email to her at *maafocus@maa.org*.

Authors of Teaching Materials to Be Honored

MAA is pleased to announce that the Daniel Solow Author's Award has been established to honor those who have created exceptional resources for undergraduate education in the mathematical sciences.

Daniel (Danny) Solow, professor in the Department of Operations in the Weatherhead School of Management at Case Western Reserve University and longtime member of MAA, worked with MAA to set up the award, which will bestow $2,500 annually, beginning at MAA MathFest 2016.

The deadline for nominations is October 15. See box for information.

Solow says that the motivation for creating such an award stems from his undergraduate years. Learning proofs was challenging for him. He says, "I came to the conclusion that a large part of the problem was that my teachers were doing things in their heads that they had not made explicit in their lectures." Solow has spent a large portion of his time as a faculty member developing materials to make mathematical thinking more explicit for students learning advanced mathematics.

In particular, in 1982 he published *How to Read and Do Proofs*. *The Keys to Advanced Math* continued this theme in 1995, as did *The Keys to Linear Algebra* in 1998.

In Sync with the MAA

When deciding to contribute to the MAA, Solow says, "As mathematics education has always been an important—and integral—part of my professional career, I found that the MAA was the organization that best represented those values and concerns."

According to Michael Pearson, MAA executive director, "The Solow Award is very much consistent with the mission and vision of the MAA and allows us to recognize those in our community whose work contributes to the highest-quality experiences for students in the mathematical sciences."

Solow received his BS in math from Carnegie Mellon University, MS at Berkeley, and PhD at Stanford in operations research. He has been at Case Western Reserve since 1978.

While spending much time in his career creating resources for undergraduate students, Solow also focused on mathematical research and has published more than 25 research papers. These are in the areas of complex adaptive systems and deterministic optimization. He is most proud of "Factors that Affect the Optimal Amount of Central Control in Complex Systems" (with Joseph Szmerekovsky in *Naval Research Logistics* 55 [5]), which applies mathematical modeling to general complex systems; and "Setting Leadership Goals and Getting Those Goals Accomplished: Insights from a Mathematical Model" (with Szmerekovsky, *Computational and Mathematical Organization Theory* 20 [1]), which models visionary and charismatic leadership.

—*Jacqueline Jensen-Vallin*

NOMINATION PACKETS should be submitted via email to *secretary@maa.org* by October 15, 2015. Full guidelines for submission and criteria for the award will soon be on the MAA website, *maa.org*.

As of this writing, the materials have to be in English, come from the mathematical sciences, and be published or developed within 15 years of the date of submission. Nominations must provide evidence of at least three institutions where the materials have helped student learning during the last three years.

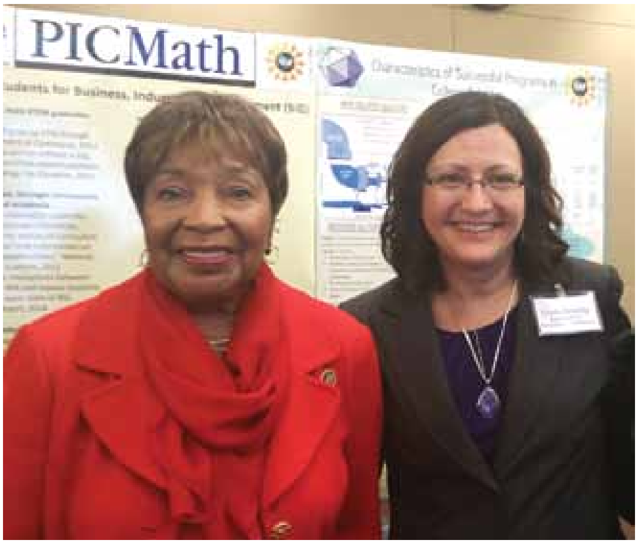
Daniel Solow decided to honor people who write outstanding educational resources for students.

MAA Supports Funding Research on Capitol Hill

Each year when members of Congress are preparing to vote on the National Science Foundation's budget, hundreds of researchers in science, mathematics, engineering, and education gather to show lawmakers the value of nationally funded studies.

Nearly 300 researchers displayed their NSF-funded research at the 21st annual Coalition for National Science Funding (CNSF) on April 29 at the Capitol in Washington, D.C. Linda Braddy, deputy executive director at the MAA, and Jessica Ellis, assistant professor in mathematics at Colorado State University, represented two projects supported by the MAA to the attending members of Congress and their staffs.

The exhibition "is a good way to demonstrate the validity of the NSF," says Samuel Rankin, chair of the CNSF and director of the American Mathematical Society's Washington office. "We want staff and members of Congress to see what projects NSF is funding by talking to the [principal investigators] and students about their research."



Rep. Eddie Bernice Johnson (Texas)(left), ranking member of the House Science, Space & Technology Committee, with Linda Braddy at the 21st CNSF on Capitol Hill.

Linda Braddy (left), Sen. James Lankford of Oklahoma, and Jessica Ellis met in the senator's office.

Calculus Programs

Ellis's material focused on a study about "Characteristics of Successful Programs in Calculus." She also spoke to lawmakers about a complementary study that identifies the features of successful Precalculus and Calculus II courses.

Braddy presented details on Preparation for Industrial Careers in Mathematical Sciences (PIC Math)—a program that prepares math students for nonacademic careers in business, industry, or government.

"This was an opportunity to highlight some of the projects and research that NSF funding is helping [the MAA] accomplish," says Braddy.

Nine members of Congress attended the event: Chaka Fattah (Pa.); Randy Hultgren (Ill.); Eddie Bernice Johnson (Texas); Rick Larsen (Wash.); Jerry McNerney (Calif.); John Moolenaar (Mich.); Mike Simpson (Idaho); Mark Takano (Calif.); and Paul Tonko (N.Y.). Staff members from other congressional offices also came by.

Meetings

The researchers who attend the event also took the opportunity to contact members of Congress who could not stop by the exhibit.

Braddy and Ellis visited with staff members for Tom Cole (Okla.); Barbara Mikulski (Md.); Jarid Polis (Colo.); Scott Peters (Calif.); Ander Crenshaw (Fla.); Ileana Ros-Lehtinen (Fla.); and James Lankford (Okla.) to talk about national funding for mathematics and education research.

Exhibitors tend to get in to see their home-state representatives because policymakers listen to constituents, says Rankin. They want to see what their constituents are involved in, which yields an opportunity to talk about why funding is important.

The overall goal of the CNSF, which is made up of more than 140 STEM-related organizations, is to increase the national investment for the NSF's research and education programs in the United States.

—Alexandra Branscombe

Supporting Pathways to Mathematics Careers

By Melanie Huggans and Ashanti Johnson

Diversity matters! At the Institute for Broadening Participation (IBP) we concur with the findings of the National Science Foundation and others that diversifying the STEM workforce is crucial to developing our nation's intellectual capital and contribute to this vast effort by providing tools and resources that guide students to success in a STEM career. We also support faculty mentors as they engage the next generation of scientists and engineers. For those who are interested in the various areas of mathematics, IBP makes available a multitude of programs and helpful tips to enhance education and professional development.

Mathematics Student Resources

IBP is committed to connecting math students throughout their academic career with multiple resources such as scholarships, fellowships, and summer research programs and later with postdoctoral and early-career support. Our Pathways to Science website at pathwaystoscience.org is IBP's open-access resource that offers search tools to find contacts and information for more than 1,500 educational and career training and funding opportunities in science, technology, engineering, and mathematics (STEM), from K-12 to postdoc. Currently, 210 summer research programs, 16 post-baccalaureate programs and 88 graduate programs are posted for mathematics and computational sciences fields.

Mathematics career opportunities posted on this website include programs that offer portable funding, study abroad components, and funding for international students. Furthermore, options are available for specific math disciplines, including statistics, operations research, and computational sciences. The Pathways to Science website allows students to sign up to receive customized information and opportunities based on individual interests and academic levels.

For those who wish to secure research opportunities and funding, the Resource Toolbox offers a breadth of information that informs undergraduate and graduate students about critical topics, including creating a winning application, a timeline for submitting applications, and how to obtain a strong letter of recommendation. Additionally, our open-access webinar archive covers a variety of topics on best practices for obtaining funding and research opportunities, building mentoring and networking support systems, and advancing professional development skills.

