Articulation and Quantitative Literacy: A View from Inside Mathematics

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Various voices concerned with K–16 educational alignment[1] recently have called for greater coherence in U.S. education to make it easier and more efficient for students to pass from one level to the next, especially from school to college. Driven in part by demands for greater accountability, about half the states have created K–16 policy units that have produced curriculum frameworks, a plethora of standards, and high-stakes testing largely aimed at the K–12 sector (Kirst, see pp. 107–120). These K–16 efforts aim at aligning higher education expectations, placement testing, and curricula with K–12 curricula, standards, and testing.

Nonetheless, U.S. colleges and universities continue to operate as 3,000 or so independent contractors that unwittingly wield considerable influence on K–12 education—on parents and students through coveted spots in freshman classes and on curricula through the influence of the academic disciplines. In no part of U.S. education are the problems caused by disunity (or lack of articulation) greater than in mathematics. Only language and writing compete with mathematics for prominence in K–16 curricula, and no other discipline creates as many difficulties for students as mathematics.

A principal cause of the transition problems in U.S. mathematics education is the lack of an intellectually coherent vision of mathematics among professionals responsible for mathematics education. Mathematicians similarly lack a coherent vision. The sometimes heated and often public disagreements about the nature of mathematics and about effective ways to teach it have led to a bewildering variety of curricular and pedagogical approaches.[2] Much of this confusion in curricula and pedagogy occurs near the critical transition from school to college.

As the United States has moved toward universal postsecondary education, mathematics education has become more critical and complex, especially in grades 11–14 and in the transition from school to college. This change has been driven largely by a rapid increase in the need for quantitative skills. Computers have created piles of data and myriad ways of interpreting these data. Almost daily, ordinary citizens confront data and numbers they need to understand for personal decisions, at the same time as they face increasing risk of being duped by those who misinterpret and misuse data. Quantitative Literacy (QL) is the ability to understand and use numbers and data in everyday life. Education for QL falls on all disciplines in K–16 but most heavily on mathematics and statistics, which are no longer tools only for scientists and engineers; everyday living requires that everyone have them.

This requirement poses daunting new challenges to mathematics[3] education —both K–12 and higher education. Most mathematics curricula, especially in higher education, are not designed to meet this requirement. Throughout high school and college, a single sequence of courses—geometry, algebra, trigonometry, and calculus (GATC)—dominates the mathematics curriculum. For several decades, success in mathematics has meant staying in this linear and hurried sequence. Those who do not stay in, approximately three of four, leave with disappointment (or worse) and frag-

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mented mathematics skills that are not readily useful in their everyday lives. In effect, the GATC sequence sifts through millions of students to produce thousands of mathematicians, scientists, and engineers. Not surprisingly, this system produces the world's best-educated and most creative scientists and engineers while at the same time yielding a quantitative literacy level that ranks near the bottom among industrialized nations (OECD 2001).

As the goal of the GATC sequence, calculus serves as a surrogate for a powerful force that controls much of school and college mathematics—the need to produce mathematicians, engineers, and scientists. Underwritten by large enrollments of science and engineering students, calculus has become the gateway to advanced mathematics. Its influence, conveyed mainly through the GATC sequence, reaches far down to middle school. Largely because of the Advanced Placement (AP) Calculus Program,[4] calculus has become the capstone of high school mathematics. As such, it is now a proxy for the American yearning for badges of excellence. As a consequence, calculus also has become a lightning rod for criticism of the lack of emphasis on general education in K–16 mathematics. Clearly, what is needed is a K–14 curriculum that prepares students both for advanced mathematics study and for using mathematics in the myriad ways that it now presents itself in everyday life. To achieve that goal, major changes are needed in mathematics curricula.

Changes also are needed well beyond the mathematics curriculum. The experiences of GATC dropouts in other disciplines—the sciences, social sciences, and humanities—likely will not repair the holes in their quantitative abilities; indeed, the data analyses and quantitative measures studied in various disciplines are isolated from one another by different terminologies and contexts. Discipline-dominated college curricula offer little synergism with quantitative education.

Many of the problems with mathematics and quantitative education are problems of articulation, mismatches that place unnecessary bumps in students' paths as they navigate through school and college. Some articulation issues are vertical issues—those associated with the fit of various components as students move from grade to grade; others are horizontal—those associated with interactions between components at approximately the same grade level. In addition, there are issues of environmental articulation between the curricula in school and college and the world external to the academy. Are our curricula up to date? Are students learning what they need to know to be successful in the outside world? Do curricula meet the needs of society?

Historically, vertical articulation has been given more attention because it involves moving from one major component of U.S. education (school) to another (college). Consequently, this paper is dominated by issues of vertical articulation. Nonetheless, horizontal and environmental articulation likely are more important levers in improving U.S. education, especially in quantitative literacy.

Forces that Shape Introductory College Mathematics

The cultures of the three components of grades 11 to 14 mathematics (high school, two-year colleges, and four-year colleges and universities) differ greatly. In spite of these differences, in mathematics the four-year sector wields considerable influence over the other two. In turn, the values of mathematics graduate programs, dominated by research, are imprinted on faculty throughout college mathematics. Consequently, the culture of research mathematics has considerable influence on college and university mathematics, even down to the introductory level.

The Culture of Mathematics

Mathematics research is the principal activity of what Paul Halmos called the "mathematics fraternity," which he described as a "self-perpetuating priesthood." "Mistakes are forgiven and so is obscure exposition—the indispensable requisite is mathematical insight" (Halmos 1968, 381). Prestige in mathematics is gained through manifestations of mathematical insight—developing new mathematics—and those who have prestige wield the greater power over academic mathematics.

Mathematics research is a demanding taskmaster requiring dedication, concentration, even obsession. Although most mathematics research does not aim at immediate applications, the history of unanticipated uses of mathematics provides strong support for its value to society. Consequently, educating mathematicians and creating new mathematics often dominate educating people to use mathematics.