Faculty Resources

The site pathwaystoscience.org gives faculty and administrators access to resources that can help them reduce barriers to participation, create environments rich in the positive factors that support student success on the STEM pathway, and conduct outreach to underserved communities and underrepresented groups. The material provides insight into topics that are not typically taught in class, but are critical to prepare for a STEM career.

IBP conducts year-round outreach to students publicizing the opportunities and resources posted on pathwaystoscience.org. Since the site receives over 100,000 visits per month during peak portions of the academic calendar, faculty and administrators can post eligible opportunities in mathematics and computational science to benefit from the student traffic.

Tools on this website equip faculty and administrators with professional development resources designed to supplement academic programs. Resources for faculty include "Designing for Success: Positive Factors that Support Student Success in STEM," "Using Social Media to Build Diversity in Your REU," and "Recruitment Strategies." Other resources to support recruitment, retention, and partnership-building efforts can also be accessed here.

The *Online Mentoring Manual,* a reference for faculty and program directors, helps faculty better understand the role of mentoring and how it affects them and their students. Contributors to the manual include PAESMEM (Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring) awardees, industry leaders, and experienced educators who have made the study and practice of mentoring an integral part of their professional lives.

In addition, the Online Diversity Reference Library offers an annotated list of peer-reviewed articles, seminal studies, and intervention studies related to broadening participation in STEM; and the Partners Directory contains more than 14,500 faculty and staff at STEM programs across the country, including NSF, AGEP, ADVANCE, GK-12, IGERT, LSAMP, McNair, MS PHD'S, PAESMEM, PIRE, RDE, REU, STC, TCUP, and NASA.

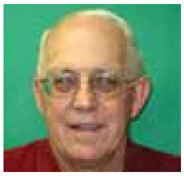
On the Pathways to Science website, there is something for everyone interested in a career in mathematics. Check it out today.

Melanie Huggans is the director of operations and Ashanti Johnson is the executive director at the Institute for Broadening Participation, pathwaystoscience.org.

2015 MAA Section Teaching Award Winners

EPaDel
Pam Gorkin
Bucknell University

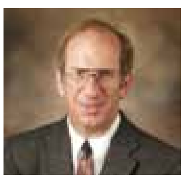
Florida
Patrick Bibby
University of Miami



Golden
Michelle Manes
University of Hawaii at Manoa

Illinois
Olcay Akman
Illinois State University

Indiana
Kenneth Constantine
Taylor University

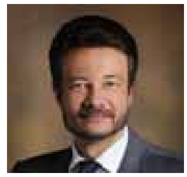


Intermountain
Matt Ondrus
Weber State University

Kansas
Cynthia Huffman
Pittsburg State University

Kentucky
Alex McAllister
Centre College

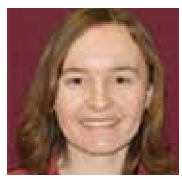
Louisiana-Mississippi
Bernd Schroeder
Louisiana Tech University



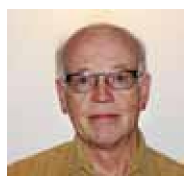
MD-DC-VA
Caren Diefenderfer
Hollins University

Metro New York
Jerry Ianni
LaGuardia Community College

Michigan
Christine Phelps
Central Michigan University



New Jersey
Denis Blackmore
New Jersey Institute of Technology



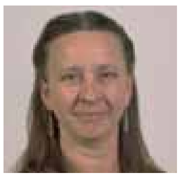
North Central
Daniel Flath
Macalester College

Northeastern
Donna Beers
Simmons College

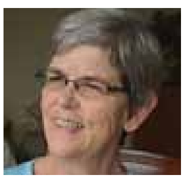
Oklahoma-Arkansas
Max Forester
University of Oklahoma

Pacific Northwest
Glen Van Brummelen
Quest University

Rocky Mountain
Janet Heine Barnett
Colorado State University-Pueblo

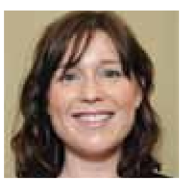


Seaway
Margaret Morrow
SUNY Plattsburgh



Southeastern
Maurice Hendon
University of Georgia

Beginning Faculty
Kristen Abernathy
Winthrop University



Southwestern
Dan Madden
University of Arizona

Texas
Rebecca Garcia
Sam Houston State University

Wisconsin
John Bean
University of Wisconsin-Oshkosh

Awards at MAA MathFest

MAA bestows several of its writing and teaching awards during MAA MathFest. Recipients and details will be listed in the October/November issue. The awards booklets for MAA MathFests and the Joint Mathematics Meetings are on our website at maa.org/awards. *Some of the honored articles, such as the Trevor Evans Award winners, are also online (*maa.org/programs/maa-awards/writing-awards*).*

Fall Section Meetings

INDIANA
October 17, Purdue North Central, Valparaiso, IN

IOWA
October 2-3, Graceland University, Lamoni, IA

OHIO
October 23-24, Capital University, Bexley, OH

SEAWAY
Nov. 6-7, St. Lawrence University, Canton, NY

Some Guidelines for Good Mathematical Writing

By Francis Edward Su

President's Message

Communicating mathematics well is an important part of doing mathematics. Many of us know from writing papers or giving talks that communicating effectively not only serves our audience but also clarifies and structures our own thinking. There is an art and elegance to good writing that every writer should strive for. And writing, as a work of art, can bring a person great personal satisfaction.

Within the MAA, we value exposition and mathematical communication. In this column, I'm sharing the advice I give my students to help them write well. There are more extensive treatments (e.g., see Paul Halmos's *How to Write Mathematics*), but I wanted a shorter introduction. So I developed the guidelines below.

Basics

Know your audience. This is the most important consideration for writers. Put yourself in your reader's shoes. What background can we assume of the reader? What terminology should we define? What kind of "voice" do we want to project: casual or professional, serious or inviting, terse or loquacious?

If you are a student writing solutions for a homework set and your professor has not specified your audience, a good rule of thumb is to assume you are writing to another student in the course who has not yet done the assignment. Though you may assume that she has attended all the same lectures and has read the same textbook, it is standard courtesy to remind your readers of any relevant items that they have learned recently from the class or textbook, or things they should know but might have forgotten.

For instance, if the concept of a rational number was only recently learned in class, you might insert "Recall that a rational number can be expressed as a fraction" before saying "since x is rational, $x = m/n$ where m and n are integers."

Set an invitational tone. It is traditional to create an inviting atmosphere in one's mathematical writing. In effect, we invite readers to join us in our reasoning process by writing in the present tense, using the pronoun "we" instead of "I" (e.g., "we construct a tangent plane..."), and directing the reader with gentle commands (e.g., "let n be . . . ," "recall that . . . ," or "consider the set of . . .").

Use complete sentences. All mathematics should be written in sentences. Open any mathematics text and you'll see that this is true. Equations, even displayed ones, have punctuation that helps you see where it fits in the context of a larger sentence. Consider this piece of writing:

$$(x-2)^2 + (x-1)^2 = 5^2\ 5^2 = 25$$
$$(x-2)^2 = x^2 - 4x + 4 + x^2 - 2x + 1 = 25.$$
$$2x^2 - 6x - 20$$
$$2(x+2)(x-5)\ x = \text{-}2, 5\ x > 0\ x = 5$$

Can you figure out what the writer is doing? What's being assumed? What's being proved? Where does one thought end and another begin? What's the relationship between these phrases? Some phrases are dangling, and others, as statements, are not even true. The reader should not have to figure out what the writer was thinking.

Now consider the work of another writer attempting the same problem:

> **Problem.** Find a point on the line $y = x$ that is distance 5 from the point (2,1) and whose x-coordinate is positive.
>
> **Solution.** The desired point is (5,5). To see this, we solve $(x-2)^2 + (x-1)^2 = 5^2$, an equation obtained from the distance formula in the plane. A little algebra turns this equation into:
>
> $$2x^2 - 6x - 20 = 0.$$
>
> Factoring the left side, we obtain
>
> $$2(x+2)(x-5) = 0,$$
>
> whose solutions are $x = -2$ and $x = 5$. Since we assumed $x > 0$, we have (5,5) as the desired point on the line $y = x$.

Here, the writer has clearly stated the problem and described her path to a solution. She has set an invitational tone, and every thought is expressed in a complete sentence. Now it is clear that x > 0 is a condition, not a result. Notice the punctuation in equations: one ended with a period because her thought was complete, the other ended with a comma because she wanted to continue the thought.