Mathematicians see great value and power in abstract mathematical structures and seek students who can master advanced mathematics. This strongly influences views of the goals of mathematics courses and curricula, and those views are reflected in school and college mathematics. Anthony Carnevale and Donna Desrochers argue that the implicit trajectory and purpose of all disciplines is "to reproduce the college professoriate at the top of each disciplinary hierarchy" (Carnevale and Desrochers, see p. 28). Mathematics, as they go on to analyze, is no exception. Lynn Arthur Steen has compared mathematics teachers' concentrated attention on the best students to hypothetical physicians who attend primarily to their healthiest patients (Steen 2002).

The efficiency of the path to calculus and advanced mathematics has led to rigid linearity of the GATC sequence. No other discipline, save perhaps foreign language, exhibits such linearity. Foreign language education is built on using the language, however, whereas students' use of mathematics is usually far in the future. Most students in the GATC sequence never get to any authentic uses for what they learn.

Fortunately, there are some signs that the mathematics fraternity is turning its attention and vast talents to issues other than its own reproduction and expansion. Among the most recent signs are three publications: *Towards Excellence: Leading a Mathematics Department in the 21st Century* from the American Mathematical Society[5] (Ewing 1999); *Adding It Up: Helping Children Learn Mathematics* from the National Research Council[6] (Kilpatrick et al. 2001); and *Mathematical Education of Teachers* from the Conference Board of the Mathematical Sciences[7] (CBMS 2001).

The First Two Years of College Mathematics

The current CBMS survey[8] reported nearly three million U.S. postsecondary mathematics enrollments in fall semester 2000. Nearly three-fourths of these were either remedial[9] (982,000) or introductory (1,123,000) enrollments. In contrast, calculus-level enrollments totaled 700,000 and advanced mathematics enrollments only 100,000. Comparable data have been reported every five years since 1980. They document that almost three-quarters of all students in college mathematics courses never take a calculus-level mathematics course and that only about 1 in 30 enrolls in a course beyond the calculus level (Lutzer et al. 2002).

Over half of the three million undergraduate mathematics enrollments are in algebra or combinations of algebra and arithmetic, trigonometry, or analytic geometry. Algebra enrollments dominate because college algebra is a prerequisite not only for calculus but also for most general education mathematics courses, casting college algebra as a general education course.

Some states, Arkansas and Mississippi for examples, have made college algebra part of state higher education policy. In Arkansas, legislation requires that mathematics courses taken for college degree credit be at least at the level of college algebra, a testament to the perceived linearity of school and college mathematics offerings. For that reason, some courses, such as mathematics for liberal arts students, were dropped from college curricula because they were judged not up to the level of college algebra. The principal criterion for judging the level of a mathematics course became the level of the mathematics taught in the course rather than the sophistication of the applications of the mathematics. That approach, of course, makes it difficult for courses aimed at the use of mathematics to measure up as college courses.

The institutionalization of college algebra as a core general education course is fraught with misconceptions. Making college algebra a requirement for some majors—e.g., for prospective elementary teachers—is even more misguided. The traditional college algebra course is filled with techniques, leaving little time for contextual problems. Students, many of whom have seen this material in prior algebra courses, struggle to master the techniques; three of four never use these skills and many of the rest find that they have forgotten the techniques by the time they are needed in later courses. No wonder the course is uninspiring and ineffective. Success rates are very low—often below 50 percent—and student dissatisfaction is high. Fortunately, many faculty and administrators realize this and reform efforts are growing. The task is nonetheless monumental.

College Influences on High School Mathematics

Multiple and complex forces shape high school mathematics. Some of these forces are matters of policy, some are circumstantial, and some are cultural. Policy forces include state and district standards for curricula and testing. Circumstantial forces include textbooks, teacher preparation, and the influence of higher education. The last is the focus of interest here.

In addition to being the locus of teacher preparation, higher education has strong influence through statements of expectations for entering students, college entrance testing (primarily the SAT and ACT), college placement testing, and national college-oriented programs. The national program with the most impact on school mathematics is the College Board's AP Program. Other national programs include the International Baccalaureate (IB) and the College Board's Pacesetter program. These national programs are discussed below.

College Statements on Expectations in Mathematics

Comprehensive and useful statements from higher education institutions about mathematics expectations for entering students are rare. In spring 2001, with the help of the Education Trust, I requested from a number of states whatever statements concerning mathematics content were available from colleges and universities about expectations for the mathematics knowledge and skills of entering students. I received responses from 11 states, seven of which had such statements. The other four states had processes or policies that addressed the transition from school to college mathematics, but these did not include statements on mathematics learning, content, or skills.

The seven statements of college expectations range from comprehensive documents that look very much like a set of complete standards for grades 9–12 mathematics to explanations of skills (mostly algebraic) needed to survive in entry-level courses. California's expectations are of the first type, Maryland's and Nebraska's of the second. The latter are focused on specific entry-level courses, for example, what students should be able to do if they begin with college algebra. Because the most likely entry-level courses are intermediate algebra, college algebra, or algebra and trigonometry, these statements necessarily are heavy on algebra-based skills.

College mathematics faculty are the natural source of statements on expectations for entering students. Very often, first attempts of this kind aim far too high; college mathematicians are inclined to describe the student they would prefer to teach rather than the student that is possible and practical to find within the education system. Very often, too, statements generated by mathematics faculty are not consistent with other institutional statements about expectations or requirements. For example, many colleges use ACT scores as a criterion for entry and sometimes for placement. ACT publishes a list of mathematics competencies that various levels of ACT Mathematics test scores indicate. In one state, the mathematical competencies described by a committee of college faculty as expected of all entering students contained competencies and knowledge that were not included on the ACT list until the mathematics score far surpassed the ACT score level chosen by that state as an indication of readiness for college mathematics. Obviously, inconsistencies of this kind confuse schools, teachers, and students.

The National Council of Teachers of Mathematics (NCTM)[10] Standards (NCTM 1989, 2000) have had considerable influence on school mathematics, even though they (or localized state versions) have been implemented in different schools in different ways. These various statements describe expectations for K–12 mathematics far better than any comparable statement for college mathematics. Reliable statements of college expectations would have great influence on school mathematics, and many in school mathematics would welcome such statements. There are pockets of efforts[11] to generate statements of college expectations and to align those with school standards and transitional testing, but as of now there are almost no models that have wide acceptance.

College Placement Testing

In recent years, college placement testing has come under increasing scrutiny as an issue in the transition from school to college mathematics. The CBMS 2000 survey reported that almost all two-year colleges (98 percent) required mathematics placement tests of first-time students. The same survey found that 70 percent of four-year colleges and universities offered placement tests and that the tests were required of first-time students by 49 percent of these institutions. Most such tests are locally written by the user departments, but some come from the Educational Testing Service (ETS), ACT, the Mathematical Association of America (MAA),[12] and other external vendors.

Critics of college placement tests argue that these tests do not measure a student's learning in high school and are too focused on algebraic skills. An analysis reported by the Education Trust showed that some nationally available placement tests do indeed focus on algebraic skills (Education Trust 1999). Further, critics point to cases in which students are not allowed to use calculators on placement tests after having used them in school. Those who defend placement tests point to the purpose of the tests: to place students in a college mathematics course that they are prepared for. The CBMS 2000 survey reported that over 85 percent of the colleges that offered placement tests periodically assessed the effectiveness of these tests. Nonetheless, some criticism is more fundamental, based on doubt that isolated examinations of isolated skills can ever be a reliable indicator of student success. Testing experts universally advise against making important judgments based only on single test scores.

Placement testing has become more controversial with widespread use of technology and the consequent potential de-emphasis on algebraic manipulation skills. Notwithstanding considerable disagreement over what manipulation skills students should possess, faculty in individual departments often decide what skills their students need to succeed in their entry-level courses. These skills then are tested on placement examinations. It turns out that, surprisingly, many students who have done well in school mathematics are weak on such skills. Add to this the timing of many placement tests (often at summer orientations), the absence of the calculator the student is accustomed to using, and the lack of any pre-test review by students, and the results may very well be both questionable and disquieting. (Of course, placement tests and placement testing conditions—e.g., with or without technology—are likely to reflect entry-level courses and teaching conditions. If so, criticisms of placement tests and testing conditions are actually criticisms of college mathematics curricula and pedagogy.)

Many colleges and universities have no systematic way of communicating their expectations about the mathematics that entering students should know and be able to do. Consequently, the content of placement tests, although narrowly aimed at basic skills for initial success in entry-level courses, takes on a broader meaning. There are, however, partial solutions to these problems. Colleges should explain clearly the purpose of placement tests, describe what material will be tested and under what conditions, and encourage students to review the material before sitting for the tests.

Teacher Preparation: From College to School

During various periods in the past, college and university mathematics faculty have played significant roles in supporting school mathematics. During the 1960s, research mathematicians were involved in developing new school curricula and in conducting workshops for in-service teachers. Shortly after, in the wake of problems with the "new math," mathematicians largely withdrew from school mathematics and the preparation of teachers. Discussions following the introduction of standards for school mathematics in 1989 by NCTM caused many mathematicians to re-engage with school mathematics. Throughout the 1990s, this re-engagement took various forms, including some rather contentious debates about fundamental approaches to mathematics education. The 2001 CBMS report on the mathematical education of teachers seemed to signal that the re-engagement is real and constructive (CBMS 2001). Further, the MAA has planned a multifaceted, multiyear effort, Preparing Mathematicians to Educate Teachers, to help implement the recommendations of the CBMS report.

For many years, stronger teacher preparation has been the headline recommendation from several national reports on how to improve mathematics and science education (National Commission 2000). If articulation issues are to be solved, and if QL education is to be improved, teachers—elementary, middle, and secondary—will need extensive training in teaching mathematics and statistics in context. Going one step beyond that, college faculty who teach these future teachers, which means most college faculty, also will need preparation for teaching in context.

From School to College: Mismatches and Overlaps

There are two very different views on the vertical articulation between school and college mathematics. One view reveals mismatches in both curricula and pedagogy. The other reveals that the content of school mathematics and college mathematics is largely the same. From this latter perspective, the articulation problem is one of repetition and ineffectiveness, not mismatches.

The Mismatches Between School and College

The NCTM standards (NCTM 2000) offer a widely accepted blueprint for both curriculum and pedagogy for school mathematics. Most state standards are generally consistent with the NCTM standards, which encourage the use of technology, promote highly interactive classrooms, and outline a reasonably broad curriculum aimed at conceptual understanding. On the other hand, college mathematics is not governed by written standards and, very often, teaching methods are determined by individual instructors. The American Mathematical Association of Two-Year Colleges (AMATYC),[13] *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* (AMATYC 1995) offers one set of college guidelines for mathematics before calculus, but its effect has been muted by the influence that four-year colleges have on two-year institutions, partly driven by the need for transferability of credit. Every 10 years since 1960, the MAA's Committee on the Undergraduate Program in Mathematics (CUPM) has issued guidelines for the mathematics major, but only in the revision now being drafted are guidelines offered for undergraduate mathematics outside the major courses, now about 95 percent of all enrollments.

Many college mathematics faculty disagree with parts of the NCTM standards, especially those that are characteristic of recent reform projects. For example, many do not think that technology—calculators or computers—helps in teaching introductory college mathematics. Further, many college faculty adhere to traditional lecture and testing methods and place heavy responsibility on students for their learning. This has increased variety in college mathematics, more so in pedagogy and the tools used in learning than in course content.