Since she assumed her audience could do algebra, she didn't bore them with algebraic manipulation, which would obscure the thread of her arguments. But she did show the crucial and most interesting piece: the factoring and its result. And she made sure she answered the original question.

Use words to give context to equations. Consider the difference in meaning between these three statements: "Let $A = 5$." "Suppose $A = 5$." "Therefore $A = 5$." Then reflect on the ambiguity of the statement "$A = 5$."

Avoid shorthand in formal writing. The many types of mathematical writing can be loosely grouped into formal and informal writing. Informal writing includes writing on a blackboard during lecture, or explaining something to a friend on a piece of scratch paper. Formal writing includes the kind of writing expected on a homework assignment or in a paper. There are differences in what is acceptable. For instance, in informal writing, it is common to use shorthand for quantifiers and implications: symbols such as $\forall$, $\exists$, $\Rightarrow$, $\Leftrightarrow$, or abbreviations such as "iff" and "s.t."

However, in formal writing, such shorthand should generally be avoided. You should write out "for all," "there exists," "implies," "if and only if," and "such that." Most other symbols are acceptable in formal writing, after defining them where needed. The membership symbol $\in$ is traditionally acceptable in formal writing, as are relations (e.g., $<$, $+$, $\cup$, etc.), variable names (e.g., x, y, z), and symbols for sets (e.g., $\mathbb{R}$). Here is an acceptable use of symbols in formal mathematical writing:

> Let A and B be two subsets of $\mathbb{R}$. We say A *dominates* B if for every $x \in A$ there exists $y \in B$ such that $y > x$.

Learn the etiquette. The above example also illustrates two common conventions of mathematical etiquette. It is customary to avoid beginning sentences or phrases with a number or symbol because that can be confusing. It is also customary to emphasize unfamiliar words that we are about to define, such as by italicizing them. Other etiquette can be learned by observing the norms used in your area of study.

Toward Elegance

Decide what's important to say. Writing well does not necessarily mean writing more. If your solution is too wordy, it can sometimes obscure the points you are making.

A well-written solution will present just enough details and highlight the most interesting or unexpected parts of the argument. What theorems or axioms were crucial in getting your solution, and where were they used? Your role as a writer is not primarily to give details (though that can be important). Your primary role is to give insight.

Highlight structure. If your argument is going to be a long one, with lots of technical details, then try to help the reader by summarizing the outline of your argument at the beginning. Then, throughout your writing, help your reader see how you are progressing through your outline.

Use paragraphs to emphasize blocks of ideas that are related. The role of the first sentences of paragraphs is crucial: imagine a reader skimming your writing and reading only the first sentences. Will she see the flow of your argument? Similarly, you might want to display only the most important equations. Replacing an oft-repeated argument by a good lemma can streamline the flow as well as highlight a key idea.

Choose good examples. A difficult idea may be easier to digest if accompanied by an example. Choose one simple enough to follow, but interesting enough to retain the salient features. A proof of a very general idea could be preceded by an example in a specific context. A long exposition might benefit from a running example—one in which the same example is used multiple times in different contexts.

Avoid red herrings! Omit details that have no bearing on the solution of the problem, because they may throw the reader off. For example, if you say "we express the rational r as m/n where m and n have no common factors," you are leaving a clue that later you will use the "no common factors" idea. So if you never use that fact, you should omit saying it. It's extraneous. Red herrings may make mystery novels fun, but in mathematical writing, your goal is to dispel mystery!

Step back and simplify. After writing a proof, step back and ask: How can I simplify this argument? Did I use every tool I pulled out to solve this problem? Can I streamline this argument? For example, consider this apparent proof by contradiction:

> **Problem.** Show that if 4 divides an integer n, then n is even.
>
> *Proof.* Suppose n were not even.
> Since 4 divides n, we have $n = 4k$ for some integer k.
> Thus $n = 2(2k)$, which is even.
> This contradicts our hypothesis that n is not even. QED.

Do you see why this is not really a proof by contradiction? The contradictory hypothesis in the first sentence was never used! Strip away the first and last sentence, and you have an elegant, direct proof.

Refine, refine, refine. Good writing is a process of successive approximations. You should not expect your first draft to be perfect. You will find that when you review your writing, you will see ways to shorten an argument or say something in a better way. This is the

part of the writing process that will help clarify your own thinking as well.

Often, after completing a draft, a writer may notice that a particular choice of notation or definition was not optimal. A lazy writer would leave things as they are, but a thoughtful writer will take the time to go back and make changes.

Observe the culture. Good communication is inseparable from the culture in which it takes place. This realization may unsettle budding mathematicians who are attracted to the logical absolutes of mathematics. But even these absolutes are expressed differently by mathematicians of different eras, as can be seen by comparing Newton's writings with any of today's calculus texts. The rules of mathematical etiquette have evolved.

Although these guidelines attempt to draw up some common principles for formal writing, there will always be exceptions—because some mathematical field may have a slightly different norm. The best way to get a sense of what is acceptable in your context is to browse several highly regarded texts or papers related to the document you are writing.

Enjoy the art of writing. Writing is an occasion to reflect on beautiful ideas and paint them on a paper canvas with great artistic care.

Acknowledgments. It is appropriate to acknowledge any support you received. My revisions of these guidelines benefited greatly from the helpful feedback of Jon Jacobsen and Lesley Ward.

Francis Edward Su is president of the MAA. His email is su@math.hmc.edu. And he is on Twitter: @mathyawp.

Interview » Richard A. Gibbs

Professor of Mathematics Emeritus, Fort Lewis College

Interviewed by Kenneth A. Ross

Kenneth A. Ross is an emeritus professor in the Mathematics Department at the University of Oregon.

As a project for the 2015 centennial of the MAA, members of the history subcommittee of the MAA centennial committee have been interviewing prominent members of the mathematical community. The full interviews will be available on the MAA website at some point; excerpts from selected interviews are appearing in MAA FOCUS.

The excerpts here are based on an interview that took place in October 2008 in Salt Lake City.

MAA 100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

When did you get interested in mathematics?

My granddad, who was a merchant and ran a grocery store, was the first influence. He would give me problems and so would my father, especially when we were driving from where we lived in Midland, Michigan, to the Lake Michigan coast where my grandfather lived.

I was blessed with good math teachers from the eighth grade on. As far as I can recall, they were all men. When I was in eighth grade, I wanted to be an eighth-grade teacher; when I was in ninth grade, I wanted to be a ninth-grade teacher, etc. My desire was always to teach, and since I was best at math, I wanted to be a math teacher. I ended up being a college teacher because it was easier than going through the education requirements for teaching high school.

Where did you go to school?

I was an undergraduate at Notre Dame for three years. Then I switched to Michigan State University for my senior year, and I stayed on for my PhD. My thesis adviser was J. Sutherland Frame, who was an excellent teacher and a "matrix magician." He was a good adviser, though I had to search him out, and, if I wasn't making much progress, it was easy to avoid him. I was interested in graph theory after taking a fine course by Ed Nordhaus. Frame was primarily interested in matrix theory, but the two areas have a significant overlap, so Frame and I were able to work out a good program.

I earned my PhD in 1970 and received several job offers. I accepted a position at Hiram Scott College in Scottsbluff, Nebraska. This was a challenging teaching job because this school specialized in students who had been turned down elsewhere, many of whom were from the east. They even started their school year a couple of weeks late to accommodate late students. During the year we learned that the school was about to fold. I stayed optimistic, but finally realized that I should look for another job. That was 1971, and that's when I took a job at Fort Lewis College in Durango, Colorado.

Did your apparent lifelong interest in problem solving come from your interaction with Professor Frame?

Only incidentally. At some point when I was a graduate student, I was given a free MAA membership. A problem posed in *Mathematics Magazine* grabbed my attention, and I got hooked on solving problems in the various MAA journals. In particular, I solved one problem and my solution was published; the poser later became infamous: Ted Kaczynski. The problem was about match sticks in a grid (see box).

Here's another problem-solving activity that I was involved with. I wrote a Sunday "Puzzle Corner" column for our local Durango *Herald* between January 1982 and June 2006 (there was a gap of a few months when I stopped writing it, then started again). In all, there were 1,000 puzzles—that seemed a good number to end with, although some thought I should continue to 1,024! I was motivated to write this column by Lee Younker, a high school math teacher in the Chicago suburbs who, at an NCTM [National Council of Teachers of Mathematics] meeting that I attended in 1980 or 1981, talked about doing a similar thing.

How did you come to be so heavily involved with AMC?