Introductory course content is pretty standard. Students moving from school to college are likely to find the content of courses familiar, although the material may be presented in a different way and at a faster pace; technology may be used, tolerated, or banned; and students likely will be left more on their own to learn and demonstrate that learning on traditional tests. The mismatch in the articulated curriculum between school and college consists primarily of a narrowing of broader school mathematics to a limited set of introductory college courses dominated by algebra and pre-calculus. The narrowing is most notable in the absence of geometry, data analysis, and probability in mainline introductory college mathematics.

The Dilemma of School Mathematics Tracks

The strongest cultural force shaping school mathematics is the widespread tradition of tracking, which is especially prevalent in high schools. There often is no easy way to move from the "lower" track to the college preparatory track. Unfortunately, especially for QL education in which applications of everyday mathematics can be quite challenging, the use of the adjective "consumer" or "general" as a code for second-rate courses has done general education a grave disservice.

There are some glimmers of hope. Elimination of the "general mathematics" track was one of the major goals of the 1989 NCTM standards (NCTM 1989). Data from the 1999 State Indicators of Science and Mathematics Education showed that the proportion of U.S. high school students enrolled in general or consumer mathematics dropped from about 20 percent in 1990 to about 8 percent in 1998 (CCSSO 2002). The 2000 version of the NCTM standards reinforced the 1989 standards by prescribing a "common foundation of mathematics to be learned by all students" (NCTM 2000).

About 20 years ago when I was chair of the department of mathematical sciences at the University of Arkansas, I was struggling with ways to reduce enrollments in intermediate algebra, the one remedial mathematics course we taught. The state was pressuring us to reduce remedial enrollments, but my most pressing reason was to reduce the range of courses we had to cover. We were the only doctoral and research institution in the state and our resources were stretched very thin, covering responsibilities from high school algebra to postgraduate seminars.

My local school system, which had one high school (from which both my son and daughter later graduated), was revising its mathematics offerings and I was invited to meet with the superintendent and associate superintendent to give them advice. I took the opportunity to talk about how they could help reduce our remedial enrollments.

Typically, they were offering two tracks of mathematics. One was a college preparatory track with the usual courses—geometry, Algebra I and II, trigonometry, and AP Calculus—actually a very strong offering. The second track was general or business mathematics, I don't remember the exact terminology. I asked why they offered this clearly weaker track and why they didn't keep all the students on the track that would prepare them for college-level mathematics, since at the time, any student who graduated from high school could enroll at the University of Arkansas. Because we did not require that they had followed a college preparatory track, students from this weaker track would almost surely land in remedial algebra. My superintendent and his associate were very frank: they were not going to take the heat for students failing. I noted that they were passing that heat on to us at the university and they did not disagree.

Unfortunately for QL education, the college preparatory track has preparation for calculus as its goal and does not include significant contextual uses of mathematics. Measurement, geometry, data analysis, and probability—all parts of most school mathematics curricula—have strong QL themes, but with calculus as the goal these get shortchanged. By attempting to articulate well with colleges, schools narrow the coverage of mathematics to what is needed to succeed in calculus. The majority of high school students who never make it through a calculus course—about three of four—never reap the benefits of this narrowed mathematics curriculum.

On the other hand, students who are in a noncollege preparatory mathematics track are often shortchanged by the lower level of the courses and find themselves unprepared for college mathematics. When they arrive at college, as many do, they are likely to enter the wasteland of remedial courses.

Overlaps: Too Much of the Same Thing

A second way of looking at school and college mathematics shows that there is enormous overlap, especially in the content of college courses with large enrollments. As indicated above, the CBMS 2000 survey showed that approximately 60 percent of the mathematics enrollments in four-year colleges and 80 percent of those in two-year colleges were in courses whose content is taught in high school. (Although calculus is taught in high schools, it is not included in these calculations. If it were, the 80 percent would rise to 87 percent and the 60 percent to 77 percent.) On the other hand, the fastest-growing enrollment in high school mathematics is in courses for college credit. Though seemingly antithetical, these two phenomena are related, and aspects of this overlap are seriously impeding students' learning of mathematics.

The GATC topics covered in high school geometry, algebra, and trigonometry align quite well with the corresponding college sequence, especially elementary, intermediate, and college algebra. In one sense, there is too much agreement, because many if not most students repeat much mathematics in moving from high school to college. This repetition is not only inefficient, it is discouraging to many students, and learning suffers. Other students mistakenly welcome the repetition, thinking it will lead to an easy A. As described earlier, much of the repeated material is devoted to algebraic and trigonometric methods, with little time for applications because the students are already deemed to be behind schedule. Because most of this material is preparation for later study that eludes most students, such courses are often dull and depressing for both teachers and students.

Remedial mathematics in college—accounting for one of three enrollments—is often the most depressing of all. Remedial mathematics is almost always arithmetic or high school algebra. Consequently, except for returning students who have been away from school for some time, students in remedial courses are repeating material they failed to learn in earlier, possibly multiple, efforts. Having to repeat work, not making progress toward a degree, and studying uninspiring—and to students, illogical—subject matter makes remedial mathematics courses unusually dreary. The subject matter of these courses is the kind of content—much of it algebraic methods—that appears to be best learned with attentive practice the first time through. Misunderstanding and bad habits are hard to undo. Consequently, the proportion of students who are unsuccessful in remedial mathematics courses is often high, in the range of one-half to two-thirds.

In the mid-1970s, I was named director of the mathematics component of the Academic Skills Enhancement Program (ASEP) at Louisiana State University. The goal of the program was to increase the success rate of students in remedial mathematics. We instituted a moderately complex system of four courses, each a half-semester long, whereby students would progress to the next course or start over based on the results of the previous course. I taught several of these classes, including one section of the first course in which all the students had failed to progress on their first try.