Steve Maurer, chair of CAMC [Committee on the American Mathematics Competitions] in the mid-1980s, noted my interest in problems and invited me to be a panelist about 1985. Then in 1987 I was asked to be on the AHSME [American High School Mathematics Examination] committee.

At this point, Leo Schneider was chair of both CAMC and AHSME. After his six-year term, Leo stepped down and they split the jobs. I became chair of CAMC and served 1993–2002.


L. McHUGH

Gibbs was involved for many years with MAA competitions.

During this period, I was also coeditor of the problem department of *School Science and Mathematics*, a publication of SSMA [School Science and Mathematics Association]. This was during 1985–1998, and my coeditor was my colleague Laszlo Szuecs.

At the end of my second three-year term, there were some questions regarding the structure of the CAMC and the position of executive director of the AMC. Tina Straley was the new MAA executive director, and Walter Mientka had stepped down as AMC director to become executive director of IMO 2001 USA. So I agreed to chair the CAMC for another term.

Tina was working to structure the CAMC so that it was more clearly seen to be a program of the MAA, with voting members appointed by the MAA. Previously, representatives of the other sponsors constituted a majority of the CAMC. Also, there was some question as to whether Walter was going to return as executive director of the AMC after the IMO.

How was it working with Walter Mientka and others?

I had great affection and admiration for Walter, and I feel honored to have had the opportunity to work with him. It was quite an experience! Walter was always fighting for more autonomy for the AMC and consequently rubbed some of the people in the MAA the wrong way. But I always thought we got along just fine.

I was also greatly impressed with Leo Schneider. He had an amazing capacity for hard work and was very efficient. The same is true of his predecessor, Steve Maurer.

What is your best memory of your work in the MAA?

The most enjoyable experiences were the USAMO awards ceremonies in Washington and serving as the emcee for the receptions beforehand at MAA headquarters.

An enjoyable outgrowth of work in the MAA has been the Colorado Mathematics Awards program that started up in 1996. I am cochair of the organizing committee with David Carlson, who is now retired from the Colorado Department of Agriculture.

Every year we recognize outstanding mathematics students from eighth grade through high school, based on their performances on AMC exams. In addition, we honor top collegiate performers on the Putnam exams and on the MCM [Mathematical Contest in Modeling].

What personalities have stood out in the mathematical community, both in the MAA and at large?

I've already mentioned Walter Mientka and J. S. Frame. Another was Paul Erdős. A group of us from Fort Lewis College met Erdős at the joint winter meetings in Dallas in 1973. At the meeting, I approached him to ask whether he'd consider visiting Durango. He knew about Mesa Verde [National Park], so he arranged to visit Durango while traveling from Boulder, Colorado, to UCLA.

He wanted to visit Mesa Verde, and I asked for volunteers to go with him. No one was available, so I had the pleasure of spending the entire day with him alone. It was quite a day. I was really afraid for him, climbing dangerously in those notorious sandals of his just to get a better view down some canyon!

And I remember asking him, when we were having lunch, if he took many vacations. He said that he took a couple of weeks off a few years earlier, but never again because he found it too depressing.

A Match Stick Problem

From *Mathematics Magazine* **44** (January-February 1971), problem 787 on page 41. Proposed by T. J. Kaczynski, Lombard, Illinois.

Suppose we have a supply of matches of unit length. Let there be given a square sheet of cardboard, n units on a side. Let the sheet be divided by lines into n^2 little squares. The problem is to place matches on the cardboard in such a way that: (a) each match covers a side of one of the little squares, and (b) each of the little squares has exactly two of its sides covered by matches. (Matches are not allowed to be placed on the edge of the cardboard.) For what values of n does the problem have a solution?

Two solutions were published in *Mathematics Magazine* **44** (November-December 1971), page 294. Richard A. Gibbs solved the problem invoking Pick's theorem. Richard L. Breisch solved an m by n generalization.


COME FOR THE
MATH.
STAY FOR THE
STORIES.
Visit QuantaMagazine.org for the latest developments in mathematics, theoretical physics and the life sciences.
QUANTA MAGAZINE
illuminating science
An editorially independent publication of the Simons Foundation

USAMO Winners Watch World Turn Inside Out

By Alexandra Branscombe

There is no question that the 12 high school students gathered in the MAA Carriage House in early June see mathematics differently than their peers. For these top scorers of the USA Mathematical Olympiad, mathematical problems are not just a challenge, but a clever mystery to solve.

After hours of poring over a difficult problem, the answer that comes into view can be elegant, said Kritkorn Karntikoon, a senior from Windsor, Connecticut. "When I solve [Olympiad problems], I'm proud of myself and smile because of the beauty of the problems," he said.

To celebrate their mathematical accomplishment, the USAMO winners, along with their parents and mentors, spent a day at MAA headquarters in Washington, D.C., on June 2. The morning program included remarks from MAA leaders and donors, followed by a lecture from a mathematician.

This year's presenter was Bruce Torrence, professor of mathematics at Randolph-Macon College in Ashland,

Virginia, who wanted the guests to see how a special tripod, some photo-stitching software, and a passion for math can change the viewable world.

What you can see from your single vantage point (up, down, all the way around) is called the viewable sphere, explained Torrence. Most photos capture one perspective—looking in one direction. Even with a wide-angle lens, a single photograph cannot capture the full scene.

To capture the viewable sphere, a photographer must take a series of photographs in every direction from the same point in space, he said. Special software can stitch together the photo series to create a seamless panorama.

On a computer, this panorama can be explored interactively, like Google Street View—a panorama application that most people are familiar with, he said.

However, when panoramas are placed on a two-dimensional plane, the image can be easily distorted. This issue can be solved with a mathematical property called a conformal map, which will preserve the angles in a photograph at the local level. Conformal mapping means that the details do not get skewed or stretched, said Torrence.

With an understanding of mathematics, a photographer can harness the conformality property of the projection to create a whole new scene that is also accurate at a detailed level.

One striking version of a conformal mapping is called stereographic projection, and images that use it have been charmingly dubbed “Little Planets” in the photography world. The horizon of the panorama, and everything below it, gets projected into a circle, looking like the equator of a tiny world. →

But change the conformal map and suddenly the picture can be inverted so the viewer has the sensation of looking through a tunnel. In fact, one can apply any conformal mapping to a spherical image to create different kinds of photographic effects, Torrence said.

Conformal mapping.

"This presentation is one example of how mathematics gives us new ways to look at the world," said MAA President Francis Su as the morning activities concluded.

Much like Torrence flips his panoramas into small worlds and tunnels, the USAMO winners are adept at changing their perspectives in order to solve a problem.

"Solving Olympiad questions takes a different set of skills," said Celine Liang, who recently won a gold medal at the European Girls' Mathematical Olympiad. "Students who have mastered thinking deeply about problems and keeping track of multiple threads of thought are successful at proof-based Olympiad contests."

Keeping a toolkit of methods ready is key to solving high-level Olympiad problems, but equally as important is knowing when to switch gears and take a new approach, said Yang Liu, a senior from St. Louis, Missouri.

"I think that is an interesting mix of techniques you have to know, and keeping an open mind to know when a certain approach fails to try another one," said Liu, who received a gold medal at last year's International Mathematical Olympiad. "If you try different approaches, one of them will work in the end."

The USAMO award ceremony took place at the U.S. Department of State, where each winner was presented with a medal of achievement. Su's address, "Combinatorial Fixed Point Theorems," involved an homage to the late John Nash and his contributions to game theory.

The reception and three-course dinner followed, including remarks from John Holdren, director of the White House Office of Science and Technology Policy. Holdren also read a personal letter from President Barack Obama congratulating the winners on their accomplishment. The evening concluded after leading donors presented financial prizes to the top 12 scorers.

When asked how the MAA and the USAMO award has affected his life, David Stoner said, "One way the MAA has benefited me is that they have given me a lot of great problems to think about, and a lot of great people to think about them with."

Alexandra Branscombe is MAA FOCUS *staff writer.*

Awards and Prizes

Samuel L. Greitzer/Murray S. Klamkin Award for Mathematical Excllence: David Stoner, Allen Liu

Robert P. Balles USA Mathematical Olympiad Prize: USAMO Winners

Akamai Foundation Scholarship Awards: First Place: Allen Liu, David Stoner

USAMO Sponsors

The MAA acknowledges the generous support of donors who help sustain its USAMO, IMO, and American Mathematics Competitions.

Patrons Circle – Akamai Foundation, Simons Foundation.

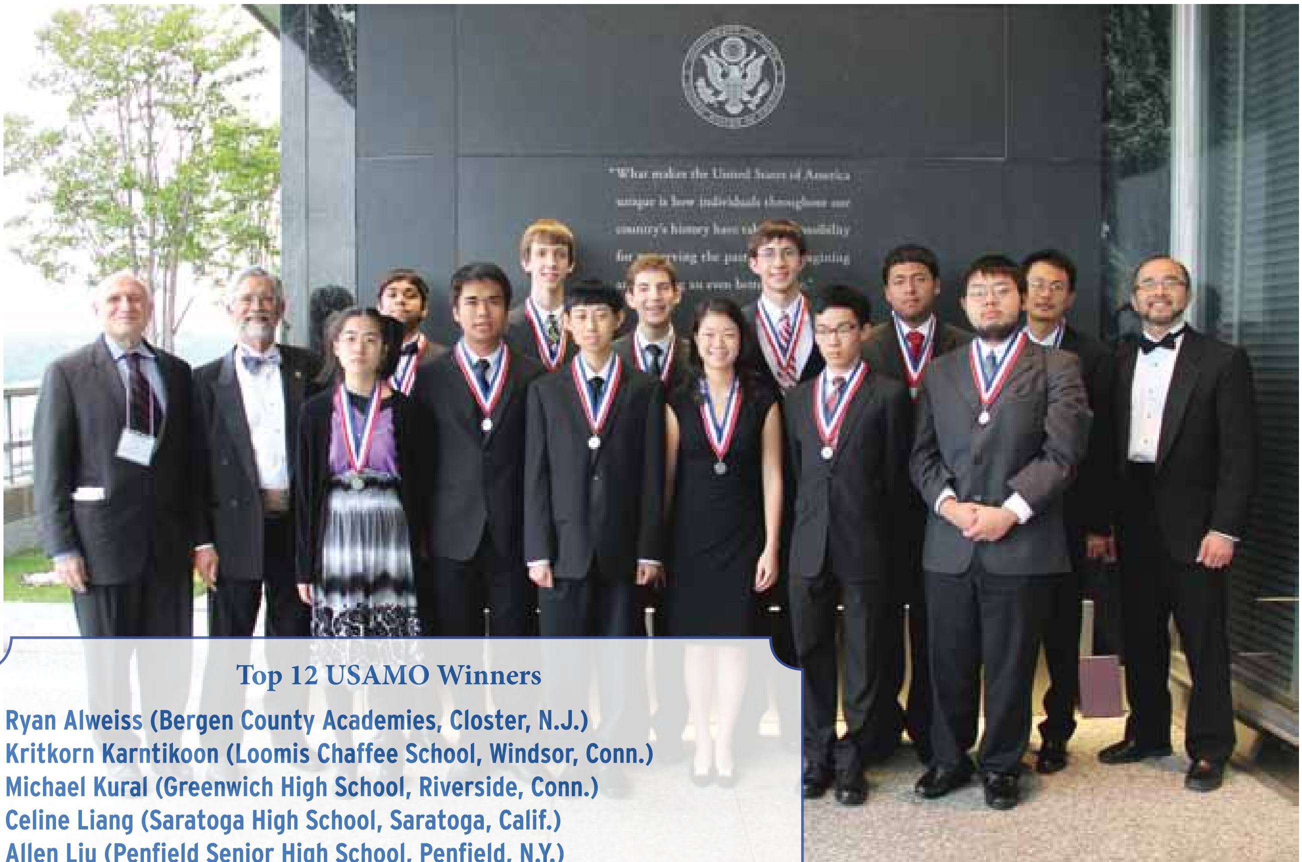
Winners Circle – American Mathematical Society, The D.E. Shaw Group, MathWorks Inc. (in support of Curriculum Inspirations), Tudor Investment Corporation, Two Sigma.

Achiever's Circle – Art of Problem Solving, Jane Street Capital, Math for America. Sustainer's Circle -- Academy of Applied Science, American Statistical Association, Army Educational Outreach Program.

Collaborator's Circle – Casualty Actuarial Society, Conference Board of the Mathematical Sciences, Expii Inc., IDEA MATH LLC, Mu Alpha Theta, National Council of Teachers of Mathematics, Society for Industrial and Applied Mathematics, Star League.

Thanks are due to Robert Balles for his support of the 2015 Balles Prize awards.

TUNNEL PHOTO: BRUCE TORRENCE; PRECEDING AND OPPOSITE PHOTOS: L. MCHUGH



Top 12 USAMO Winners

Ryan Alweiss (Bergen County Academies, Closter, N.J.)
Kritkorn Karntikoon (Loomis Chaffee School, Windsor, Conn.)
Michael Kural (Greenwich High School, Riverside, Conn.)
Celine Liang (Saratoga High School, Saratoga, Calif.)
Allen Liu (Penfield Senior High School, Penfield, N.Y.)
Yang Liu (Ladue Horton Watkins High School, St. Louis, Mo.)
Shyam Narayanan (Blue Valley West High School, Overland Park, Kan.)
Kevin Ren (Torrey Pines High School, San Diego Calif.)
Zhuo Qun "Alex" Song (Phillips Exeter Academy, Exeter, N.H.)
David Stoner (South Aiken High School, Aiken, S.C.)
Kevin Sun (Phillips Exeter Academy, Exeter, N.H.)
Danielle Wang (Andrew Hill High School, San Jose, Calif.)

Above, the USAMO winners pose at the State Department with (far left) Mark Saul (MAA's director of competitions) and John Holdren (director of the White House Office of Science and Technology Policy) and (far right) Francis Su, MAA president. Below: Winners enjoy a full day in D.C., including activities at the MAA Carriage House before a formal dinner at the State Deparment (preceding pages).

U.S. Team Takes Top Honors at IMO

A six-member team representing the United States captured the top spot at the International Math Olympiad held June 4–13 in Thailand. It is the first time in 21 years the U.S. team has won first. Stories about the victory ran on National Public Radio (*http://n.pr/1DnSHnp*) and in the *Washington Post* (*http://wapo.st/1TH12vK*). Because of production deadlines, a feature in *MAA FOCUS* will appear in the October/November issue. Until then, you can check out details on the MAA website, *http://bit.ly/1HuTaFS*.

Naked Sudoku

By Laura Taalman

Puzzle Page

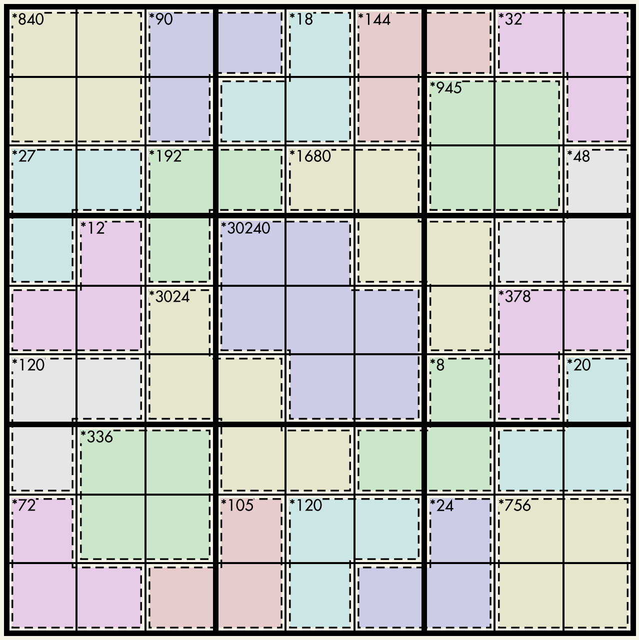
In the April/May 2012 issue of *MAA FOCUS*, we discussed Sudoku puzzles with a minimum number of clues. In 2011–2012, Gary McGuire used 7 million hours of supercomputing time to prove by exhaustive search that the fewest number of clues that a valid (unique solution) Sudoku puzzle could have is 17. For puzzles with rotationally symmetric clue locations, the fewest number of clues for a valid Sudoku puzzle is thought to be 18. But we're not here to talk about any of that today. Today, we are going to look at puzzles that have no clues at all. Naked Sudoku.