Never have I had a more challenging assignment. I was helping college-age (and older) students to succeed in ninth-grade mathematics after they had all failed to do so in the previous eight weeks. It was there that I learned the many different reasons why students have trouble with elementary algebra. I also learned why remedial algebra in universities faces almost insurmountable obstacles given the levels of success expected in most academic enterprises. Perhaps 30 years later, with the use of technology, the obstacles can be overcome.

Dual Credit Courses

The enormous overlap between college and high school mathematics has fueled the recent growth of dual credit[14] courses in high schools. The expansion is typified by this common scenario: A two-year college enters into an agreement with a high school to give college credit to high school students for specific courses taken in the high school that also count for high school credit (whence the term "dual" credit). Agreements of this type have been made for college credit in most disciplines. In mathematics, dual credit is being awarded in courses from beginning algebra up through calculus. These agreements are generating considerable college credit in courses taught in high schools by high school teachers, and most dual credit programs have nothing similar to the AP Examinations to validate their quality. The CBMS 2000 survey reported that 15 percent of all sections of college algebra (or algebra and trigonometry) taught in two-year colleges in fall 2000 were for dual credit.

A recent national survey estimated that one-half of all juniors and seniors in U.S. high schools (approximately 3.5 million) are enrolled in courses that carry credit for both high school graduation and college degrees (Clark 2001). Some of these courses are in the examination-based programs of AP and IB in which college credit depends on scores on national or international examinations and not merely on high school grades. According to the data in this report,[15] however, most dual credit enrollees (57 percent) are in courses that, unlike AP and IB, have no uniform examination. Because the recent growth of dual credit has been so large, there are no good data on how students with this credit fare in college, but if nothing significant has changed except the awarding of college credit, the knowledge gained by many of these students will be insufficient for success in subsequent college courses. Will they then re-enroll in courses for which they already have credit? Standards for this practice are urgently needed, lest we push the line between college and school mathematics—if there still is to be one—well below what it should be.

The largest examination-based dual credit program is AP,[16] a 50-year-old program of the College Board aimed at providing opportunities for advanced study in high school with the possibility of receiving credit or advanced placement in college. AP has been growing by about 10 percent per year for the past 20 years and now offers 34 courses and examinations. Approximately 1.5 million AP Examinations will be given in 2002 to over a million high school students, mostly juniors and seniors. About 200,000 of these will be in AP Calculus and about 50,000 in AP Statistics.

AP Calculus has become the goal of ambitious mathematics students because it is a hallmark of high school success. To enroll in AP Calculus by grade 12, students must take Algebra I by grade 8. The lure of AP Calculus has accelerated the high school mathematics sequence and consequently reduced the time for teaching mathematics in context. Although contextual teaching was one of the goals of calculus reform, and the AP Calculus Course Description issued in 1998 represented a consensus on a reformed calculus course, AP Calculus is still short on the kinds of contextual problems needed to develop QL.

The AP science and social science courses do offer contextual problems, but like the college disciplines they emulate, these AP Course Descriptions and examinations are developed independently with no special efforts toward synergism in learning. Because AP courses constitute a large portion of college general education core requirements for many students, AP courses need to contribute significantly to crosscutting competencies such as QL. This will clearly require closer coordination among the various AP courses.

Notwithstanding its public prominence, AP Calculus represents only a fraction of high school calculus courses. Enrollments in all kinds of high school calculus courses are approximately 600,000 each year, roughly half of which are in courses called AP Calculus, but only about 200,000 students take an AP Calculus examination and about two-thirds of these qualify for college credit. That leaves about 450,000 students with a calculus course that likely will be repeated in college. Contrary to what we might expect, a high school calculus experience that does not result in college credit or advanced placement is likely to cause the student difficulty in college mathematics.

This problem of calculus articulation was addressed years ago by a CUPM Panel on Calculus Articulation consisting of four high school teachers and three college teachers (CUPM 1987). Their report concluded that a successful high school calculus course requires a qualified teacher with high but realistic expectations, a full year of study based on something equivalent to the AP Course Description, and students who are willing and able to learn. The report described two models of high school calculus courses that are unsuccessful: one is a partial year "highlights of calculus" course and the second is a year long, watered-down version that does not deal with the concepts of calculus in any depth. One of the panelists was quoted as describing the effects in college of the highlights course as "like showing a 10-minute highlights film of a baseball game, including the final score, and then forcing the viewer to watch the entire game from the beginning—with a quiz after every inning." Reports such as this one provided background for a joint statement from the MAA and NCTM in 1986[17] recommending, in part, "that all students taking calculus in secondary school who are performing satisfactorily in the course should expect to place out of the comparable college calculus course." A 2002 National Research Council study reached a similar conclusion, recommending that "all calculus taught in high school should be at the college level" (Gollub et al. 2002, 537).

The huge overlap between school and college mathematics complicates the school-to-college transition, partly because the line between the two systems is so blurred. There is nothing inherently wrong with students learning calculus in high school or learning algebra in college. There is something very wrong with students repeating the same material, whether it is arithmetic, algebra, or calculus. Repeating and failing are the culprits in this overlap. Schools and colleges must concentrate more effort on students' learning, success, and progress. There is little value in weak courses that do not lead to progress. Moreover, repeating courses when previous experience has failed is often a barrier to success.

College Mathematics as a Filter

One of the headlines of the calculus reform movements was the phrase "a pump, not a filter," expressed by National Academy of Engineering President Robert White in his opening remarks to the Calculus for a New Century Colloquium in 1987 (White 1988). Unfortunately, college mathematics still is widely used as a filter.