Each of these puzzles comes from the Brainfreeze Puzzles book *Naked Sudoku* (Philip Riley/Laura Taalman), published by PuzzleWright Press. You can read more about the mathematics behind and concerning Sudoku in the Jason Rosenhouse/Laura Taalman book *Taking Sudoku Seriously,* published by Oxford University Press. For more on Gary McGuire's work on the minimum number of clues problem, see his "Sudoku Checker" website at math.ie/checker.html. For a look at some of the Sudoku-related mathematics being investigated without computers, try the paper by Elizabeth Arnold, Rebecca Field, Stephen Lucas, and myself called "Minimal Complete Shidoku Symmetry Groups" from the *Journal of Combinatorial Mathematics and Combinatorial Computing* (November 2013).

Laura Taalman is editor of the Puzzle Page column. To suggest puzzles for the column, contact maafocus@maa.org.

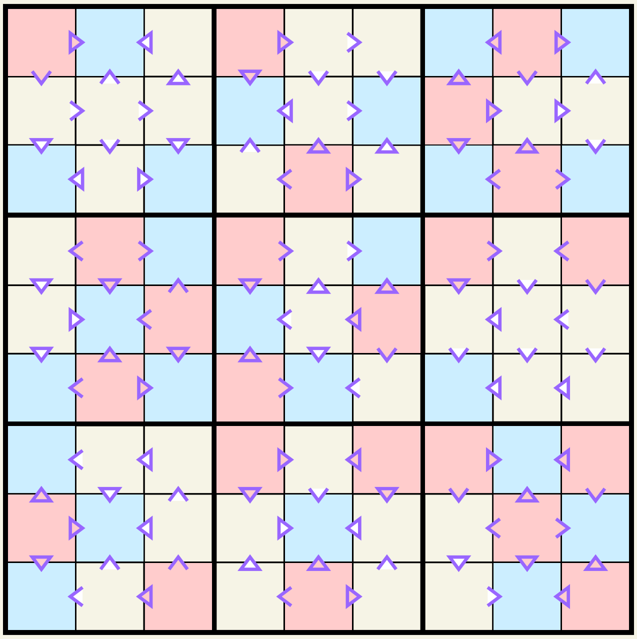
Puzzle 1: Product Sudoku

By adding structure to the grid itself, we can provide enough information so that no initial clue values are needed at all. In Product Sudoku, the rules are just like regular Sudoku (1–9 in each row, column, and 3×3 block), but in addition there are no repeated entries in any colored region, and the small numbers at the top of each region indicate the product of the entries within. Hints for getting started: 5s and 7s are easier to place than other numbers. Use divisibility facts to your advantage.



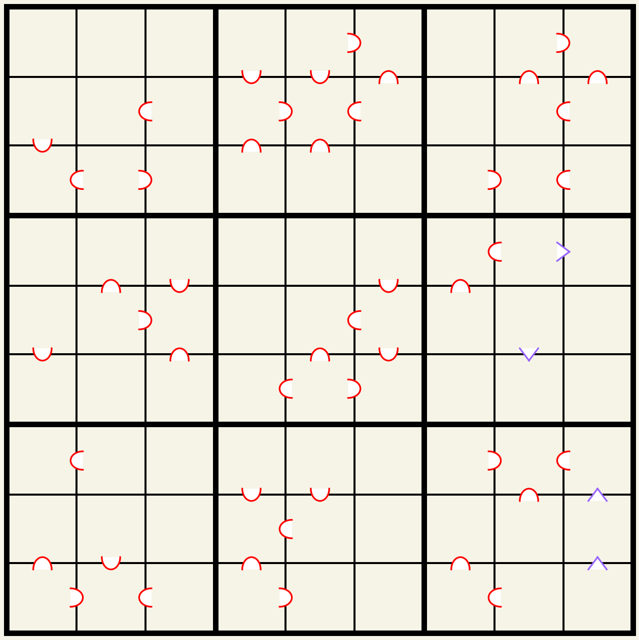
Puzzle 2: Greater Than Greater Sudoku

In this puzzle we again place significance on the actual numerical values of the entries, but add structure a different way: this time, by indicating whether cells are larger or smaller than their neighbors. In Greater Than Greater Sudoku, the usual rules of Sudoku apply, but in addition the "greater than" signs indicate which of two neighboring entries is larger, and the "triangular greater than" signs indicate that one entry is more than one unit greater than its neighbor. Hints for getting started: The pink and blue cells can help you place some 1s and 9s; then try placing 2s and 8s, and so on.



Puzzle 3: Division Sudoku

In a sort of hybrid of the previous two puzzles, Division Sudoku uses symbols to indicate when one cell is a divisor of its neighbor, where A ⊂ B means that A divides B. Some cells are also separated by a "greater than" sign to indicate that one cell is larger than another. Bonus question: This type of puzzle will always need to have at least one "greater than" symbol; why?



MAA American Mathematics Competitions

Middle School and High School Levels

The MAA American Mathematics Competitions provides:

- Great math problems that engage your students and connect with the Common Core State Standards
- Print and video teaching resources found in Curriculum Inspirations by award-winning mathematics teacher and scholar James Tanton
- MAA K-12 teacher membership – electronic subscriptions to *Math Horizons* and *MAA FOCUS* magazines, *AMC Math Messenger* and *MAA Math Alert* monthly electronic newsletters plus discounts on national meetings, books and videos

Students use words like “fun,” “challenging,” “great math,” and “bringing math to life” when describing the MAA American Mathematics Competitions.

Register

for the MAA American Mathematics Competitions (AMC), the oldest, largest, and most prestigious mathematics competition in the United States.

AMC 8 Contest Date
Tuesday, November 17, 2015

AMC 10 and AMC 12 Contest Dates
Tuesday, February 2, 2016
Wednesday, February 17, 2016

To register and for more information visit amc.maa.org.
Questions: Email amcinfo@maa.org or call 800 527-3690.

MAA 100
MATHEMATICAL ASSOCIATION OF AMERICA
CELEBRATING A CENTURY OF ADVANCING MATHEMATICS

EXCERPT » A Century of Advancing Mathematics

As part of the MAA's celebration of its 100th anniversary, we have assembled a volume of essays from esteemed mathematicians. Every MAA member has received an electronic copy of A Century of Advancing Mathematics *(see p. 2). Physical copies of the book can be purchased on* maa.org. *Here is an excerpt of one of the articles. Read the complete essay in* A Century of Advancing Mathematics, *an MAA Press publication.*

Defying God: The Stanley-Wilf Conjecture, Stanley-Wilf Limits, and a Two-Generation Explosion of Combinatorics

By Eric S. Egge

In March of 2005, at the Third International Conference on Permutation Patterns in Gainesville, Florida, Doron Zeilberger declared that "Not even God knows $a_{1000}(1324)$." Zeilberger's claim raises thorny theological questions, which I am happy to ignore in this article, but it also raises mathematical questions. The quantity $a_{1000}(1324)$ is the one-thousandth term in a certain sequence $a_n(1324)$. God may or may not be able to compute the thousandth term in this sequence, but how far can mortals get? If we can't get beyond the 40th or 50th term, can we at least approximate the one-thousandth term? How fast does $a_n(1324)$ grow, anyway? And what does $a_n(1324)$ even mean?

The answers to these questions involve fast computers, fascinating mathematics, and remarkable human ingenuity. But their stories, which are ongoing, also reflect important undercurrents and developments that have influenced all of mathematics, but especially combinatorics, over the past two generations and more.

Knuth's Railroad Problems

The story of $a_{1000}(1324)$ begins with a gap in the railroad literature, which Donald Knuth began to fill in 1968 in the first edition of the first volume of his masterpiece, *The Art of Computer Programming.* In the second section of chapter 2, Knuth included several exercises exploring a problem involving sequences of railcars one can obtain using a turnaround. One of Knuth's exercises is equivalent to the following problem.

> At dawn we have n railroad cars positioned on the right side of the track in Figure 1, numbered 1 through n from right to left. During the day we gradually move the cars to the left side of the track, by moving each car into and back out of the turnaround area. There can be any number of cars in the turnaround area at a time, and at the end of the day the cars on the left side of the track can be in many different orders. Each possible order determines a permutation of the numbers 1, 2, . . . , n. Show that a permutation $\pi_1; \ldots; \pi_n$ (this time reading from left to right along the tracks) is attainable in this way if and only if there are no indices $i < j < k$ such that $\pi_i < \pi_k < \pi_j$.