There are two different reasons for using mathematics as a filter. Some disciplines require particular analytical and critical thinking skills that are best learned in mathematics courses. In such cases, mathematics courses are legitimate prerequisites and necessarily serve as filters. In many cases, however, mathematics is used as a filter only because the courses are difficult and only the best prepared and most dedicated survive. This type of filtering misuses mathematics and abuses students.

> When I was a new chair of the department of mathematical sciences at the University of Arkansas, I was introduced at a social event to the dean of the college of business administration. As we chatted, I mentioned the recent increase in the mathematics requirements for business majors to two courses—one in finite mathematics and one in polynomial calculus. I said that I hoped the students would do well and that we didn't want these courses to reduce his college's enrollment. He immediately said that reduction of enrollment, that is, filtering out students, was a major purpose of the requirement. So, like my school superintendent, the business dean was passing the heat on to me.
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> Later, as dean of my college for 10 years, I learned a lot more about the role of mathematics courses as filters. I heard about it from faculty and administrators in architecture, business, engineering, agriculture, and education, and from my own faculty, including premedical advisers, science faculty, humanities faculty, and fine arts faculty. Some were for filtering and some were against it, but all recognized it as a key role played by mathematics.

Unfortunately, many mathematics faculty accept the long tradition of their discipline as a filter and expect a large number of students to fail. This expectation casts a pall that hangs over many mathematics classrooms, causes additional students to fail, and increases resentment toward mathematics.

Statistics Articulation

In most colleges, statistics courses are spread across several departments including statistics, mathematics, engineering, social sciences, agriculture, and business. By and large, college statistics is taught to support majors in other disciplines, often by faculty whose appointments are in the disciplines served. Statistics has been viewed as a research method in agriculture and the social sciences—consistent with Richard Scheaffer's characterization of statistics as "keeper of the scientific method" (Scheaffer et al., see p. 145). In many institutions, there is little interaction or synergy among the statistics courses taught in various disciplines. Partly because of this dispersion, college statistics departments have never had sufficient enrollments to justify large departmental faculties. Measured by degree programs, statistics is largely a graduate discipline.

But now statistics is also a high school discipline. The AP Statistics course, first offered in 1997, has grown remarkably fast, with about 50,000 examinations in 2002. Ten years ago, when the College Board's AP Mathematics Development Committee was first asked to make a recommendation about developing AP Statistics, they were stymied because there was no typical first college course in statistics, which was necessary for the standard prototype of an AP course. This lack of a standard first college course was indicative of the dispersion of statistics teaching in higher education. Therefore, in a reverse of the traditional pattern, AP Statistics, which was developed by college and school faculty outside the muddled arena of college statistics, has become a model for a first college course in statistics. This history illustrates the degree of difficulty in changing college curricula without outside impetus.

The position of AP Statistics in school and college curricula differs from AP Calculus in that the former does not sit in an established sequence of prerequisite and succeeding courses. This freedom promotes access to AP Statistics and does not affect students' course choices nearly as dramatically as does AP Calculus. AP Statistics has been well served by the introduction of a strand on data analysis and probability in the K–12 mathematics standards, which has increased the visibility of statistics to students and teachers.

One explanation for the weakness of quantitative literacy at the college level is that many undergraduate degrees do not require statistics. Even when there are program requirements, they are aimed at using statistics as a research method (in the social sciences and agriculture) or are very bound up with the jargon and practices of professional education (in business and engineering). Rarely are statistics courses required for general education, where their goal would be to help students use statistics to make decisions concerning public issues or personal welfare.

A little bit of elementary statistics—perhaps a chapter or two—does appear in some introductory mathematics courses. Some appears in courses on finite mathematics that often are required for business majors and sometimes are part of a general education core. There also is some in the mathematics courses for prospective elementary school teachers; it is essential there because of the presence of the data analysis and probability strand in the K–12 mathematics standards. As we have noted, however, college courses change very slowly and college faculty are neither attuned to changes in K–12 curricula nor much inclined to be guided by forces external to their discipline or department.

In 1991, CUPM recommended that every mathematical sciences major should take at least one semester of probability and statistics at a level that requires calculus as a prerequisite (CUPM 1991), but this recommendation by mathematicians for mathematics majors is somewhat inconsistent with statisticians' data-oriented view of statistics. The CUPM report acknowledged that in one course it is difficult to cover an introduction to probability and also convey an understanding of statistics. Consequently, a mathematics graduate is likely to have very little statistics education, and many graduate programs in mathematics do not correct this deficiency. Thus both secondary school mathematics teachers and college mathematics faculty are likely to have weak training in statistics, leaving them unprepared to teach courses in data analysis and probability. These deficits in articulation, along with the virtual absence of statistics in statements of college expectations for mathematics preparation (including the content of placement tests), weaken significantly the emphasis on data analysis and probability in school mathematics. To improve statistics education in the schools, it must be strengthened in colleges and become a more prominent part of general education.

Teaching Across the Curriculum: Synergism in Education

The most important area of horizontal articulation in education is teaching crosscutting competencies in all curricular components. The most notable example of this is "writing across the curriculum," a practice that has been successfully implemented in a number of colleges and universities. Many believe that a similar model will be required for effective QL education.

Teaching QL across a college curriculum will require considerably more coordination among the disciplines than currently exists at most institutions. The independence of disciplines is strong. According to Carnevale and Desroches (see p. 21), "academic specialization that creates virtually impregnable barriers between the discrete disciplinary silos of mathematics, science, and the humanities."

> My experience confirms these barriers. I was a double major in college, in mathematics and physics. I took 12 or 13 mathematics courses and an equal number of physics courses. Mathematics was a part of all the physics courses, and occasionally some physics concept would emerge in a mathematics course. Aside from elementary applications of calculus concepts—mostly the derivative—I rarely recognized any of the mathematics from my mathematics courses in the mathematics I saw in physics courses. They were two parallel worlds, occasionally touching but never merging or synergistically promoting understanding.
>
> In my Ph.D. studies in mathematics I minored in physics, taking 12 hours of graduate work. As in my undergraduate experience, physics and mathematics were still worlds apart. And physics and mathematics should be the easiest subjects to integrate. My years of college teaching tell me that my experience is not unusual; there is very little synergy in teaching mathematics across college disciplines.