The solution to this problem is a fun exercise in careful bookkeeping. If such a subsequence exists, then

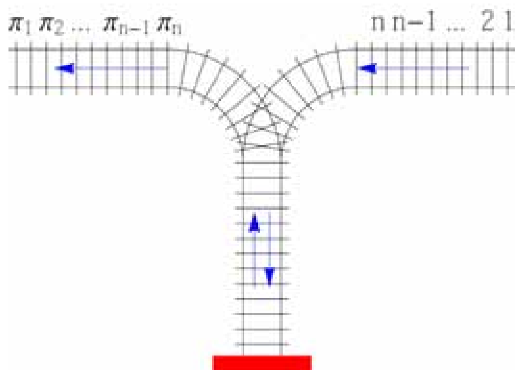


Figure 1. Knuth's railroad tracks.

consider the situation when car π enters the turnaround. Since π_i is the smallest of our three car numbers, cars π_j and π_k have already entered the turnaround, in that order. Furthermore, in order for car π_i to appear to the left of cars π_j and π_k, cars π_j and π_k must both still be in the turnaround when car π_i enters. But now cars π_j and π_k will leave the turnaround in the wrong order.

Conversely, suppose we have a target permutation $\pi_1, \ldots, \pi_n$ with no subsequence of the forbidden type. We can always move π_1 into position, and when car π_1 leaves the turnaround, the cars in the turnaround are, from bottom to top, n; $n - 1, \ldots, \pi_1 + 1$. Now notice that π_2 cannot be larger than $\pi_1 + 1$, since this would mean π_1, π_2, and $\pi_1 + 1$ form a forbidden subsequence. So if π_2 is in the turnaround, then it is the top car there. Either way, we can move car π_2 into position. In general, if we have just moved car π_j into position, and b is the smallest entry greater than π_j which has not yet left the turnaround, then $\pi_{j+1} \leq b$, since otherwise π would have a forbidden subsequence π_j, π_{j+1}, b. Therefore, if π_{j+1} has entered the turnaround then it is the top car there, and we can move it into place.

Knuth was interested in this railcars problem because it models the data structure commonly called a stack, which arises in numerous programming problems, so he introduced no particular notation for the permutations he obtained. Indeed, no general notation for these permutations appeared in print until 1985, when Simion and Schmidt published the first systematic study of permutations with forbidden subsequences of the type Knuth uses.

Today, if π and σ are permutations of lengths n and k respectively, then we say a subsequence of π of length k has *type* σ whenever its entries are in the same relative order as the entries of σ.

For example, the subsequence 829 of the permutation 718324695 has type 213, since its smallest entry is in the middle, its largest entry is last, and its middle entry is first. In this context, we say π avoids σ, or π is σ-avoiding, whenever π has no subsequence of type σ, and we write $S_n(\sigma)$ to denote the set of all permutations of length n which avoid σ. We might also say that σ is a *forbidden subsequence* or a *forbidden pattern*. With this terminology, the permutations Knuth obtains with his railcars are exactly the 132-avoiding permutations, and the term $a_{1000}(1324)$ that Zeilberger's God finds so perplexing is none other than the size of $S_{1000}(1324)$. In table 1 we have the first 10 values of $|S_n(1324)|$.

The rest of this essay is in A Century of Advancing Mathematics, published by MAA Press.

Eric S. Egge is a member of the Department of Mathematics and Statistics, Carleton College, Northfield, Minnesota; eegge@carleton.edu.

n	0	1	2	3	4	5	6	7	8	9
$\lvert S_n(1324)\rvert$	1	1	2	6	23	103	513	2762	15793	94776

Table 1: The first 10 values of $|S_n(1324)|$.

FOUND MATH
Columnar basalt found by Adriana Salerno while on a hike through Maui, Hawaii. The formations are created by the quick cooling of basaltic lava flow. The column cross sections are often hexagonal, but may also have five sides.

See math in the world around you? Send your photos to *Editor@maa.org*.

Interdisciplinary Class: A Welcoming Environment

By Gwen Spencer

Despite a number of spectacular failures in the history of so-called scientific management, fields related to ecology and sustainable management increasingly employ computational and quantitative techniques. Interdisciplinary offerings around sustainability can offer a simultaneous benefit. First, such offerings equip environmental studies/sciences students with a wider range of quantitative skills. Second, such electives engage math and computer science majors with exciting and societally important applications. Offering accessible interdisciplinary electives can provide critical preparation for interdisciplinary graduate study and future jobs in green consulting.

In the winter quarter of 2014, I designed an unusual new elective for the Environmental Studies Department at Dartmouth College. I was at Dartmouth as a Neukom postdoctoral fellow (the Neukom Institute is an interdisciplinary computational institute). The course was called Computational Toolbox for Environmental Sustainability and was listed only through the Environmental Studies (ENVS) Department. It attracted mainly senior ENVS students with little or no college-level math or computer science experience. Developing and teaching the course was a very engaging project for me, and the student feedback was strongly positive. Both the content and the form of the course were novel. From talking with colleagues, particularly at smaller liberal arts colleges, I believe there is a broader interest in how such a course can be developed. I offer some reflections here.

Deciding on Content

I began by wondering whether it was possible to push a taste of research to an earlier place in the course sequence. I wanted to communicate to students that sustainable management of natural systems is a space with many open questions and vigorous debates, and that mathematical and computational tools will give them new ways to participate. But how could we go

EACH UNIT

	In Class	At Home
2 weeks	***Introduction to Mathematical Topic (5 or 6 Lectures)*** • Small application-motivated examples • General formalism and mathematical assumptions • Exploring assumptions • Computational demonstrations to develop intuition	***Short Worksheet*** • Ground basic Matlab skills • 2–4 hours • Individual • Examples that demonstrate key properties
1 week	**Literature Review Days** • Modeled on “reading group” in a scientific lab • 5–6 classic and contemporary research papers • Variety of applications • Student presenters: summary plus discussion prompts • Hybrid critique: mathematical, ecological, policy strengths vs. weaknesses	**Extended Take-Home Lab** • 10–15 hours • Focused on major application • Exploratory • Investigation of assumptions • Students generalize code; collaborate in pairs; interpret computational results; produce a lab write-up; suggest future work
Final Project		• Teams of three • 15–20 hours expected per person • Computational inquiry on a sustainability-related topic • Must consult at least two primary sources • 20-minute presentation
Traditional Final Exam		

from "0" to "research" in one quarter?

There is a vast pool of applications to choose from, but I worried that students would feel lost in a sea of anecdotes. Instead, I opted to emphasize generalizability: to show how a single mathematical concept can be extended to reason about a range of applications. This is a potent, alluring message for students whose prior math experience has focused on memorization. Thus, I organized the course around three mathematical units: Markov models, linear and integer programming, and

Creating opportunities for students to act as explainers is essential.

dynamic programming. Each unit lasted three weeks and connected to several applications. Each three-week unit followed a common model, summarized in the table on the preceding page. We tied up the course with a week of project presentations and a traditional final exam.

Particularly when reading recent research articles (from venues such as *PNAS* and *Science*), the level was significantly beyond our expertise (as is often the case in research settings). I told students that they were not meant to understand every detail in these primary sources. Rather, the point was to use our quantitative foundation to understand scientific context, engage with main technical points, and reason about potential weaknesses. Given our basic knowledge about the technique and its assumptions: Does the application seem responsible? What is missing from the approach that the authors take?

A major ingredient in the success of the literature review days was casting the class as a cooperative and interdisciplinary enterprise. In approaching a complex system, we each arrive with valuable disciplinary expertise. We have different views on even the most foundational questions. For example: What are the primary issues? How they can be understood? What policy interventions may be possible?

A student with coursework in economics (e.g., in the valuation of ecosystem services) will be concerned with very different questions than a student who embraces conservation as an ethical imperative.

Creating opportunities for students to act as explainers is always a good technique to increase buy-in, but in teaching an interdisciplinary course, it is essential. To lower the stakes for this sharing, I started the semester with contrasting "Why to Model" readings from a landscape ecologist and an ecological economist.

I pointed out that in their coursework so far, students have already built complex connotations about words that may appear to be simple English to their classmates. These subtle and crafted vocabularies have allowed students to reason in new ways: our joint purpose in the course is to harness this expertise while also developing a shared quantitative vocabulary that extends what we can learn about complex systems.

In lectures, I emphasized how mathematical concepts allow us to think more precisely about themes the students have already encountered in other courses. For example, a major argument in ecological economics is that natural landscapes provide amenities (aesthetic, recreational, and in providing ecosystem services like water filtration) that are not explicitly valued in the market. Ignoring this value provision can lead to socially inefficient policies.