Articulation with the Environment

In his paper "Mathematics for Literacy" (see pp. 75-89), Jan de Lange makes several observations about what is needed to gain mathematical or quantitative literacy:

- The mathematics that is taught should be embedded in the real world of the student.

- Mathematical literacy will lead to different curricula in different cultures.

- The content of [mathematics] curricula will have to be modernized at least every five to 10 years.

U.S. mathematics curricula, both in high school and college, fail badly in meeting de Lange's criteria. Although high school and introductory college mathematics do include some so-called real-world problems, these very often are not embedded in the world of any student. Some national needs are cited as reasons for stronger mathematics education, but the duties of citizenship in a democracy—perhaps the most fundamental need of the country—are rarely considered when teaching mathematics. The school curriculum may have been modernized once in the past 50 years, depending on the interpretation of "modernize," and introductory college mathematics currently may be undergoing some reform, but there is no systematic way to modernize college offerings. Every five to 10 years seems beyond the pale.

Beyond a lack of connection to real-world applications, there is an additional mismatch between the mathematics curriculum and available jobs. According to Carnevale and Desrochers, "too many people do not have enough basic mathematical literacy to make a decent living even while many more people take courses such as geometry, algebra, and calculus than will ever actually use the mathematical procedures taught in these courses in high school" (see p. 25).

How Did We Get Here and How Do We Get Out?

The foregoing paints a clear picture of an enormously inefficient and ineffective system of introductory college mathematics. The GATC sequence, driven by the needs of scientists and engineers, controls the system, but the system now serves—or more accurately, disserves—a much larger population. In the interest of efficiency, we have gathered together largely uninspiring algebraic methods and created courses with a singular, dominating goal of preparation for calculus, the gateway to the use and further study of mathematics. Those who do not survive are left on the side of this narrow road with fragmented and often useless methodological skills. The system produces millions of such students every year, at least three of four entering college students.

Two major corrections are needed. First, the rigid linearity of the route to advanced mathematics must be abandoned. Second, college mathematics courses must have independent value and not be only routes to somewhere else.

Similar to mathematics research, learning mathematics at the college level need not be linear. Students can learn mathematical concepts and reasoning through combinatorial mathematics, through data analysis, and through geometry, as well as through calculus. Even fundamental concepts of calculus—rate of change, approximation, accumulation—can be understood outside the infrastructure of calculus methodology. A major impetus for the calculus reform movement was a 1983 conference convened to discuss discrete mathematics as an alternative gateway to college mathematics (Douglas 1986). By developing multiple interconnecting pathways to the advanced study of mathematics, introductory college mathematics can become more appealing and more useful to students. Further, a broader view of college mathematics can support a broader school mathematics curriculum and remove much of the emphasis on a failed system of courses dominated by algebraic methodology.

Because of their easy experience learning mathematics, most mathematicians do not relate well to a student struggling with factoring quadratics or mangling the addition of algebraic fractions. We mathematicians see the larger algebraic architecture and the logic underlying the operations; however, some of us can identify with that bewildered student by reflecting on how we first use a new graphing calculator or software package. Here the architecture and underlying logic of the hardware or software are obscure. So what do we do? We begin to use the calculator or the software package and refer to the manual primarily when needed. No one would first spend days pouring over the manual trying to commit to memory procedures or keystrokes to accomplish thousands of unconnected operations. Many of our students see college algebra and trigonometry in this same illogical light. Every operation is new and independent, making retention of skills until the end of the semester unlikely and until the next year almost impossible.

Just as computer software and calculators are useful to all of us, so is algebra. For education to be effective, these uses of algebra must be given priority over techniques, not only to accomplish tasks that use algebra but also to master algebra. This approach may help break the rigid GATC verticality and can increase access to and success in both mathematics and its applications. And technology can surely help.

Much of the GATC sequence consists of learning skills that can be performed by technology. Unfortunately, mathematicians do not agree on what manual (paper-and-pencil) skills are essential or on how technology helps; some even ban technology. Mathematicians know their own algebraic skills served them well, so when they see students falter because of poor algebraic skills it reinforces the beliefs that help maintain the GATC stranglehold.

Both the NCTM Standards and AMATYC *Crossroads* have fully endorsed using technology in mathematics education. Nonetheless, the mixed attitudes of college and university mathematics faculty toward technology have created a dual system in school mathematics: first teach and test it with technology, then teach and test it without technology. The AP Calculus Examinations display this duality—one part with calculators and one part without. No doubt this model has strengths, but we can no longer afford these strengths; there is too much else to do. We can teach and test mathematical skills and concepts using graphing and computer algebra systems (CAS) technology. Computers are part of the world of our students. It is past time to use them regularly in teaching mathematics.

By focusing introductory college mathematics courses on learning by using, especially learning by using technology, these courses can extend school mathematics at the same time they fill in gaps in learning. We can stop the treadmill of repeated failures in repetitious courses. We can stop telling students that they will need algebra later, perhaps in calculus and its applications. Instead, we can show students why algebra is important and what they need to master. With wise use of technology and learning-by-doing, the GATC sequence in college can be replaced by courses that enhance the use of mathematics in other disciplines, prepare students for the quantitative demands of everyday life, and support the study of advanced mathematics. In this way, introductory college mathematics can become a pump, not a filter.

Notes

1. The Bridge Project, housed at the Stanford Institute for Higher Education Research, has as its aim "to improve opportunities for all students to enter and succeed in postsecondary education by strengthening the compatibility between higher education admissions and placement requirements and K–12 curriculum frameworks, standards, and assessments."

   The Education Trust was created to promote high academic standards for all students at all levels, kindergarten through college. The Education Trust publishes *Thinking K–16,* an occasional newsletter that contains discussions of issues in K–16 education and how they are being addressed by various coalitions. See www.EdTrust.org.

   The American Diploma Project (ADP) is aimed at aligning high school academic standards with higher education and the needs of the new economy. ADP is sponsored by Achieve, Inc., the Education Trust, the Thomas B. Fordham Foundation, and the National Alliance of Business.
2. Personal communication. Attributed to William Schmidt by Alfred Manaster.
3. Because statistics is a part of the mathematics curriculum in K–12, mathematics at this level is often interpreted to include statistics. In this paper, the more inclusive "mathematical sciences" often will be abbreviated to "mathematics."
4. Advanced Placement Calculus is a program of the College Board that provides a course description and national examinations whereby students can earn college credit or advanced placement in college courses while still in high school.
5. The American Mathematical Society (AMS) is a professional society of mathematicians that focuses on issues in research and graduate study in mathematics.
6. The National Research Council (NRC) is the operating arm of the National Academy of Science, the National Academy of Engineering, and the Institute of Medicine.
7. The Conference Board of the Mathematical Sciences (CBMS) is a confederation of presidents of 17 professional organizations in the mathematical sciences.
8. Every five years since 1965, CBMS has surveyed college and university mathematical sciences departments on curricula, enrollments, and instructional practices. The CBMS 2000 survey was conducted in fall 2000.
9. Remedial mathematics often is called developmental mathematics and consists of courses in arithmetic, beginning algebra, and intermediate algebra. "Remedial" often indicates that college degree credit is not awarded.
10. NCTM is the National Council of Teachers of Mathematics, a professional organization that focuses on K–12 mathematics education.
11. The American Diploma Project cited above is one such effort.
12. MAA is the Mathematical Association of America, a professional organization that focuses on undergraduate mathematics. The MAA Placement Test Program, established in 1977, was discontinued in 1999 but some of the tests still are being used.
13. AMATYC is the American Mathematical Association of Two-Year Colleges, a professional association primarily of two-year college faculty.
14. Other terms used to describe these courses are "dual enrollment" and "concurrent enrollment."
15. The report gave the number of students in AP as 1.2 million in 2000; however, this was the number of examinations taken. The number of students was closer to 800,000. The estimate of 300,000 U.S. students in IB also seems too large. Using these better estimates, the percentage of students in courses that do not have national examinations is probably higher than the 57 percent cited.
16. The author has considerable experience with the AP Program, including a term as Chief Faculty Consultant for AP Calculus (1995–1999) and as a member of the Commission on the Future of the Advanced Placement Program (1999–2001).

17. Reprinted as Appendix B of the Statement on Competencies in Mathematics Expected of Entering College Students, endorsed by the Intersegmental Committee of the Academic Senates, California Community Colleges, California State University, and University of California. Sacramento, California, 1997.

References

American Mathematical Association of Two-Year Colleges (AMATYC). 1995. *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges.

Clark, Richard W. 2001. "Dual Credit: A Report of Programs and Policies that Offer High School Students College Credits." Philadelphia, PA: Pew Charitable Trusts.

Committee on the Undergraduate Program in Mathematics (CUPM). 1987. "Report of the Panel on Calculus Articulation: Problems in the Transition from High School Calculus to College Calculus." *American Mathematical Monthly* 94: 776–85.

Committee on the Undergraduate Program in Mathematics (CUPM). 1991. "The Undergraduate Major in the Mathematical Sciences." Reprinted in *Heeding the Call for Change,* edited by Lynn Arthur Steen, 225–47. Washington, DC: Mathematical Association of America, 1992.

Conference Board of the Mathematical Sciences (CBMS). 2001. *Mathematical Education of Teachers*. Providence, RI and Washington, DC: American Mathematical Society and Mathematical Association of America.

Council of Chief State School Officers (CCSSO). 2002. *State Indicators of Science and Mathematics Education 2000*. Washington, DC: Council of Chief State School Officers.

Douglas, Ronald G. 1986. *Toward a Lean and Lively Calculus*. Washington, DC: Mathematical Association of America.

Education Trust. 1999. *Thinking K–16: Ticket to Nowhere*. Washington, DC: Education Trust.

Ewing, John, ed. 1999. *Towards Excellence: Leading a Mathematics Department in the 21st Century*. Providence, RI: American Mathematical Society. http://www.ams.org/towardsexcellence/.

Gollub, Jerry P., Meryl W. Bertenthal, Jay B. Labov, and Philip C. Curtis, eds. 2002. *Learning and Understanding: Improving Advanced Study of Mathematics and Science in U.S. High Schools*. Washington, DC: National Academy Press.

Halmos, Paul. 1968. "Mathematics as a Creative Art." *American Scientist* 56(4): 375–89.

Kilpatrick, Jeremy, Jane Swafford, Bradford Findell, eds. 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Lutzer, David, et al. 2002. *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 2000 CBMS Survey*. Washington, DC: Mathematical Association of America.

National Commission on Mathematics and Science Teaching for the 21st Century (National Commission). 2000. *Before It's Too Late*. Washington, DC: U.S. Department of Education.

National Council of Teachers of Mathematics (NCTM). 1989. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Organization for Economic Cooperation and Development (OECD) Center for Educational Research and Evaluation. 2001. *Education Policy Analysis 2001*. Paris: Organization for Economic Cooperation and Development.

Steen, Lynn Arthur. 2002. "Achieving Mathematical Proficiency for All." *College Board Review.* (Spring)196: 4–11.

White, Robert M. 1988. "Calculus of Reality." In *Calculus for a New Century*, edited by Lynn Arthur Steen, 6–9. Washington, DC: Mathematical Association of America.