In an in-class exercise on applying Bellman's equation to compute the optimal rotation age for a forest, students suggested adding a term to describe the aesthetic damage caused by clear-cutting. Making that simple change, we could see how this stretched the optimal periods between clear cuts. Next, we explored how this feature interacted with discounting.

In their homework, students proposed additional terms to reflect benefits from ecosystem services and predicted how the optimal results would change. Finally, students altered the code from class and interpreted their results. Several students reported that implementing and testing their ideas in Matlab felt empowering.

Questioning Theory

The first take-home lab investigated the maximum sustainable yield (MSY) principle. This classical theory was embraced in international fisheries treaties throughout the mid-20th century. Unfortunately, maximum sustainable yield ignores that reproduction rates are often forced by annual environmental fluctuations.

The first part of the lab paralleled the influential mid-'90s critique of MSY that appeared in the *Proceedings of the National Academy of Sciences* after the collapse of the Newfoundland fisheries. Students engaged with Gambler's Ruin on a massive scale: when a stochastic process has absorbing states (corresponding to a species's "minimum viable population"), adding annual expected growth gives a misleading picture of long-term behavior.

For the final group presentation, students opened a computational inquiry about a sustainability-related

Continued on p. 38

Some People Have All the Luck . . . Or Do They?

By Richard Arratia, Skip Garibaldi, Lawrence Mower, and Philip B. Stark

The three mathematicians among us have answered questions about the lottery for reporters before, but the query from Lawrence Mower of the *Palm Beach Post* was different.

He didn't ask whether winning the lottery twice was less likely than being struck by lightning while being eaten by a shark. Instead, he had a list of big prizes ($600 or more) and the people who had claimed them. Some had received hundreds of big prizes, and this seemed to require too much luck to be legitimate. To convince officials that things were fishy, Mower wanted help figuring out just how lucky the claimants must have been—assuming they were honest gamblers—to win.

The list was of all the prizes given out by the Florida Lottery worth $600 or more, numbering just over 1 million prizes awarded between January 4, 1993, and August 7, 2013, encompassing hundreds of thousands of winning players. It is hard to give an exact number of how many people won more than one prize, because there are not unique identifiers in the data; we just have names (which may be recorded with some variations like with or without a middle initial) and hometowns (people can move from city to city during a 20-year period). The people we investigated all claimed more than 80 prizes.

Even though lottery odds are known, to answer this question required some nontrivial math, in part because there's no way to know how many bets any given gambler made, or even what games he or she bet on. And some combinations of bets are probabilistically dependent. This was a delicious mathematical excursion, using recent pure mathematics. One ingredient, the BKR inequality, was proved in the 1990s; we also relied on convex optimization.

We confirmed Mower's suspicion that most of the claimants were implausibly lucky. The chance of winning as often as they did was essentially zero, even under very generous assumptions. (Our calculations showed that two of the claimants could have been plausibly lucky, innocent—but heavy—gamblers.)

This might be the first time that a theorem launched criminal investigations: Mower's story about our probability calculations prompted Florida police to raid some of the stores involved. The story is online at mypalmbeachpost.com/gaming-the-lottery.

If they didn't win legitimately, what had the claimants been up to? You might imagine they had hacked the game somehow, for instance, figuring out which scratcher tickets are winners without scratching them. But they had claimed prizes in all sorts of different games, not just scratchers.

Ways to Cheat

Mower's investigation revealed many possible scams and confirmed that the suspicious claimants had been up to some of them. Store clerks can dupe legitimate gamblers to get their winning tickets, for instance, by telling a gambler that a high-value ticket is a loser (or a lower-dollar winner) and then collecting the high-value prize.

Some gamblers don't want to collect high-value prizes because they owe back taxes or child support that would be subtracted from the winnings, so they pay someone else, called an aggregator, to turn in their ticket for them. That's a crime in some states, but profitable for the aggregator.

Some aggregators with other criminal enterprises use the lottery to launder money: they pay the gambler with

PALM BEACH POST

A look into lottery mathematics found illegal doings.

"dirty" cash and get paid by the lottery with a "clean" check, making a profit in the process. This is a brilliant double chisel, since laundering money is usually expensive.

The aftermath of our work included arrests and seizures of Lotto terminals from the stores involved. In Florida, to date, 52 stores across the state have lost their ability to sell lottery tickets, and six clerks have been arrested. Investigations and policy changes were made in 10 other states as well. Mathematics has the power to excite, delight, and indict!

The mathematical details involved in these lottery scams can be read in the June *Mathematics Magazine*. To access the journal, go to your MAA member page, click on "My Subscriptions," and select *Mathematics Magazine*.

Skip Garibaldi is associate director of the UCLA Institute for Pure & Applied Mathematics (skip@garibaldibros.com). Richard Arratia is a professor in the USC Department of Mathematics. Lawrence Mower is an investigative reporter, Palm Beach Post. *Philip B. Stark is a professor and chair of the UC Berkeley Department of Statistics.*

MAA Awards

A list of the awards given by the MAA is at *maa.org/awards*. Look online to learn more about awards' history, nomination process, and prizes. The awards include these: Alder Award • Allendoerfer Award • Beckenbach Book Prize • Certificate of Merit • Chauvenet Prize • Dolciani Award • Euler Book Prize • Evans Award • Halmos-Ford Award • Gung and Hu Award for Distinguished Service • Hasse Prize • Haimo Award for Distinguished Teaching • Hedrick Lectures • High School Sliffe Awards • James R. C. Leitzel Lecture • JPBM Communications Award • MAA-NAM David Blackwell Lectures • Meritorious Service • Morgan Prize • Pólya Award • Pólya Lectures • Putnam • Robbins Prize • Selden Award

Interdisciplinary, *from p. 36*

topic. First, they discussed the scientific context and related policy questions. Second, they needed to explain what assumptions they used to formulate a model, what difficulties they encountered, and what they believed the strong and weak elements of their analysis were.

I asked students to emphasize how their initial questions evolved during the project. Each presentation ended with a list of directions they would pursue if the project continued. One student told me that this was the first technical assignment she had ever had where she didn't feel pushed toward a particular "right answer."

The computational lab, literature review, and project were designed to emphasize the iterative nature of scientific progress. After proposing a first approach, we explored the results: Do they have the qualitative properties appropriate to the application? Are there unexpected things happening? Do these strange features stem from the application or from our modeling and implementation choices? How can we refine our original model?

Pointing out that similar refinements occur in the scientific literature seemed to resonate with my students. This provides an important message about how science works (in contrast with the apparent authority of scientific facts) and carves out a legitimate space for the novice.

Before students graduate with the idea that they are not math people, I want them to know that novice-hood is a regular step on the path to understanding. Two of my most mathematically self-conscious students finished the course with top grades.

In talking with colleagues and students, it seems that many smart people never reach the mathematical topics that would interest them because of demanding prerequisites. Drawing on primary sources (from other fields) may seem intimidating, but if expectations are framed appropriately, it can be an invigorating component of an interdisciplinary course.

Finally, in addition to cultivating some practical mathematical and programming skills, students leave a course of this type with substantial practice interrogating model assumptions, the ability to ask productive questions about generalizations, and the confidence to continue building a rich toolbox to grapple with the issues they care about.

Gwen Spencer works in combinatorial optimization, operations research, and theoretical computer science. She has just joined the Math and Statistics Department at Smith College.


FASTER FORWARD TO WEB EXPERIENCES THAT KNOWS NO BOUNDS.
With organizations under more pressure than ever to provide an instantaneous, personalized and secure online experience on any device, we need to re-think not only our content, but how it's delivered.
From content delivery to transaction security and fending off cyber attacks, Akamai provides solutions for the world's most recognizable brands - in categories like entertainment, sports, gaming, finance, retail, software, social media, education and others.
• We helped NBC Sports Digital deliver gold during the 2014 Winter Olympics.
• We enabled RedHat to expand their global reach.
• We've managed Black Friday web traffic for the biggest retailers on the planet.
LET AKAMAI HELP YOU MOVE FASTER FORWARD.
AKAMAI.COM
Media Delivery
Web Performance
Cloud Security
Akamai
FASTER FORWARD

The Mathematical Association of America
1529 Eighteenth Street, NW
Washington, DC 20036

Periodicals Postage Paid at
Washington, DC, and
additional mailing offices